Damping Characteristics of Flexural Vibration for Partially Covered Beams with Constrained Viscoelastic layers*

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A beam partially covered with a constrained viscoelastic layer shows superior damping capacity to that of a fully covered beam, depending on the beam geometry and the properties of the viscoelastic layer. This damping effect, the so-called partial cover effect, however, had not been examined in detail until now. In this paper, the strain energy analysis of the three types of partially covered five-layered beam is carried out in order to explain the damping mechanism. The results of analysis for the center-covered beam and the end-constrained beam indicate that the partial cover effect is caused by the shear strain energy dissipation at the outer edges of the viscoelastic layer, and also, such energy dissipation depends on the tensile stiffness of the constraining layer. The separately covered beam has a superior damping capacity to that of the center-covered beam with the same cover ratio, due to the energy dissipation of the inner edges of the viscoelastic layers. These damping characteristics of the three types of beam presented are confirmed by experimental results.

Key Words : Vibration, Damping, Partial Cover Effect, Partially Covered Beams, Loss Factor, Strain Energy Analysis, Viscoelastic Layer

1. Introduction

Laminated steel sheets composed of elastic and viscoelastic layers, especially the sandwich type steel sheets with viscoelastic core well known as the "damping steel sheet", have been utilized as an excellent damping structural material against acoustic noise and mechanical vibration. These damping steel sheets are now used up to 40 percent in the oil pan of small diesel engine trucks⁽¹⁾ in the domestic automobile industries.

Since the earliest studies on the damping characteristics of the two layered beam by Oberst⁽²⁾ and the sandwich beam by Kerwin⁽³⁾, a large number of studies on sandwich beams and plates have been reported^{(4)~(6)}. In these previous studies, the partial cover effect which a partially covered treatment by the constrained viscoelastic layer on the main beam is superior to the fully covered beam in damping capacity, had been indicated by Nokes and Nelson⁽⁷⁾. This

effect was subsequently confirmed by more exact theories and detailed experiments by a few authors for the beam^{(8)~(10)}, plate⁽¹¹⁾ and circular plate⁽¹²⁾.

The damping mechanism for this effect, however, is not explained clearly so far, even though the presumptions⁽⁷⁾⁽⁹⁾ that this effect is attributed to the increase of shear deformations in the outer edges of the viscoelastic layer has been given.

In this paper, a strain energy analysis was performed for the center-covered beam and the end-constrained beam, as shown in Fig. 1(a) and Fig. 1(b), in order to explain the partial cover effect based on the above presumptions.

Furthermore, in the symmetric mode of flexural vibration, a neighboring part of anti-node point of the viscoelastic layer almost does not attribute to strain energy dissipation because of a small shear deformation. If we assume that the partial cover effect depends on the above presumptions, the energy dissipation of the separately covered beam, as shown in Fig. 1(c), seems to increase more than that of the center-covered beam due to the energy dissipation at the inner edges. Then, we also examined the case for the separately covered beam.

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These damping characteristics estimated from the above analytical results are verified by some experimental results.

2. Analysis

The approximate analysis by Nokes and Nelson⁽⁷⁾, in which they assumed that the partially covered beam keeps the normal modes as a homogeneous beam, does not agree well with the experimental results if the bending stiffness of the constraining layer as well as the shear modulus of the viscoelastic layer become greater. However, this analytical procedure is simple, and it seems that the damping characteristics of partially covered beam can be estimated roughly by this approximate analysis.

Then, we applied the approximate analysis to the three types of partially covered beam, as shown in Fig. 1. The center covered beam in Fig. 1(a) is the usual partially covered beam, and the end-constrained beam in Fig. 1(b) is the beam which constrains shear deformations of the viscoelastic layer ends. The separately covered beam in Fig. 1(c) is the beam which is divided the viscoelastic layer only into two parts at its beam center.

The following assumptions are made through the analysis :

- (1) No slippage occurs in the interface between each layer.
- (2) Young's modulus of the viscoelastic layer is small in comparison with that of the constraining layer ; that is, the axial force of the viscoelastic layer can be ignored.
- (3) The transverse displacement in each layer is

Fig. 1 Partially covered five-layered beams

the same, and the flexural vibration keeps the normal mode.

(4) The damping of the beam depends on only the shear strain energy dissipation in the viscoelastic layer.

(5) The elastic and constraining layers are treated as Bernoulli-Euler beam, then the inertia of rotation of the beam is ignored.

