Rapid Position Control using Feedforward Control based on Inverse Dynamics* (The Case of the Mechanical System of Linear DC Motor and Beam)

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A position control system consisting of a linear DC motor and a beam, based on inverse dynamics, is proposed. The rapid and exact control performance is confirmed experimentally and theoretically. In the experiment, the linear DC motor is made of an aluminum coil frame or an acrylic resin coil frame. It is confirmed that, by the present control method, the moving part of the linear DC motor can be controlled accurately to the position control command for an arbitrary settling time without undesirable mechanical vibrations. When the settling time is comparatively long, the wave shape of the control current of the motor is affected mainly by the bending force of the beam. Then, with a decreasing settling time, the inertial force has significant effects on the wave shape of the control current. Furthermore, the numerical simulation results obtained with the finite element method agree well with the experimental results.

Key Words: Computer Control, Mechatronics, Control Device, Inverse Dynamics, Feedforward Control, Linear DC Motor, Beam

1. Introduction

In the position control system, a feedback loop of the position of the mechanism is used frequently, and undesirable mechanical vibrations may be generated remarkably with an increasing value of feedback gain. Therefore, studies have been reported on such subjects as the following: the effects of mechanical resonance systems on a control loop⁽¹⁾, the design method of a positioning control system with mechanical resonant characteristics⁽²⁾, positioning control of a servomotor mechanism with an oscillatory load based on the modern control theory⁽³⁾, and PI and I-P velocity controls in a position control system that considers oscillatory mechanism⁽⁴⁾.

Recently, the feedforward control method based

on inverse dynamics has attracted special interest, and it has been used effectively in the control system of a robot^{(5),(6)}. This feedforward control method has several advantages of motion control in comparison with usual feedback control methods, and it can be expected to be used widely hereafter.

A study on the position control system consisting of electromagnets and metallic springs has also been reported⁽⁷⁾. In this control system, feedforward position control by an open-loop-type control is possible because the springs act as a restoring force. If the feedforward-type position control system is constructed with an electromagnetic actuator and a beam, based on inverse dynamics, rapid and effective control performance can be expected. However, this study seems to have been little investigated.

In this paper, first, the outline of the mechanical system consisting of a linear DC motor and a beam is shown, and the design method of the feedforward position control system based on inverse dynamics is described. Then, the experimental results and the numerical simulation results obtained with the finite element method are demonstrated, and the rapid and exact control performance is confirmed. Furthermore,

* Received 9th July, 1991. Paper No.89-0941B

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the effects of the settling time on the shape of the feedforward voltage of the linear DC motor are clarified.

2. Outline of Mechanical System

Figure 1 shows the outline of the mechanical system consisting of a linear DC motor and a beam. The dimensions of the beam are as follows: the length is $l = 27.5$ cm, the cross-sectional area is $l = 2.75$ cm, the cross-sectional area is $A = 3.40 \times 10^{-5} \text{ m}^2$, Young's modulus is $E = 157$ GPa, and the second moment of area is $I_b = 7.25 \times 10^{-12} \text{ m}^4$. Figure 2 shows the structure of the linear DC motor in the case of an aluminum coil frame. In this type of linear DC motor, the electromagnetic damping force can be generated by the eddy current in the aluminum coil frame due to the relative motion between the moving coil and the stator. Table 1 shows the specifications of the linear DC motor. From this table, it is seen that the electromagnetic damping force can not be generated in the

case of the acrylic resin coil frame.

3. Feedforward Position Control System Based on Inverse Dynamics

Figure 3 shows the outline of the feedforward position control system based on inverse dynamics. The feedforward voltage control command is computed by a BASIC program, and it is applied to the linear DC motor through a machine language program, a D/A converter and a power amplifier. In the position control system, it is desirable that the acceleration of the mass be continuous and smooth in terms of the dynamics of the mechanical system. Therefore, in the present experiment, the position control command based on the sine-wave velocity curve is utilized as follows:

$$T_c \geq t \geq 0 : \dot{Y}_c = Y_d(1 - \cos \omega t)/T_c \quad (1)$$

$$t \geq T_c : \dot{Y}_c = 0 \quad (2)$$

where

$$\omega = 2\pi/T_c \quad (3)$$

and Y_d is the desired value of the position, Y_c is the velocity control command, t is time, and T_c is the settling time. Then the position control command Y_c and the acceleration one $\ddot{Y}_c$ are expressed as

$$T_c \geq t \geq 0 : Y_c = Y_d \{ t - (\sin \omega t)/\omega \} / T_c \quad (4)$$

