Development of Spatial Parallel Manipulators with Six Degrees of Freedom*

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In the present paper, kinematic structures of spatial 6-degree-of-freedom (dof) parallel manipulators are obtained by a number synthesis of multiloop linkages which consist of pairs with 1, 2 and/or 3 dof and have no moving closed loops. Concerning simple and practical manipulators among the obtained ones, that is, concerning the parallel manipulator consisting of six arms, each of which has three revolute pairs and a spheric pair, analytical equations for inverse and direct kinematics are derived and singular positions where the configurations of the manipulator cannot be defined are discussed. Moreover, taking account of allowable pressure angles and maximum swing angles of spheric pairs, the relation between the working space and parameters of the manipulator is investigated, revealing that the manipulator can operate with larger vertical motions than the Stewart Platform.

Key Words: Robotics, Mechanism, Spatial Parallel Manipulator, Number Synthesis, Kinematics, Singular Position, Working Space

1. Introduction

Serial robot manipulators have good flexibility of motion within large working space, but they often have the disadvantages of low power and low rigidity. Although the Stewart Platform, which is a typical parallel manipulator, has high power and high rigidity, it cannot operate within a large working space. Thus, some possible structures of parallel manipulators have been generated⁽¹⁾, and combinations in which the manipulator is partly serial and partly parallel have been presented⁽²⁾. These are useful for manipulator design to some extent.

In the present paper, a kinematic study is carried out in order to provide spatial 6-dof parallel manipulators best suited to design requirements on the characteristics of output power, working space and rigidity. That is, kinematic structures of spatial 6-dof parallel manipulators are clarified by a number synthesis of spatial multiloop linkages taking the simplicity of both design and control into consideration. Moreover, concerning simple and practical mechanisms among the obtained ones, singular positions are

discussed and angular displacements of pairs are analyzed. Taking account of the link parameters, pressure angles and allowable swing angles of spheric pairs, the working space is investigated.

2. Number Synthesis

The manipulators are supposed here to be driven by rotary or linear motion actuators installed on the frames. The structures of the manipulators are limited to those that make the design and control simple; rapid calculations of actuator coordinates may be essentially desired. Thus, the manipulator mechanisms are synthesized under the next three conditions.

(1) The mechanisms consist of pairs with 1, 2 and/or 3 dof, and they have six arm links which are connected to the frame by revolute or prismatic pairs.

(2) All of the subchains which connect the output link to the frame have the same kinematic structure.

(3) There are no moving closed loops in the mechanisms, that is, all of the closed loops contain the frame.

The mechanisms which satisfy these conditions are classified into three types, as shown in Fig. 1, where the top plates express the output links and the dashed lines express serial link chains or arms. In type A, which is the fundamental structure, the output link is supported by six arms. In type B, six lower arms are gathered into three by ternary links and the output link is supported by three upper arms. In type C, the

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lower arms are gathered into two upper arms by which the output link is directly supported.

Denoting the numbers of pairs with $f(=1,2,3)$ dof composing each of the lower and upper arms P_1P_2 and P_3P_4 in Fig. 1 by J_f and J'_f , respectively, the total number of links N in each mechanism is given as follows :

$$N = m \left(\sum_{f=1}^3 J_f - 1 \right) + m' \sum_{f=1}^3 J'_f + 2, \quad (1)$$

in which m and m' are numbers of lower and upper arms, respectively, that is, $m=6$ for all of the types and $m'=0, 3$ and 2 for types A, B and C, respectively.

