Sensitivity Calculation Methods for Conducting Modal Frequency Response Analysis of Coupled Acoustic-Structural Systems*

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The previous reports by the authors presented analytical methods for determining the sensitivities of the eigenvalues and eigenvectors and the sensitivity of the direct frequency response (DFR) of coupled acoustic-structural systems, with the aim of reducing vehicle interior noise levels. As a continuation of the previous studies, this paper deals with an investigation of the modal frequency response (MFR) of coupled systems and proposes methods for analyzing their MFR sensitivities. Two types of analytical methods are derived for determining the MFR sensitivities; one is based on the eigenpair sensitivities of a coupled system and the other on the direct differentiation of the modal frequency response. It is shown that the latter method can also be derived from the DFR sensitivity and that it is a more effective method in terms of calculation accuracy and efficiency. A coupled acoustic-box model and a coupled acoustic-vehicle model are analyzed to confirm the applicability of the proposed methods to actual noise analysis problems.

Key Words: Coupled Acoustic-Structural System, Modal Frequency Response, Sensitivity Analysis, Frequency Response Analysis, Acoustic Analysis, Structural Analysis, FEM

1. Introduction

Previous studies conducted by the authors examined sensitivity analysis methods for application to coupled acoustic-structural system for the purpose of reducing vehicle interior noise levels. An analytical method was presented in Ref.(1) for determining the eigenmode sensitivities of coupled systems and similar analytical methods were presented in Ref.(2) for determining the direct frequency response (DFR) of coupled systems and for analyzing their DFR sensitivities. As a continuation of the previous work, this paper presents an investigation of the modal frequency response (MFR) of coupled acoustic-structural systems and proposes methods for calculating their MFR sensitivities.

One of the major difficulties in analyzing coupled acoustic-structural systems is that their coefficient matrices are asymmetrical. This means that conven-

tional methods developed for treating structures having symmetrical matrices cannot be applied to coupled systems without making modifications. In order to resolve this difficulty, a method was first proposed in 1980⁽³⁾ for transforming asymmetrical matrices into symmetrical ones so that analyses could then be carried out. Although this method is still being used today⁽³⁾⁻⁽⁵⁾, it has several drawbacks. With this method, the calculation becomes rather complex and the sensitivity coefficients cannot be determined easily.

The authors sought to overcome those drawbacks by developing an eigenmode sensitivity analysis method applicable to coupled acoustic-structural systems. Calculation theories and methods were derived for treating asymmetrical matrices directly, and the application of those methods led to the first success in obtaining the sensitivity coefficients of coupled acoustic-structural systems⁽¹⁾.

This paper presents an investigation of methods for conducting MFR analyses of coupled acoustic-structural systems, involving the direct treatment of asymmetric matrices, and for calculating their MFR sensitivities. Using the orthogonality and normaliza-

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tion conditions introduced in Ref.(1) for the eigenmode vectors of coupled acoustic-structural systems, formulations are derived for analyzing the MFR of such systems. Based on those formulations, two methods for calculating MFR sensitivities are examined and proposed here. One method is based on the eigenpair sensitivities of a coupled system and the other method is based on the direct differentiation of the modal frequency response. Further, it is shown here that the second method can also be derived from the DFR sensitivity⁽²⁾ and that it is a more accurate and efficient calculation procedure. An evaluation is made of the sound pressure level within a certain given frequency range and an assessment is made of the overall rate of change in the sound pressure level in relation to changes in design variables. The validity of the methods presented here has been confirmed by using them to analyze a coupled acoustic-box structure and a coupled interior sound field-vehicle body system.