$$\ddot{Y}_c = (\omega Y_d \sin \omega t) / T_c \quad (5)$$

Fig. 1 Schematic model of experimental apparatus

Fig. 2 Linear DC motor with aluminum coil frame

Table 1 Specifications of linear DC motor

Frame		Aluminum	Acryl
Force constant	G	4.64 N/A	4.66 N/A
Inductance	L	3.42 mH	4.09 mH
Resistance	R_L	4.20 Ω	4.26 Ω
Number of coils	N_c	360	365
Damping coefficient C_d		2.37 Ns/m	0.30 Ns/m
Mass of moving part M_L		312 g	255 g

Fig. 3 Feedforward position control system based on inverse dynamics

$$t \geq T_c : Y_c = Y_d, \dot{Y}_c = 0. \quad (6)$$

In the present experiment, the mass of the moving part of the linear DC motor (312 g, in the case of aluminum coil frame) is comparatively bigger than the mass of the beam (712 g). Therefore, the beam can be treated as a spring element, and the equation of motion for the position control command Y_c is expressed as

$$M\ddot{Y}_c + C\dot{Y}_c + KY_c = GI_c \quad (7)$$

where

$$M = M_L + 33\rho A l^3 / 140, K = 3EI_b / l^3 \quad (8)$$

and M_L is the mass of the moving part of the linear DC motor, G is the thrust coefficient, and I_c is the electric current control command. On the other hand, the equation of the electric current of the linear DC motor is expressed as

$$L\dot{I}_c + RI_c + \bar{G}\dot{Y}_c = V_c \quad (9)$$

where V_c is the voltage control command, L is the inductance, $\bar{G}$ is the back electromotive force, and R is the sum of the motor electric resistance R_L and the external electric resistance. By substituting Eqs. (1) - (6) into Eq. (7) and using the electric current control command I_c and Eq. (9), the feedforward voltage control command based on the sine-wave velocity curve is derived as follows:

$$V_c = \{ML\ddot{Y}_c + (MR + CL)\dot{Y}_c + (KL + CR + G\bar{G})Y_c + KRY_c\} / G. \quad (10)$$

4. State Equation of Dynamics-Control Coupled System

In this study, the effect of the flexural vibrations of the beam on the control performance is investigated by means of the numerical simulation of the dynamics-control coupled system. In the numerical simulation, the state equation of the dynamics-control coupled system are derived by the finite element method in consideration of the flexural vibrations of the beam. By using the finite-element model of the beam, as shown in Fig. 4, the equation of motion of the beam is expressed as

$$M_0\ddot{y} + C_0\dot{y} + K_0y = b_0I \quad (11)$$

where

$$y = \{W_1, \phi_1, \dots, W_4, \phi_4\} \quad (12)$$

$$b_0 = \{1, 0, \dots, 0\} \quad (13)$$

Fig. 4 Finite element model of a beam

and M_0 is the mass matrix in consideration of the mass of the moving part of the linear DC motor, C_0 is the damping matrix for the magnetic damping coefficient C_m shown in Table 1, K_0 is the stiffness matrix. On the other hand, the equation of electric current is expressed as follows:

$$L\dot{I} + RI + \bar{G}\dot{Y} = V_c \quad (14)$$

where I is the electric current, and $Y (= W_1)$ is the displacement of the moving part of the linear DC motor. Therefore, by using Eqs. (11) and (14), the dynamics-control coupled system is derived as

$$\dot{x} = A_0x + b_1V_c \quad (15)$$

where

$$x = \{\dot{y}_0^T, y^T, I\}^T \quad (16)$$

$$A_0 = -A_1^{-1}A_2, \quad b_1 = A_1^{-1}b_2 \quad (17)$$

$$A_1 = \begin{bmatrix} A_3 & 0 \\ 0 & L \end{bmatrix}, \quad A_2 = \begin{bmatrix} A_4 & b_3 \\ b_4 & R \end{bmatrix} \quad (18)$$

$$A_3 = \begin{bmatrix} M_0 & 0 \\ 0 & E \end{bmatrix}, \quad A_4 = \begin{bmatrix} C_0 & K_0 \\ E & 0 \end{bmatrix} \quad (19)$$

$$\begin{aligned} b_2 &= \{b_0^T, 0^T\}^T \\ b_3 &= \{-G, 0, \dots, 0\}^T \\ b_4 &= \{\bar{G}, 0, \dots, 0\}^T \end{aligned} \quad (20)$$

and E is the unit matrix.