Thus, the degrees of freedom F of the mechanisms become as below according to the Grübler's equation in the case where the mechanisms have no special geometric conditions :

$$F = 6(N-1) - \sum_{f=1}^3 (6-f)(mJ_f + m'J'_f). \quad (2)$$

Substituting Eq. (1) into the above equation, the next equation is obtained,

$$F = mK + m'K' - 6(m-1), \quad (3)$$

in which

$$K = \sum_{f=1}^3 f J_f, \quad K' = \sum_{f=1}^3 f J'_f. \quad (4)$$

2.1 Type A

Substituting $m=6$, $m'=0$ and $F=6$ into Eq. (3), we have

$$3J_3 + 2J_2 + J_1 = 6. \quad (5)$$

Solving Eq. (5) under the condition of $J_3, J_2 \geq 0$ and $J_1 \geq 1$, we have the five solutions shown in Table 1.

2.2 Type B

Substituting $m=6$, $m'=3$ and $F=6$ into Eq. (3), we have

$$2K + K' = 12. \quad (6)$$

Because the loop L_1 must have at least 2 dof when it is cut off at point P_3 from the upper arm in Fig. 1(b),

Fig. 1 Types of 6-dof parallel manipulators

$K \geq 4$ is obtained by substituting $m=2$, $m'=0$ and $F \geq 2$ into Eq. (3). On the other hand, with $K' > 0$, we have $K < 6$ from Eq. (6).

Thus, putting $K=4, 5$ in Eq. (6), we have

$$(K, K') = (4, 4) \text{ and } (5, 2). \quad (7)$$

Substituting these values into Eq. (4) and solving for J_f and $J'_f (f=1, 2, 3)$, 16 structures are obtained, as listed in Table 1.

2.3 Type C

Substituting $m=6$, $m'=2$ and $m=6$ into Eq. (3), the next equation is obtained :

$$3K + K' = 18. \quad (8)$$

With $K' > 0$ in the above equation, $K < 6$. Because the subchain made by loops L_2 and L_3 has at least 3 dof when it is cut off at point P_3 from the upper arm in Fig. 1(c), $K \geq 5$ is obtained by substituting $m=3$, $m'=0$ and $F \geq 3$ into Eq. (3). Therefore, we have

$$K = 5. \quad (9)$$

Then, we have $K'=3$ from Eq. (8). Accordingly, solving Eq. (4) for J_f and J'_f , 8 structures are obtained, as shown in Table 1.

In the above synthesis, we omitted structures which have redundant freedom in closed loops L_1 , L_2 and L_3 composed of the lower arms.

3. Displacement Analysis and Singular Positions

3.1 Displacement analysis

Among the mechanisms listed in Table 1, the mechanism of $(J_1, J_2, J_3, J'_1, J'_2, J'_3) = (3, 0, 1, 0, 0, 0)$ is cited as an example of the parallel manipulators. Revolute and spheric pairs are adopted as the pairs with 1 and 3 dof, respectively.

As shown in Fig. 2, the revolute pairs $P_{i1} (i=1 \sim 6)$ on the frame are arranged on a circle O whose radius is r_{BOT} . An orthogonal coordinate system $O-XYZ$ is fixed on the frame so that the Z -axis may intersect perpendicularly with the circle at its center point O ; the directions of X - and Y -axes are arbitrarily set. Then, the position of points P_{i1} is expressed by the

Table 1 Results of number synthesis

	(J ₁ , J ₂ , J ₃ , J ₁ ['] , J ₂ ['] , J ₃ ['])											
Type A	(6,	0,	0,	0,	0,	0)	(4,	1,	0,	0,	0,	0)
	(2,	2,	0,	0,	0,	0)	(3,	0,	1,	0,	0,	0)
	(1,	1,	1,	0,	0,	0)						
Type B	(4,	0,	0,	4,	0,	0)	(2,	1,	0,	4,	0,	0)
	(4,	0,	0,	2,	1,	0)	(2,	1,	0,	2,	1,	0)
	(4,	0,	0,	0,	2,	0)	(2,	1,	0,	0,	2,	0)
	(4,	0,	0,	1,	0,	1)	(2,	1,	0,	1,	0,	1)
	(5,	0,	0,	2,	0,	0)	(3,	1,	0,	2,	0,	0)
	(1,	2,	0,	2,	0,	0)	(2,	0,	1,	2,	0,	0)
Type C	(5,	0,	0,	0,	1,	0)	(3,	1,	0,	0,	1,	0)
	(1,	2,	0,	0,	1,	0)	(2,	0,	1,	0,	1,	0)

