A Study on the Running Accuracy of an Externally Pressurized Gas Thrust Bearing* (Rotor Run-Out Characteristics)

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To establish a fundamental concept for the precision design of a gas bearing, the static and dynamic run-out characteristics of a rotor supported by an externally pressurized gas-lubricated thrust-collar bearing are investigated in this paper. The concept is based on the theoretical results developed in the previous papers on load capacity fluctuation, which is caused by machining errors such as the out-of-perpendicularity of the rotor-end surface and the size deviation of supply holes of the bearing, as well as on the fundamental bearing characteristics such as stiffness and damping coefficient. The "averaging effect" for the rotor run-out characteristics is evaluated quantitatively. The experimental results for the amplitude of the rotor vibration coincide well with the theoretical results, which may confirm the validity of the theoretical model used in this study.

Key Words: Hydrostatic Bearing, Design Engineering, Accuracy, Vibration, Thrust Gas Bearing, Run-Out Characteristics, Deviation of Forms, Precision Design

1. Introduction

In order to achieve a successful design of externally pressurized gas-lubricated bearings applied to such devices as high-precision spindles of measuring instruments or machine tools, it is very important to obtain the informations on the running accuracy characteristics of the bearing.

In the course of this study, we discuss and investigate theoretically the run-out characteristics of an externally pressurized thrust gas bearing with special attention to the effect of machining errors of the rotor and the bearing.

As the run-out is one of the vibration properties of the rotor, its characteristics may be evaluated on the basis of the solutions of the dynamics of the rotor. Thus, in the previous papers, we investigated the load capacity fluctuation of the bearing⁽¹⁾, which may produce exciting force to the rotor, as well as the fundamental bearing characteristics (stiffness and

damping coefficient)⁽²⁾, in connection with the machining errors such as the out-of-perpendicularity of the rotor-end surface and the size deviation of the supply holes.

In this paper, the run-out characteristics of the rotor supported by an externally pressurized gas-lubricated thrust-collar bearing is investigated on the basis of the results developed in the preceding studies.

2. Fundamental Equation of Motion

Consider a rotor which has a machining error of out-of-perpendicularity in its end surface, and a mating bearing which also has a machining error of size deviation of supply holes. Figure 1 shows the schematic diagram of the bearing. The reference clearance is denoted as h_o , and the amount of maximum form deviation at the outer radius of the shaft is represented by $\epsilon h_o (= \Delta h_o / 2)$, taking a ratio to the reference clearance. The angular position of the maximum form deviation is measured from a particular supply hole, and is designated as α .

If the shaft rotates under such a situation, a change of gap height distribution in the bearing area occurs so that the pressure distribution variation,

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hence, the gas film force fluctuation, may result. The analytical dimensionless gas film force (load capacity) of the bearing, W , was obtained in the previous paper⁽¹⁾, and it can be formulated as follows by applying the "perturbation solution" :

$$W = a_0 + \varepsilon a_{11} \cos \alpha + \varepsilon^2 (a_{20} + a_{22} \cos 2\alpha) + \varepsilon^3 (a_{31} \cos \alpha + a_{33} \cos 3\alpha) + \dots \quad (1)$$

It is presumed that an axial vibration of the rotor is caused by this gas film force fluctuation. The equation of motion of the rotor is expressed by

$$(M\lambda^2/144\pi)\ddot{X} + B\dot{X} + KX = W - W_0 \equiv \Delta W \quad (2)$$

$$X = x/h_0, \quad \dot{} = \partial/\partial\tau, \quad \tau = \omega t,$$

where K : dimensionless bearing stiffness, B : dimensionless damping coefficient, W_0 : external load applied to the rotor, M : mass parameter [$= (m p_a / \mu^2 r_o) (h_0 / r_o)^5$], λ : bearing number [$= (12 \mu \omega / p_a) (r_o / h_0)^2$], m : rotor mass, r_o : radius of the rotor, p_a : ambient pressure, μ : viscosity coefficient, x : axial displacement of the rotor, ω : angular velocity of the rotor, and t : time. The reference gap height, which corresponds to $x=0$, is designated as the equilibrium position of the rotor such that the bearing load W_0 is equal to the load capacity component independent of the rotation angle α ; that is

$$a_0 + \varepsilon^2 a_{20} + \dots - W_0 = 0. \quad (3)$$

The run-out characteristics of the rotor can be evaluated by solving Eq. (2). It is noted that the bearing stiffness and the damping coefficient are also dependent on x , but a conventional method which uses constant bearing characteristic values obtained by assuming a steady-state gap height may give a considerably good approximation, as shown in the later section.