2. Eigenvalue Problem of Coupled Acoustic-Structural Systems

The following FEM equation is used here to treat a coupled acoustic-structural system⁽³⁾⁻⁽⁵⁾:

$$M\ddot{u} + Ku = f, \quad (1)$$

where

$$K = \begin{bmatrix} K_{ss} & K_{sa} \\ 0 & K_{aa} \end{bmatrix}, \quad M = \begin{bmatrix} M_{ss} & 0 \\ M_{as} & M_{aa} \end{bmatrix},$$

$$u = \begin{Bmatrix} u_s \\ u_a \end{Bmatrix}, \quad f = \begin{Bmatrix} f_s \\ f_a \end{Bmatrix} \quad (2)$$

and u_s is the displacement vector of the body structure, u_a is the sound pressure vector, f_a is the sound pressure of the sound in the sound field, M_{ss} and K_{ss} are the mass and stiffness matrices of the body structure, M_{aa} and K_{aa} are the mass and stiffness matrices of the sound field, and M_{as} and K_{sa} are the coupled matrices. The subscripts s and a denote the body structure and the sound field, respectively, and "'' represents a double differentiation according to time. Further, we obtain

$$K_{sa} = -M_{as}^T. \quad (3)$$

From Eq.(1), the following eigenvalue problem is obtained:

$$(K - \lambda_i M)\phi_i = 0, \quad (4)$$

where λ_i is the eigenvalue and ϕ_i is the corresponding eigenvector. Also,

$$\phi_i^T = \{\phi_{si}^T, \phi_{ai}^T\}. \quad (5)$$

Here, ϕ_{si} is the component pertaining to the structure of ϕ_i and ϕ_{ai} is the component pertaining to the sound field of ϕ_i . For the sake of simplicity, it is assumed here that Eq.(4) does not have any zero or repeated eigenvalues.

As explained in Ref.(1), since the coefficient

matrices, K and M , of Eq.(4) are not symmetrical, the following orthogonality conditions do not generally hold true for their eigenvector ϕ_i .

$$\phi_i^T K \phi_j = 0 \text{ and } \phi_i^T M \phi_j = 0 \quad (\text{for } i \neq j) \quad (6)$$

This means that a conventional mode superposition technique cannot be used when only the value of ϕ_i is known. This problem was overcome in ref.(1) by introducing the following orthogonality and normalization conditions for the eigenvector system.

⟨Orthogonality conditions⟩

The following orthogonality conditions hold true with respect to the coupled acoustic-structural system in Eq.(4):

$$\bar{\phi}_i^T K \phi_j = 0, \quad \bar{\phi}_i^T M \phi_j = 0 \quad (\text{for } i \neq j), \quad (7)$$

where $\bar{\phi}_i$ is referred to as the left eigenvector and

$$\bar{\phi}_i^T = \left\{ \phi_{si}^T, \frac{1}{\lambda_i} \phi_{ai}^T \right\} \quad (\text{for } \lambda_i \neq 0). \quad (8)$$

⟨Normalization condition⟩

The normalization condition for the mass of the eigenvector ϕ_i is

$$\phi_{si}^T M_{ss} \phi_{si} + \frac{1}{\lambda_i} (\phi_{ai}^T M_{as} \phi_{si} + \phi_{ai}^T M_{aa} \phi_{ai}) = 1 \quad (\text{for } \lambda_i \neq 0). \quad (9)$$

In the next section, a mode superposition technique for coupled acoustic-structural systems will be derived based on these orthogonality and normalization conditions.

3. Mode Superposition Technique for Coupled Acoustic-Structural Systems

Employing the orthogonality conditions presented in the previous section for the eigenvectors of a coupled system, it is possible to apply a conventional mode superposition technique to a vibration mode analysis of the coupled system in Eq.(1). This is accomplished as explained below.