2.1 Basic equation

The damping characteristics of the laminated beam with viscoelastic layers depend on factors such as the geometrical beam composition, the material constants, the boundary conditions, and the mode numbers. In order to explain a complicated damping mechanism of the partial cover effect, simpler beam conditions should be provided. We consider thus the case of a simply supported beam and its fundamental mode.

In Fig. 2, the axial force of the elastic layer becomes $F_3=0$ due to the geometrical symmetry of the beam. If we consider assumption (2), the equation of equilibrium for the axial forces per unit beam width in the x -direction is given as

$$2E_c t_c \left(\frac{du_1}{dx} + \frac{du_5}{dx} \right) = 0. \quad (1)$$

Taking the origin of the coordinates at the geometrical center of the beam, the following equation with respect to the axial displacements of the constraining layers, u_1 and u_5 , is derived from Eq.(1) and the geometrical relationship in Fig. 2 :

$$-u_1 = u_5 = \{ t_3 \phi - 2t_v (\psi - \phi) + t_c \phi \}.$$

Then, the shear strain ψ in the viscoelastic layer is as follows,

$$\psi = \frac{1}{4t_v} (u_1 - u_5 + 2h\phi) = -\frac{1}{2t_v} (u_5 - h\phi), \quad (2)$$

where $h = t_3 + 2t_v + t_c$.

Considering Eq.(2) and the equilibrium of the axial force of the upper constraining layer and the shear force of the viscoelastic layer ; $dF_5/dx = -G_v\psi$, the following equation is given :

$$\frac{d^2 u_5}{dx^2} = -\frac{G_v}{4t_v t_c E_c} (u_5 - h\phi). \quad (3)$$

Fig. 2 Deformation of the five-layered beam

Rewriting Eq. (3) by using the dimensionless constant $R = G_v L^2 / (4 t_v t_c E_c)$ and the x co-ordinate $\xi = x/L$, the following basic equation for u_s is obtained :

$$\frac{d^2 u_s}{d\xi^2} - R u_s = -R h \phi. \quad (4)$$

From assumption (3), the following boundary conditions at the fundamental mode are applied to the present beam :

$$w=0, \quad d^2 w / d\xi^2 = 0 \quad (\xi = \pm 1/2) \\ dw/d\xi = 0, \quad d^3 w / d\xi^3 = 0 \quad (\xi = 0).$$

Then, the transverse displacement w and the slope ϕ are easily given as

$$w = w_0 \cos \lambda \xi, \quad \phi = -\frac{\lambda w_0}{L} \sin \lambda \xi. \quad (5)$$

A general solution of Eq. (4) is obtained by substituting ϕ into Eq. (4) as the following :

$$u_s = A_1 \sinh \sqrt{R} \xi + A_2 \cosh \sqrt{R} \xi - a \sin \lambda \xi, \quad (6)$$

where $a = R h \lambda w_0 / L (\lambda^2 + R)$, $\lambda = \pi$, and w_0 is the transverse displacement at the steady state.

In the following, the strain energy components for three types of partially covered beam are determined based on the axial displacement u_s in Eq. (6).

2.2 Center-covered beam

The boundary conditions for the constraining layer in Fig. 1 (a) are given as follows (denoting the cover ratio ; $\theta = l/L$),

$$u_s = 0 (\xi = 0), \quad du_s/d\xi = 0 (\xi = \pm \theta/2),$$

since there is no axial displacement at the anti-node and no normal stress at the layer ends, respectively. Considering the above conditions, u_s and ϕ are determined from Eq. (6) and Eq. (2) as follows :

$$u_s = a \left(\frac{\lambda q}{\sqrt{R}} \sinh \sqrt{R} \xi - \sin \lambda \xi \right), \quad (7)$$

$$\phi = \frac{\lambda a}{2 t_v} \left(\frac{q}{\sqrt{R}} \sinh \sqrt{R} \xi + \frac{\lambda}{R} \sin \lambda \xi \right), \quad (8)$$

where

$$q = \frac{\cos(\lambda \theta/2)}{\cosh(\sqrt{R} \theta/2)}.$$

The strain energy components of the center-covered beam, such as the shear strain energy $\bar{U}_s$ in the viscoelastic layer, the tensile strain energy $\bar{U}_e$ and that of $\bar{U}_f$ due to bending in the constraining layer, and that of $\bar{U}_F$ due to bending in the elastic layer, are determined from the following equations, using Eqs. (7), (8), and Eqs. (5) :