central angles β_{i1} measured from the X -axis. All rotation axes of the pairs P_{i1} are here supposed to lie on the XY -plane. The rotation axes are described by the two angles β_{i2} and $\beta_{i3}(\equiv 90^\circ)$ measured from the axes parallel to the X - and Z -axes, respectively, as shown in Fig. 2.

As for the top plate or the output link, six spheric pairs are arranged on a circle P of radius r_{TOP} . An orthogonal coordinate system $P-x_P y_P z_P$ is set on the top plate so that the z_P -axis may intersect perpendicularly with the circle at its center point P . The positions of the spheric pairs $P_{i4}(i=1\sim 6)$ are expressed by the central angles β_{i4} measured from the x_P -axis, as shown in Fig. 2.

In Fig. 3, the 3R serial link chain $P_1-P_2-P_3-P_4$ generally represents each of the arms $P_{i1}-P_{i2}-P_{i3}-P_{i4}(i=1\sim 6)$ of the manipulator shown in Fig. 2. Orthogonal coordinate systems $P_j-x_j y_j z_j(j=1\sim 3)$ are fixed on each link so that z_j -axes may coincide with the rotation axes of the revolute pairs P_j , and x_j -axes except $j=1$, may be the common normals between z_{j-1} - and z_j -axes. The x_1 -axis is placed parallel to the Z -axis defined in Fig. 2. At the spheric pair P_4 , an orthogonal system $P_4-x_4 y_4 z_4$ is also fixed so that the x_4 -axis may be parallel to the z_3 -axis, and the z_4 -axis may coincide with the normal to the z_3 -axis. The letters a_j, d_j, α_j and θ_j express the link lengths, perpendicular distances and twist angles between the axes of neighboring pairs and relative angular displacements of links.

In order to describe the angular displacements of the end link P_3P_4 relative to the top plate, a system $P_4-x_P' y_P' z_P'$ is fixed at the pair P_4 parallel to the system $P-x_P y_P z_P$, and angles ϕ_1, ϕ_2 and ϕ_3 are defined as follows.

- ϕ_1 : the angle between z_4 -axis and $x_4 z_P'$ -plane
- ϕ_2 : the angle between x_4 -axis and $x_P' y_P'$ -plane

Fig. 2 An example of a parallel manipulator

ϕ_3 : the angle between x_P' -axis and $x_4 z_P'$ -plane. Moreover, a system $P-X'Y'Z'$ is fixed at the point P parallel to the system $O-XYZ$, and Euler's angles (Θ, Φ, Ψ) between the two systems are used to express the orientation of the top plate. That is, Θ is the angle between the z_P -axis and the Z' -axis, Φ is the angle between the X' -axis and the $Z'z_P$ -plane and Ψ is the angle between the x_P -axis and the $Z'z_P$ -plane.

Next, the angular displacement θ_1 of the input link is determined so that the prescribed position and orientation of the output link may be satisfied. Supposing that θ_2 and θ_3 can vary from 0° to 360° , the movable region of the point P_4 with respect to the system $P_2-x_2 y_2 z_2$ in Fig. 3 becomes a torus. Therefore, when the position of the point P and the orientation of the output link are given as (X_P, Y_P, Z_P) and (Θ, Φ, Ψ) in the system $O-XYZ$, transforming the equation of the torus into the coordinate system $P_1-x_1 y_1 z_1$, we have the next equation:

$$\begin{aligned} & [(Z_0 \cos \theta_1 + B \sin \theta_1 - a_1)^2 \\ & + \{(-Z_0 \sin \theta_1 + B \cos \theta_1) \cos \alpha_1 \\ & - (A + d_1) \sin \alpha_1\}^2 + \{(Z_0 \sin \theta_1 - B \cos \theta_1) \sin \alpha_1 \\ & - (A + d_1) \cos \alpha_1 - d_2\}^2 - (a_2^2 + a_3^2 + d_3^2)]^2 \\ & = 4a_2^2[a_3^2 - \{((Z_0 \sin \theta_1 - B \cos \theta_1) \sin \alpha_1 \\ & - (A + d_1) \cos \alpha_1 - d_2 - d_3 \cos \alpha_2) / \sin \alpha_2\}^2], \end{aligned} \quad (10)$$

where

$$\begin{aligned} A &= A(X_P, Y_P, \Theta, \Phi, \Psi) \\ B &= B(X_P, Y_P, \Theta, \Phi, \Psi) \\ Z_0 &= Z_0(Z_P, \Theta, \Phi, \Psi) \end{aligned} \quad (11)$$

As an example, a manipulator, whose kinematic parameters are listed in Tables 2 and 3, is cited here, which is determined so that the 3R serial link chain shown in Fig. 3 may have large-volume working space⁽³⁾.

The relative angular displacements of the links

Fig. 3 The 3R serial link chain

are shown in Fig. 4 in the case where the output link makes a dwell-rise-dwell parallel motion from $Z_P=244$ to $Z_P=394$ mm holding $\Theta=\Phi=\Psi=0^\circ$ and $X_P=Y_P=0$ mm; the displacement Z_P is given by a 5th-order function of a nondimensional time τ . All of the displacements are observed to vary gently and monotonously. All of the angular displacements for $i=2, 4$ and 6 have the same magnitude with those for $i=1, 3$ and 5, but $\theta_{i1}, \theta_{i3}, \phi_{i1}$ and ϕ_{i3} have the opposite signs to those for $i=1, 3$ and 5.

3.2 Singular positions

Singular positions where the configurations of manipulators cannot be defined are discussed here by means of motor algebra⁽⁴⁾. Denoting the velocity vectors of the pairs on the frame and the pairs on the moving links by $\dot{\theta}$ and $\dot{\theta}'$ respectively, the velocity motor S of the output link is expressed as follows :

$$D_1 \dot{\theta} + D_2 \dot{\theta}' = S \quad (12)$$

In each closed loop, the total summation of the relative velocities of pairs is zero. That is, we have

$$D_3 \dot{\theta} + D_4 \dot{\theta}' = 0 \quad (13)$$

in which D_1, D_2, D_3 and D_4 are matrices of $6 \times 6, 6 \times 30, 30 \times 6$ and 30×30 , respectively, and their elements are functions of the link lengths, perpendicular distances and twist angles between the axes of neighboring pairs, and relative angular displacements of links.

When $\dot{\theta}=0$ in Eq. (13), the determinant of D_4 should not be zero. Otherwise, $\dot{\theta}'$ may have finite values, which means that the configurations of the mechanism cannot be defined. Thus, singular positions can be investigated by the value of $\det D_4$.

Figure 5 shows the variation of $\det D_4$ in Eq. (13) due to the arrangement of the pairs P_{i1} on the frame relative to the positions of the pairs P_{i4} on the top plate when the manipulator has the parameters listed in the caption of the figure and the top plate is located so that the z_P -axis may coincide with the Z -axis. The parameters a_j, d_j, r_{BOT} and r_{TOP} are proportional to those listed in Tables 2 and 3; the value of $a_1 + a_2 + a_3$

Table 2 Parameters of subchain

Link j ($j=1\sim 3$)	1	2	3
a_j [mm]	80.0	160.0	160.0
d_j [mm]	0.0	0.0	0.0
α_j	90.0°	90.0°	—

Table 3 Positions and orientations of pairs on fixed and output links

Chain i ($i=1\sim 6$)	1	2	3	4	5	6
β_{i1}	51.9°	68.1°	171.9°	188.1°	291.9°	308.1°
β_{i2}	150.0°	330.0°	270.0°	90.0°	30.0°	210.0°
β_{i3}	90.0°	90.0°	90.0°	90.0°	90.0°	90.0°
β_{i4}	15.0°	105.0°	135.0°	225.0°	255.0°	345.0°
r_{TOP} [mm]	77.0		r_{BOT} [mm]		84.8	

here is unity.