3. Static Run-Out Characteristics

Now consider a situation where the angular velocity of the rotor is very low. In this case, the first and

Fig. 1 Schematic diagram of the bearing

second terms of the left-hand side of Eq. (2) can be neglected, yielding a solution of the rotor displacement as

$$X = (W - W_0)/K. \quad (4)$$

This solution is called "static run-out", for it ignores the dynamic terms in the fundamental equation.

The static run-out characteristics can be easily evaluated by using the results of the previous papers^{(1),(2)}. Figure 2(a) is one example of the calculated static run-out characteristics. The dimensions used in the calculations are $r_i/r_o=0.5$, $r_a/r_o=0.75$ and $r_s/r_o=0.0075$, where r_i : inner radius of the bearing, r_a : radial position of the supply holes, and r_s : radius of the supply holes. Other notations are k : number of supply holes, P_{sg} : dimensionless supply pressure (gage pressure) [$= (p_s - p_a)/p_a$], p_s : supply pressure, Γ : feeding parameter defined by

$$\Gamma = (24 \mu C_D r_s \sqrt{RT}) / (p_a h_0^2) \quad (5)$$

and A_0 : full amplitude of rotor displacement. The ε_r is a parameter for hole size accuracy, and is defined by

$$\varepsilon_r = (r_s' - r_s) / r_s, \quad (6)$$

where we suppose that one supply hole has a size

(a)

(b)

Fig. 2 Static run-out characteristics

deviation in its diameter of $2r_s'$.

In the figure, we assume for convenience, to show the results clearly, that $\varepsilon=0.3$ and $\varepsilon_r=0.3$, which may be rather large values compared with the practical situations. However, the obtained amplitude of the rotor displacement becomes considerably smaller, by about an order of magnitude, than the machining error levels, and decreases with the increase of the number of supply holes. This feature is considered to be a quantitative verification of the so-called "averaging effect" of the externally pressurized gas bearings.

In Fig. 2, the positive value of A_o corresponds to the case where the gas film force, hence the gap height, takes the maximum value when $\alpha=0$ (the case when the direction of maximum form deviation of the rotor end is just towards the particular supply hole with size deviation), while the negative value indicates the opposite situation. As can be seen in the figure, such a phase depends on the Γ value, and a good rotation accuracy of the shaft may be attained if we design the bearing system at a Γ value which makes A_o nearly zero. It should be noted that this optimum Γ coincides considerably well with the one under the condition for maximum bearing stiffness, which is shown by a small circle in the figure.

Figure 2(b) shows the effect of supply pressure on the static run-out characteristics for the case of $k=3$. Although the magnitude of gas film force fluctuation increases with increase of supply pressure, the bearing stiffness also increases, resulting in a rather small effect on the static run-out. The optimum Γ value increases with increase of the supply pressure, which is similar to the feature of the maximum bearing stiffness.

4. Dynamic Run-Out Characteristics

For a case with an appreciable rotational velocity of the shaft, the problem should be analyzed as a forced-vibration system considering that the right-hand side of Eq. (2) is variable with time. This case is called "dynamic run-out" in this paper.

$$\begin{aligned} \Delta W &= \varepsilon a_{11} \cos \tau + \varepsilon^2 a_{22} \cos 2\tau \\ &+ \varepsilon^3 (a_{31} \cos \tau + a_{33} \cos 3\tau) + \dots \\ &= \sum F_j \cos j\tau. \end{aligned} \quad (7)$$

If we use a conventional method in which K and B in Eq. (2) can be approximated by constant values irrespective of rotating angle α , the rotor displacement solution at the steady state is given by

$$X = \sum_j X_j \cos(j\tau - \delta_j), \quad (8)$$

where

$$X_j = F_j / \sqrt{[(K - j^2 \lambda^2 M / 144 \pi)^2 + (jB)^2]}$$

$$\delta_j = \arctan[jB / (K - j^2 \lambda^2 M / 144 \pi)]$$

The dynamic run-out characteristics can be obtained from the full amplitude of Eq. (8).

Examples of the theoretical results of the dynamic run-out for the bearing with $k=3$ are shown by solid curves in Fig. 3. The amplitude at $\lambda=0$ corresponds to the static run-out discussed above.

For a small Γ value case, for example, when $\Gamma=0.2$, there is a peak of the amplitude at nearly $\lambda=2.2$. This is a resonant state where the frequency of excitation due to the gas film force fluctuation coincides with the angular velocity of the rotor. Rather small peaks of rotor amplitude can also be seen nearly at $1/2$ and $1/3$ of the resonant velocity. These are caused by resonances with higher-order terms of $\cos 2\alpha$ and $\cos 3\alpha$ in Eq. (6). The $1/3$ harmonic vibration is rather clearly observed in the figure because this fraction corresponds to the number of supply holes of the bearing ($k=3$).