$$\left. \begin{aligned} \bar{U}_s &= 4 t_v G_v L \int_0^{\theta/2} \phi^2 d\xi \\ \bar{U}_e &= \frac{4 E_c t_c}{L} \int_0^{\theta/2} \left(\frac{du_s}{d\xi} \right)^2 d\xi \\ \bar{U}_f &= \frac{2 E_c I_c}{L^3} \int_0^{\theta/2} \left(\frac{d^2 w}{d\xi^2} \right)^2 d\xi \\ \bar{U}_F &= \frac{E_3 I_3}{L^3} \int_0^{1/2} \left(\frac{d^2 w}{d\xi^2} \right)^2 d\xi \end{aligned} \right\} \quad (9)$$

(I_c and I_3 are the second moments of inertia of the

constraining and the elastic layer, respectively).

The composite loss factor η_c of the partially covered beam is determined from the ratio of the imaginary to the real part of the total strain energy stored in the entire beam after replacing the shear modulus G_v with the complex shear modulus $G_v^* = G_v(1 + i\eta)$, where η is the loss factor of the viscoelastic material. However, such calculations are more complicated, so we applied the approximate method in which the maximum strain energy dissipated in the entire beam per cycle is equivalent to those in the viscoelastic layers. Then, the composite loss factor is determined from the following equation :

$$\eta_c (\bar{U}_s + \bar{U}_e + \bar{U}_f + \bar{U}_F)_{\max} = \eta \bar{U}_{s \max}. \quad (10)$$

2.3 End-constrained beam

The end-constrained beam mounted with small massless chips at its viscoelastic layer ends is assumed to constrain shear deformation only of the viscoelastic layers. In this case, the boundary conditions are given as

$$u_s = 0 (\xi = 0), \quad \phi = 0 (\xi = \pm \theta/2).$$

Considering these conditions in Eq. (6), and using Eq. (2) and Eq. (5), the axial displacement u_s and the shear strain ϕ are given as

$$u_s = -a \left(\frac{\lambda^2 p}{R} \sinh \sqrt{R} \xi + \sin \lambda \xi \right), \quad (11)$$

$$\phi = \frac{\lambda^2 a}{2 t_v R} (p \sinh \sqrt{R} \xi + \sin \lambda \xi), \quad (12)$$

where

$$p = \frac{\sin(\lambda \theta/2)}{\sinh(\sqrt{R} \theta/2)}.$$

Both strain energies $\bar{U}_f$ and $\bar{U}_F$ in this beam are equivalent to those of the center-covered beam from the assumption (3), and $\bar{U}_e$ and $\bar{U}_s$ as well as η_c are determined from Eqs. (9) and (10), respectively.

2.4 Separately covered beam

In the separately covered beam in Fig. 1 (c), the girder part of the constraining layer is subjected to a bending moment due to the shear force arising at the interface. If we ignore such bending moment under an assumption of a small amplitude of vibration, the axial force in the girder is kept constant. Then, u_s can be represented as

$$\bar{u}_s = A_3 \xi + A_4. \quad (13)$$

Denoting the locations of both covered ends in Fig. 1 (c) as $\theta_1 = l_1/L$ and $\theta_2 = l_2/L$, the boundary conditions are as follows :

$$\bar{u}_s = 0 (\xi = 0), \quad du_s/d\xi = 0 (\xi = \pm \theta_2/2).$$

The coefficients A_1 , A_2 and A_4 in Eq. (6) and Eq. (13) are determined from above conditions, and also A_3 in Eq. (13) is given by the following condition of continuity in the constraining layer :

$$\bar{u}_s = u_s (\xi = \pm \theta_1/2).$$

Then, the displacement in the constraining layer is given by u_5 and $\bar{u}_5$ as the following :

$$u_5 = a \left(\frac{\lambda q_2}{\sqrt{R}} \sinh \sqrt{R} \xi - \sin \lambda \xi \right), \quad (14)$$

$$\bar{u}_5 = \frac{2a}{\theta_1} \left\{ \frac{\lambda q_2}{\sqrt{R}} \sinh(\sqrt{R} \theta_1/2) - \sin(\lambda \theta_1/2) \right\} \xi, \quad (15)$$

where

$$q_2 = \frac{\cos(\lambda \theta_2/2)}{\cosh(\sqrt{R} \theta_2/2)}.$$

Substituting u_5 of Eq.(14) into Eq.(2), the shear strain ψ is given as

$$\psi = \frac{\lambda a}{2t_v} \left(\frac{q_2}{\sqrt{R}} \sinh \sqrt{R} \xi + \frac{\lambda}{R} \sin \lambda \xi \right). \quad (16)$$