When $\delta=|\beta_{i1}-\beta_{i4}|=0$, that is, when the pairs P_{i1} and P_{i4} are arranged in the same directions on the bottom and top circles, respectively, $\det D_4$ vanishes, and we know that the position is singular. As δ increases, $|\det D_4|$ increases greatly in the case of $\alpha_1=\alpha_2=90^\circ$, but it increases very little in the case of $\alpha_1=0^\circ$ and $\alpha_2=90^\circ$. Thus, the latter case should be avoided; the mechanism specified by Tables 2 and 3 satisfies this condition.

4. Working Space

Using Eq. (10), we shall analyze the working space of the output link of the manipulator whose parameters are listed in Tables 2 and 3. As the factors which limit the working space, the next two angles are considered. One is the allowable maximum pressure angle, and it is assumed here as 60° , which is slightly larger than high-speed linkages in common use. The other is the maximum swing angle between

Fig. 4 Angular displacements of pairs ($Z_P=244\sim 394$ mm, $i=1, 3, 5$, Solid line of θ_{i1} : desired value, Dashed line of θ_{i1} : experimental value)

Fig. 5 Relationship between determinant of D_4 and δ ($\beta_{i4}=0^\circ, \beta_{i1}=\beta_{i4}+\delta, \beta_{24}=60^\circ, \beta_{21}=\beta_{24}-\delta, \beta_{34}=120^\circ, \beta_{31}=\beta_{34}+\delta, \beta_{44}=180^\circ, \beta_{41}=\beta_{44}-\delta, \beta_{54}=240^\circ, \beta_{51}=\beta_{54}+\delta, \beta_{64}=300^\circ, \beta_{61}=\beta_{64}-\delta, a_1=0.2, a_2=0.4, a_3=0.4, r_{TOP}=0.19, r_{BOT}=0.21, d_j=0.0 (j=1\sim 3), \beta_{i3}=90^\circ (i=1\sim 6), X_P=Y_P=0.0, Z_P=0.8, \Theta=\Phi=\Psi=0.0^\circ$, Solid lines: $\beta_{i2}=\beta_{i1}$, Dashed lines: $\beta_{i2}=\beta_{i1}-30^\circ$, Chain lines: $\beta_{i2}=\beta_{i1}-90^\circ$)

the pairing elements of spheric pairs, an example of which is shown in Fig. 6: a ball and socket and a universal joint with one ball bearing on the market. The ψ_1 and ψ_2 are the maximum swing angles about two axes intersecting at right angles. As for the ball and socket, these two axes are taken so that they may intersect at right angles with the central axis of the socket, as shown in Fig. 7(a). As for the universal joint, the two axes are taken to be coincident with the rotation axes of revolute pairs, as shown in Fig. 7(b). As is seen in Fig. 6, the universal joint can produce greater swing angle than the ball and socket. The minimum limit angle of the universal joint is about 60° at the positions of $\psi_2 = \pm \psi_1$.

Adopting the universal joints as the spheric pairs in the mechanism specified in Tables 2 and 3, we obtain the working space shown in Fig. 8. Figure 8(a) is a schematic view of the reachable region of point P. Figures 8(b) and (c) are, respectively, the cross sections of the working space on the XZ-plane and the planes parallel to the XY-plane. In Fig. 8(b), the upper part of the outlines is the limit due to the maximum reachable distance between the output link and the frame, and the lower part of the outline is given by the pressure angles at the pairs P_{i4} . Moreover, the curves within the working space show the maximum orientation θ of the output link in the direction of the Y-axis when the angles ϕ and ψ are held zero.