5. Experimental Results and Numerical Results

5.1 Frequency response

Figure 5 shows the experimental results of the thrust characteristics of the linear DC motor in the present study. From this figure, it is seen that the thrust characteristics have sufficient linearity. Figure 6 shows the frequency responses of the displacement of the moving part and the electric current of the linear DC motor. From the frequency responses of the displacement, the resonance amplitude is damped dramatically by the magnetic damping force in the case of the aluminum coil frame. Then, from the frequency responses of the electric current, antiresonance is generated due to the back emf, and the

Fig. 5 Thrust characteristics of linear DC motor

tendency becomes marked in the case of the acrylic resin coil frame. Consequently, it indicates that the design method of the control system in consideration of the electric circuit of the linear DC motor is important.

5.2 Step response

Figure 7 and 8 show the experimental results and the numerical results of the step response of the system when the desired value of the position is $Y_d = 1.5$ mm, and the step voltage (the aluminum coil frame, 6.77 V; the acrylic resin coil frame, 6.75 V) is applied to the linear DC motor. In the present study, the settling time is defined as the time required for the step response to stay within 5 percentage of its final value. In the case of the linear DC motor with the aluminum coil frame, the settling time is approxi-

mately 1 s. On the other hand, in the case of the acrylic resin coil frame, the residual vibrations continue for a long time, and the settling time becomes considerably larger in comparison with that in the case of the aluminum coil frame. Consequently, it is seen that the damping force of the linear DC motor with the aluminum coil frame is remarkably strong. Then, in the step responses of the electric current in Figs. (7) and (8), the vibrating behavior due to the back emf is denoted, and the interaction between the mechanical system and the electrical system can not be neglected.

5.3 Transient response of position control system

Figures 9-11 show the experimental results and the numerical results of the transient response of the feedforward position control system for the desired value of the position $Y_d = 1.5$ mm. The undesirable mechanical vibrations are not generated, and, as a result, the moving part of the linear DC motor can be controlled rapidly and precisely along the position control command. When the settling time is long, the wave shape of the control electric current is affected considerably by the bending force of the beam. Then, with decreasing settling time, the inertial force

Fig. 6 Frequency response of the system

Fig. 7 Experimental results of the step response in the case of linear DC motor with aluminum coil frame

Fig. 8 Experimental results of the step response in the case of linear DC motor with acrylic resin coil frame

significantly affects the wave shape of the control electric current, and the maximum value of the control electric current becomes large. The experimental results agree well with the numerical results, and the validity of the model of the dynamics-control coupled system is proved. Furthermore, it is recognized that the limit of the settling time is determined by factors such as the maximum value of the output current of the power amplifier. Figure 12 shows the transient response in the case of the acrylic resin coil frame, and it is observed that rapid control performance is realized; the maximum value of the electric current is slightly larger than that in the case of the aluminum coil frame.

In the present study, position control is realized merely by feedforward control based on inverse dynamics. However, in practice, feedback control as well as feedforward control should be utilized to guarantee the control performance for the disturbance force.

6. Conclusions

In the present paper, a feedforward position control system consisting of a linear DC motor and a beam, based on the inverse dynamics, is proposed. Then, rapid and exact control performance is confirmed experimentally and theoretically. The results may be summarized as follows:

(1) The linear DC motor composed of an aluminum coil frame has a comparatively strong magnetic force due to the eddy current.

(2) From the experimental results, it is confirmed that, by means of the present feedforward control method, the moving part of the linear DC motor can be controlled exactly to the position control command for an arbitrary settling time, without mechanical vibrations.

(3) When the settling time is long, the feedforward control electric current wave is chiefly affected by the bending force of the beam. Then, with decreasing settling time, the inertial force significantly affects

Fig. 9 Transient response of the system in the case of linear DC motor with aluminum coil frame when the settling time is $T_c = 0.6$

Fig. 10 Transient response of the system in the case of linear DC motor with aluminum coil frame when the settling time is $T_c = 0.2$ s

Fig. 11 Transient response of the system in the case of linear DC motor with aluminum coil frame when the settling time is $T_c=0.08$ s

Fig. 12 Transient response of the system in the case of linear DC motor with acrylic coil frame when the settling time is $T_c=0.08$ s

the wave shape of the control electric current, and the maximum value of the control electric current becomes large.

(4) The simulation results of the dynamics-control coupled system, in consideration of the flexural vibrations, agree well with the experimental results. Then, the validity of the modeling of the dynamics-control coupled system is confirmed.

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