Then, the strain energy components $\bar{U}_s$ and $\bar{U}_e$ are given as the following :

$$\left. \begin{aligned} \bar{U}_s &= 4t_v G_v L \int_{\theta_1/2}^{\theta_2/2} \psi^2 d\xi \\ \bar{U}_e &= 4t_c E_c L \left\{ \int_0^{\theta_1/2} \left(\frac{d\bar{u}_5}{d\xi} \right)^2 d\xi \right. \\ &\quad \left. + \int_{\theta_1/2}^{\theta_2/2} \left(\frac{du_5}{d\xi} \right)^2 d\xi \right\} \end{aligned} \right\} \quad (17)$$

The other strain energy components $\bar{U}_F$ and $\bar{U}_f$ are equivalent to those of the center-covered beam in Eq.(9), and η is given also by Eq.(10).

In this beam, if the girder part has a longer span, a coupled transverse vibration arises between the part and the elastic layer. But we ignore such vibration under a consideration of the fact that both layers are transversely coupled with each other, as mentioned in the later experimental procedure.

3. Calculation Results of the Shear Strain Energy and the Composite Loss Factor

3.1 Center-covered beam

In the following calculations, the elastic and the constraining layers are of the same material ($E_3 = E_c$

Fig. 3 Composite loss factor of the center-covered beam

=206 GPa), and the other numerical values are given as: $t_3 = 1$ mm, $t_c = t_v = 0.5$ mm, $L = 900$ mm, and $\eta = 0.4$. Then, the value of parameter R depends on only the shear modulus G_v of the viscoelastic layer.

The calculated results of η_c for the center-covered beam are shown in Fig. 3 as a function of the cover ratio θ . The low value of R ($R=1$) does not show a peak value of η_c ; however, the higher value of R ($R>5$) maximizes η_c at the cover ratio lower than the full cover ratio.

In order to explain the difference of the above damping characteristics, the calculated results for the total strain energy components ($\bar{U}_s$, $\bar{U}_e$, $\bar{U}_F$, and $\bar{U}_f$) and those densities (U_s , U_e , U_F , and U_f) per unit beam length of the fully covered and the 60%-covered

(a) In the case of $R=1$

(b) In the case of $R=10$

Fig. 4 Strain energy and its densities of the center-covered beam

beams are shown in Fig. 4(a) and Fig. 4(b) for the typical cases of $R=1$ and $R=10$. The values of total energy ratio of the table in Fig. 4 give the normalized values divided by $\bar{U}_F$ as shown by those suffixes. The former case ($R=1$) shows no peak value of η_c , and the later case ($R=10$) maximizes η_c at the 60%-cover ratio as shown in Fig. 3.

In Fig. 4(b), the value of U_s at the covered end ($\xi=0.3$) of the 60%-covered beam increases more than two times that at the same location of the fully covered beam. Furthermore, the value of U_e of the 60% covered beam decreases more than that of the fully covered beam over the entire covered range. These results are attributed to decrease of the axial displacement of the constraining layer due to the cutting off of the viscoelastic layer ends of the fully covered beam, and this decrease causes, to a great extent, the increase of the shear strain in the remaining viscoelastic layer ends. Here, the values of U_F and U_f are relatively smaller than those of U_s and U_e , as shown in Fig. 4(b). Therefore, the damping characteristics mainly depend on strain energies of both $\bar{U}_s$ and $\bar{U}_e$.

On the other hand, in the case of $R=1$, as shown in Fig. 4(a), the values of U_s at $\xi=0.3$ of both the 60%-covered and the fully covered beam are very close. Such a small difference is caused by the fact that the axial displacement u_s becomes sufficiently small due to the low shear stress in the viscoelastic layer; then, cutting off the covered ends result in little effect on the axial displacement of u_s .

From the comparison of the damping characteristics between the cases for both $R=1$ and $R=10$, the partial cover effect can be explained to be caused by the shear strain energy dissipation in the neighbor the viscoelastic layer ends. This energy dissipation also depends on both the shear and the tensile stiffnesses of the viscoelastic and the constraining layers.