For the case with a large Γ value, the resonance frequency increases as the bearing stiffness increases, and the static run-out is improved, as previously shown in Fig. 2. The case with $\Gamma=1.0$ is a nearly optimum design, as judged from the view of static run-out in Fig. 2, and the dynamic run-out characteristics are also improved, although the fractional resonance amplitude increases somewhat.

Larger Γ value, for example, $\Gamma=5.0$, gives less bearing stiffness, hence a smaller resonance frequency, and worse dynamic run-out characteristics. This region of the parameter is not recommended for practical design.

Now, we consider the dynamic state of the rotor. If the rotor moves axially, the reference clearance varies with time, accompanied by a change of Γ in the theoretical treatment. Then, the values of coefficient a_{ij} in Eq. (1) and the bearing characteristics, K and B , as well as the parameter of out-of-perpendicularity, ε , are variable with time. A strict treatment of Eq. (2) thus requires a solution of a differential equation

Fig. 3 Dynamic run-out characteristics

with time-dependent coefficients. In order to verify the solution by a conventional time-independent coefficient model, the strict solution is calculated by a direct numerical integration. The dotted curves in Fig. 3 show the results of dynamic run-out. It can be concluded that the conventional treatment may become a good approximation, judging from the agreement of the solid and dotted curves in the figure.

5. Experiments

5.1 Experimental apparatus and method

In order to verify the theoretical results, experiments were carried out using a bearing system with specified machining errors. Figure 4 is the schematic diagram of the experimental apparatus.

An externally pressurized gas-lubricated thrust-collar bearing is constructed of the thrust plate ① and the rotor ②. The outer diameter of the bearing is 30 mm, which is identical to the rotor diameter, $2r_o$, while the inner one is 15 mm, which is the exhaust hole diameter of the bearing plate. Several kinds of thrust plates are prepared with different numbers of supply holes, i.e., $k=3, 4$ or 6 . The radial position of supply holes, r_a , is 11.7 mm, and the diameter of supply holes, $2r_s$, is 0.2 mm. One of the supply holes has a hole size deviation, whose diameter, $2r_s'$, is 0.3 mm, that is, $\epsilon_r = 0.5$.

The compressed air is fed to the bearing from the reservoir. The supply pressure is measured by a fine pressure gage, and is set at 0.49 kPa.

The rotor ② is 100 mm in length, and its mass is 540 g. The rotor has a machining error in perpendicularity at its end surface. The amount of out-of-perpendicularity is $\Delta h = 10 \mu\text{m}$ according to the definition in Fig. 1. The rotor end and the bearing

Fig. 4 Experimental apparatus

plate are ground so that the flatness is better than $0.5 \mu\text{m}$.

The rotor is supported radially by an externally pressurized gas journal bearing ③, and can rotate in a range of 0–20 000 rpm with an air jet through the turbine supply ④. The rotational speed is measured by a photoelectric tachometer ⑤. The axial displacement of the rotor is measured by an eddy-current-type non-contact transducer through the probe ⑥. It is noted that the radial movement of the rotor is also detected and confirmed to be sufficiently small.

The air exhausted from the journal gas bearing is introduced into a space composed of the upper cap ⑦. The pressure in the space is controlled by a valve so that an appropriate axial load is applied to the rotor resulting in a desired gap height at the tested thrust bearing.

5.2 Experimental results and discussions

Figures 5 and 6 are typical results of experiments, in which the full amplitude of the axial vibration of the rotor is shown versus the rotational speed of the rotor. The solid and dotted curves are the theoretical results explained in the previous section. The experimental conditions as well as the theoretical gap height (reference clearance), $h_o(\mu\text{m})$, feeding parameter, Γ , and out-of-perpendicularity parameter, ϵ , are listed in Table 1.

Figure 5 shows the results for the bearing of $k=3$ with applied load as a parameter. In a light load case, for example, at $W_o=0.075$, the occurrence of resonance can be seen at 9 800 rpm. Fractional resonances with rather small peaks occur at 4 900 rpm and 3 300 rpm. The theoretical results coincide well qualitatively and even quantitatively with the experimental ones, although the theoretical resonance frequency is 8 900 rpm, which is slightly smaller than the experimental one.

For a larger load case, the resonance frequency increases and the magnitude of axial vibration decreases. The agreement between the experimental results and theoretical predictions seems to be improved.