Figure 9 shows sections of the reachable regions of point P on three different lengths of links which are shown just under the caption (the other parameters are the same as those listed in Tables 2 and 3). In the figure, in the case where each stroke of linear actuators of the Stewart Platform⁽⁶⁾ is within 40% of their maximum lengths, the working space of the Stewart Platform is shown by thin dashed lines. The arrangement of the pairs on the frame and the platform and the maximum reachable displacement of the platform in the Z-direction are the same as those of the manipulator proposed here.

Fig. 6 Maximum swing angles

With an increase of $a_1/(a_1 + a_2 + a_3)$, although the upper part is kept almost unchanged, the lower part of the working space of the manipulator proposed in the present paper falls downward. Therefore, the manipulator can operate with larger vertical motions than the Stewart Platform.

Figure 10 shows the maximum displacement of the output link when it is moved downward parallel to the XY-plane from the position where all arms are completely stretched out. In the figure, when $a_1/(a_1$

(a) Ball and socket (b) Universal joint

Fig. 7 Examples of spheric pairs

(a) Schematic view

(b) XZ-section and maximum orientation angle θ of output link

(c) Sections parallel to XY-plane
(○: positions of pairs P_{i1})

Fig. 8 Working space of the parallel manipulator

Fig. 9 Relationship between working space and link lengths

(Dashed line : $a_1/(a_1+a_2+a_3)=0.175$, $a_3/a_2=3.7$

Solid line : $a_1/(a_1+a_2+a_3)=0.3$, $a_3/a_2=3.33$

Chain line : $a_1/(a_1+a_2+a_3)=0.375$, $a_3/a_2=4.0$

Thin dashed line: Stewart Platform, $a_1+a_2+a_3=400.0$ mm)

$+a_2+a_3$) is small and a_3/a_2 is large and input links can make full rotations, $\Delta Z/(a_1+a_2+a_3)$ is proportional to $a_1/(a_1+a_2+a_3)$, as shown by the dashed lines. When a_3/a_2 is small or when both a_3/a_2 and $a_1/(a_1+a_2+a_3)$ are large, input links cannot make full rotation because the pressure angles exceed the allowable value, and $\Delta Z/(a_1+a_2+a_3)$ becomes as shown by the solid lines.

Figure 11 shows an experimental apparatus whose parameters are the same as the values listed in Tables 2 and 3. The control system of the apparatus consists of a 16-bit personal computer, PWM drivers (PWM frequency is 1.3 kHz), DC servomotors (60 W), pulse encoders (500 pulses/rev.) and 16-bit up-and-down counters. Experiments including the motion illustrated in Fig. 4 are carried out, and the above-mentioned results of analysis are confirmed to coincide with the experimental ones.

5. Conclusions

A kinematic study has been carried out in order to develop spatial 6-dof parallel manipulators. The results are summarized as follows.

(1) Kinematic structures of spatial 6-dof parallel manipulators have been obtained by a number synthesis of multiloop linkages without moving closed loops.

(2) Concerning the parallel manipulator consisting of six arms, each of which has three revolute pairs and a spheric pair, an equation relating the input angular displacements with the position and orientation of the output link has been derived. Using the equation, angular displacements of the whole pairs have been calculated for the prescribed motion of the output link. Moreover, using motor algebra, singular positions where the configurations of the manipulator cannot be defined have been discussed.

Fig. 10 Relationship between maximum displacement ΔZ and link lengths

Fig. 11 Experimental apparatus

(3) Taking account of the allowable pressure angles and maximum swing angles of spheric pairs, the working space has been investigated, and the manipulator developed here has been confirmed to have larger working space than the Stewart Platform.

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