3.2 End-constrained beam

In order to confirm the previous explanation of

Fig. 5 Composite loss factor of the end-constrained beam ($R=10$)

the partial cover effect, the calculated results of η_c and the strain energy and its density for the end-constrained beam (for the case of $R=10$) are shown in Figs. 5 and Fig. 6.

The value of η_c does not show a peak value, and also, those of the fully covered beam decrease markedly in comparison with the center-covered beam in Fig. 3. These contrasting features of η_c are clearly caused by the constraint of the viscoelastic layer ends. This constraint causes the decrease of U_s over the entire beam range, especially in the neighbor of the covered ends (here, U_s of the 60%-covered beam is not observed to be extremely small), and causes the increase of U_e depending on the increase of u_s .

3.3 Separately covered beam

It is clear by means of the previous discussion

Fig. 6 Strain energy and its densities of the end-constrained beam ($R=10$)

Fig. 7 Composite loss factor of the separately covered beam ($R=10$)

that the partial cover effect depends on the strain energy dissipation of the viscoelastic Layer ends. In order to develop such an end effect, in this paragraph, we discuss the calculated strain energy and its density for the separately covered beam in Fig. 7. The abscissa in Fig. 7 is the dimensionless location of the outer viscoelastic layer end ξ_2 ($\xi_2 = l_2/2L$), the cover ratio θ ($\theta = 2(\xi_2 - \xi_1)$, here ξ_1 is $\xi_1 = l_1/2L$) is a parameter, and the dotted line shows the results of the center-covered beam with the same outer viscoelastic layer end.

If the girder part becomes wider, the value of η_c becomes higher than that of the center-covered beam with the same cover ratio (shown as points in the dotted line), and takes a peak value at its optimum span width when its cover ratio is under range of the optimum cover ratio of the center-covered beam. In the case of $\theta = 20\%$, for example, $\eta_c = 0.198$ at the optimum separation ($\xi_2 = 0.25$), in contrast with $\eta_c = 0.07$ at the center-covered beam ($\xi_2 = 0.1$). This peak value gradually becomes closer to $\eta_{c\max}$ of the optimum center-covered beam in accordance with the increase of the cover ratio θ . On the other hand, the increase of the cover ratio (for example $\theta = 70\%$) over that of the optimum center-covered beam ($\theta = 60\%$), the value of η_c does not take the peak value and decrease in accordance with the increase of the span width.

In order to discuss these damping characteristics, the strain energies in the case for $\theta = 20\%$ are shown in Fig. 8, varying the location of the outer covered layer end such as $\xi_2 = 0.1$ (the case of no separation), 0.2, 0.3, 0.4, and 0.5 (the case locating the outer ends to the elastic beam ends). In Fig. 8, the values of U_s of the outer and the inner ends become higher due to the

separation (as shown in $\xi_2 = 0.2 \sim 0.5$) than those of the center-covered beam (as shown in $\xi_2 = 0.1$), and take the maximum values at $\xi_2 = 0.4$. Therefore, the separation of the viscoelastic layer results in a higher value of η_c than those of without separation due to the increase of U_s of both outer and inner ends. However, the separation over the optimal span length results in lowering the value of η_c due to the decrease of U_s and the increase of U_e caused by longer girder part. From the above features of both U_s and U_e , it is known that there is an optimum separation width to make η_c a peak value.

The similar calculated results for the other case with the large cover ratio (the case of $\theta = 70\%$ in Fig. 7, which is not illustrated here) indicate that the values of both U_s and U_e are almost equivalent to those without separation; then, the value of η_c in this case becomes slightly lower than those of the center-covered beam, as shown in Fig. 7, due to the increase of U_e of the girder part. Therefore, the separation for the beam with a larger cover ratio is not effective.

4. Experiment

In order to confirm the partial cover effect, we carried out the following experiments for the three types of partially covered beam.

4.1 Experimental procedure

Both the elastic and the constraining layers are used the 420 stainless steel beam (SUS420) with dimensions (width $\times$ thickness) of 10 mm $\times$ 2 mm and 10 mm $\times$ 1 mm, respectively. The viscoelastic layer material was prepared from a commercially available damping rubber (Hamadamper C-1 : YOKOHAMA Rubber Co.), and its material constants were measured by the previously reported method⁽¹³⁾ at 20°C to be $G_v = 40$ MPa and $\eta = 0.4$. These materials were bonded to the center-covered and the separately covered beams by the adhesive (Struct Bond EH-455: MITUI-TOUATU Co.) at room temperature and were cured for five hours by a hot-press device at 40°C⁽¹⁴⁾.