Fig. 5 Experimental results (1)

Table 1 Experimental conditions

Number of supply holes	k	3				4	6
Size deviation of a supply hole	ε_T	0.5				0.5	0.5
Supply pressure	P_{sg}	5.0				5.0	5.0
Dimensionless load capacity	W_o	0.075	0.125	0.175	0.225	0.175	0.175
Reference clearance	$h_o(\mu\text{m})$	21.4	16.2	13.4	11.4	15.5	18.8
Feeding parameter	Γ	0.19	0.33	0.48	0.67	0.37	0.25
Out-of-perpendicularity of rotor end	ε	0.23	0.31	0.37	0.44	0.32	0.27

Fig. 6 Experimental results (2)

Figure 6 shows the results of run-out vibration with the number of supply holes as the parameter. The theoretical and experimental results coincide well with each other irrespective of the number of supply holes. In Fig. 6, the peak amplitude at the resonant state for $k=6$ becomes larger than those for the other cases. As previously mentioned concerning Fig. 2, generally, the greater the number of supply holes, the better the run-out characteristics. It should be noted, then, that the apparent experimental results above contradict this statement. This is because the Γ value is not identical throughout the experiments, but is different for each case depending on k .

It can be concluded, from the above discussions, that the validity of the theoretical model and analysis can be confirmed for the axial run-out due to machining errors supposed in the model. Thus, the analytical treatment developed in this study offers a fundamental concept of the precision design of externally pressurized gas thrust bearings.

6. Conclusions

In this paper, the run-out characteristics of an externally pressurized thrust gas bearing is investigated on the basis of the results developed in the previous papers, such as the gas film force fluctuation due to the machining errors of the bearing and the rotor, and the fundamental gas bearing characteristics. The

obtained results are summarized as follows.

(1) The so-called averaging effect of the externally pressurized gas thrust bearing, which means that the rotating accuracy of the shaft is generally better than the machining errors of the system, is evaluated quantitatively.

(2) If we design a gas bearing according to the criterion of maximum bearing stiffness, we can also achieve a nearly optimum rotation accuracy design condition at the same time.

(3) We can effectively apply the conventional time-independent coefficient model for the analysis of the dynamic run-out characteristics.

(4) The run-out characteristics due to the machining errors of the rotor and the bearing can be theoretically evaluated by the proposed theoretical model developed in this study, which may offer a fundamental concept of the precision design of the gas thrust bearings.

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Appendix: Equation of Axial Motion of Rotor

We assume that the gas film force is expressed by the sum of two terms; one is a component independent of the gap height variation velocity (i.e., static load carrying capacity), and the other is a component proportional to the axial velocity of the rotor (i.e., viscous damping term). Then, the equation of motion can be written as

$$m(d^2x/dt^2) = f_1 + f_2(dx/dt) - w_o, \quad (9)$$

where x is the rotor displacement measured from the reference clearance state, and w_o is the load applied to the rotor.

The static load-carrying capacity term, f_1 , is a function of rotor displacement x as well as of the rotating angle α . We further assume that the term f_1

$=f_1(x, \alpha)$ can be divided into two terms ; one is independent of α and the other is dependent on it. The latter term corresponds to ΔW in Eq. (2). This term is evaluated approximately at the state of reference clearance, although it may also be variable with rotor displacement in a strict sense of analysis. Thus, the term $f_1(x, \alpha)$ is written as

$$f_1(x, \alpha) = f_{10}(x) + f_{11}(\alpha).$$

Expanding $f_{10}(x)$ into a perturbed series around $x=0$, neglecting higher-order terms, and recalling that $f_{10}(0) = w_o$ and $\partial f_{10}/\partial x = -k$ (bearing stiffness), we can express the function as

$$f_1(x, \alpha) = w_o - kx + f_{11}(\alpha).$$

We also make an approximation of the squeeze term, f_2 , to be evaluated at the reference clearance state, and denote the damping coefficient to be $-c$.

Substituting these relations into Eq. (9), we finally obtain the following equation:

$$m(d^2x/dt^2) + c(dx/dt) + kx = f_{11}(\alpha) \quad (10)$$

Defining dimensionless time by $\tau = \omega t$, where ω is the angular velocity of the rotor, and applying the definitions in which

$$K = kh_o/(\pi r_o^2 p_a)$$

$$B = c\omega h_o/(\pi r_o^2 p_a)$$

$$\Delta W = f_{11}(\alpha)/(\pi r_o^2 p_a),$$

we can obtain Eq. (2) in the text.

When we apply the time-dependent coefficient model to the analysis, the term f_1 should be treated as a function of both x and α , and f_2 should be considered as a function of x . The fundamental equation of motion can be expressed by the same form as Eq. (9).

References

- (1) Yabe, H. and Watanabe, N., A Study on Running Accuracy of an Externally Pressurized Gas Thrust Bearing (Load Capacity Fluctuation Due to Machining Errors of the Bearing), JSME Int. J., Ser. III, Vol. 31, No. 1 (1988), p. 114.
- (2) Yabe, H. and Yamamoto, M., A Study on Running Accuracy of an Externally Pressurized Gas Thrust Bearing (Bearing Stiffness and Damping Coefficient), JSME Int. J., Ser. III, Vol. 32, No. 4 (1989), p. 618.