The end-constrained beam specimens were utilized the same center-covered beams of which the covered ends are constrained by small chips (SUS 420: 10 mm in length and 2 mm in thickness), and reinforced by bonding thin phosphor bronze sheets (15 mm in length and 0.2 mm in thickness) between the chip and the constraining layer.

A coupled vibration of the elastic layer and the girder part in the separately covered beam occurred when the girder span length became wider. In this case, several small rods 1 mm in diameter were inserted and bonded between these layers.

The amplitude decay curves in the damped free

Fig. 8 Strain energy and its densities of the separately covered beam ($R = 10$, $\theta = 20\%$)

vibration of the test beams, which were simply supported end condition and 300 mm in the span length, were measured at $22 \pm 1^\circ\text{C}$ in an environmental chamber using an electromagnetic exciter and an eddy-current-type gap sensor. After recording these decay curves on a digital transient memory, the composite loss factors η_c of these beams were calculated from the output data of these curves. An example of these decay curves for the test beams ($\theta = 20\%$) are shown in Fig. 9.

4.2 Experimental results

Nine test beams were prepared for both the center-covered and the end-constrained beams ($R = 17.1$), varying the cover ratio ranging from 10% to

Fig. 9 Typical amplitude decay curves of the partially covered beams ($R = 17.1$, $\theta = 20\%$)

Fig. 10 Experimental results of η_c for both the center-covered beam and the end-constrained beam

90%, respectively. Figure 10 shows the experimental results for both beams. The value of η_c of the center-covered beam shows a peak value, but this peak value is shifted to a higher cover ratio side in comparison with the calculated result shown in Fig. 3. The reason of this difference is attributed to the fact that the calculated result is overestimated for the shear deformation of the viscoelastic layer based on assumption (3). On the other hand, the measured value of η_c of the end-constrained beam becomes over ten times greater than that of the calculated result in Fig. 5. This difference is not clear now, which may be due to the superimposition of both damping effects of an unstable constraint by the chips and of the phosphor bronze sheets. However, both calculated and measured results agree well with each other qualitatively.

The composite loss factor for the end-constrained beams with 20% and 70% cover ratio is shown in Fig. 11. The dotted line in Fig. 11 indicates the same experimental results of the center-covered beam as those in Fig. 10. These results agree well with the calculated results in Fig. 7, except for the difference of the cover ratio corresponding to the peak value of η_c . Therefore, the separately covered beam with a low cover ratio can obtain the highest composite loss factor by means of the optimum separation (for example, in the case of $\theta = 20\%$, the value of η_c at $\xi_2 = 0.35$ takes 90% of $\eta_{c \max}$ of the optimal center covered beam).

From the previous discussions, the damping generation mechanism for the partial cover effect is mainly due to the shear strain energy dissipation in the neighbor of the covered viscoelastic layer ends, and also such end effect may be explained the damping mechanism of the fully covered beam in the same

Fig. 11 Experimental results of η_c for the separately covered beam

manner.

5. Conclusions

An approximate strain energy analysis was carried out for three types of partially covered beam in order to explain the partial cover effect, and these damping characteristics were verified by some flexural free vibration experiments. The results obtained are summarized as follows :

(1) The partial cover effect does not occur for the end-constrained beam, and its composite loss factor becomes sufficiently lower than the center-covered beam. This fact indicates that the partial cover effect is caused by the shear strain energy dissipation in the neighbor of the covered viscoelastic layer ends.

(2) The above energy dissipation depends on both the shear and the tensile stiffnesses of the viscoelastic and the constraining layers, respectively. These facts suggest that the partial cover effect does not appear for the beam with a soft viscoelastic material, and also that such effect depends on the tensile stiffness of the constraining layer if both geometrical beam composition and viscoelastic material constants are fixed.

(3) A separation of the viscoelastic layer for a center-covered beam generates a higher composite loss factor than that without separation, and maximizes the composite loss factor by the optimal separation. But for the case of a larger cover ratio over the optimal one of the center-covered beam, the separation does not generate a higher composite loss factor than that without separation.

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