First we will represent the solution to u in Eq.(1) as follows:

$$u = \sum_{i=1}^n \phi_i q_i. \quad (10)$$

Substituting this expression into Eq.(1) and multiplying the equation from the left by $\bar{\phi}_i^T$ and then using the orthogonality condition in Eq.(7), we obtain

$$m_i \ddot{q}_i + k_i q_i = f_i \quad (i=1, 2, \dots, n), \quad (11)$$

where

$$\begin{aligned} m_i &= \bar{\phi}_i^T M \phi_i \\ &= \phi_{si}^T M_{ss} \phi_{si} + \frac{1}{\lambda_i} (\phi_{ai}^T M_{as} \phi_{si} + \phi_{ai}^T M_{aa} \phi_{ai}), \\ k_i &= \bar{\phi}_i^T K \phi_i \\ &= \phi_{si}^T K_{ss} \phi_{si} + \phi_{si}^T K_{sa} \phi_{ai} + \frac{1}{\lambda_i} \phi_{ai}^T K_{aa} \phi_{ai}, \\ f_i &= \bar{\phi}_i^T f = \phi_{si}^T f_s + \frac{1}{\lambda_i} \phi_{ai}^T f_a. \end{aligned} \quad (12)$$

In the case of a frequency response analysis, we let $f = F e^{i\omega t}$, $u = U e^{i\omega t}$ and $q_i = p_i e^{i\omega t}$, and then the follow-

ing expression is obtained from Eqs.(10) and (11) :

$$U = \sum_{i=1}^n \phi_i p_i = \sum_{i=1}^n \frac{\phi_i \bar{\phi}_i^T F}{m_i(\lambda_i - \omega^2)}, \quad (13)$$

where

$$p_i = \frac{\bar{\phi}_i^T F}{m_i(\lambda_i - \omega^2)} \quad (14)$$

and $\lambda_i = k_i/m_i$. Equations (11) and (13) are completely identical in form to the formulations used in a conventional mode superposition technique. It should be noted, however, that the coefficient calculation procedure in Eq.(12) differs from the conventional method.

4. Modal Frequency Response Sensitivity of Coupled Acoustic Structural Systems

The object of an MFR sensitivity analysis is to find the rate of change (i.e., sensitivity) in the modal frequency response relative to a change made in a certain design variable. We will let $\alpha_k (k=1, 2, \dots)$ denote design variables. Then, the rate of change, s_k , in the MFR of the present structure, U , in relation to α_k can be expressed in differential form as $s_k = \Delta U^k / \Delta \alpha_k$, where $\Delta \alpha_k$ is the amount of minute change made in the design variable α_k , and ΔU_k is the amount of change in the MFR due to the change in α_k . The following discussion will examine a method for finding ΔU_k .

Consider that ΔK^k and ΔM^k represent the amount of change in the coefficient matrices K and M in Eq. (1) in relation to $\Delta \alpha_k$ and are given by

$$\Delta K^k = \begin{bmatrix} \Delta K_{ss}^k & \Delta K_{sa}^k \\ 0 & \Delta K_{aa}^k \end{bmatrix}, \quad \Delta M^k = \begin{bmatrix} \Delta M_{ss}^k & 0 \\ \Delta M_{as}^k & \Delta M_{aa}^k \end{bmatrix}. \quad (15)$$

The subscript k will be omitted in the following discussion for the sake of simplicity.

4.1 Method based on eigenpair sensitivities

We will let U represent the modal frequency response of the present structure and U_1 the frequency response after the design variable is changed. Then, using the MFR calculation formula in Eq.(13), the amount of change in the frequency response, ΔU , can be given by

$$\Delta U = U_1 - U. \quad (16)$$

Then, letting ϕ_i represent an eigenvector normalized according to Eq.(9), we obtain

$$U = \sum_{i=1}^n \frac{\phi_i \bar{\phi}_i^T F}{\lambda_i - \omega^2}, \quad U_1 = \sum_{i=1}^n \frac{\phi_{1i} \bar{\phi}_{1i}^T F}{\lambda_{1i} - \omega^2}. \quad (17)$$

In Eq.(17),

$$\lambda_{1i} = \lambda_i + \Delta \lambda_i, \quad \phi_{1i} = \phi_i + \Delta \phi_i, \quad \bar{\phi}_{1i} = \bar{\phi}_i + \Delta \bar{\phi}_i, \quad (18)$$

where $\Delta \lambda_i$ is the amount of change in the eigenvalue λ_i in relation to the change in the design variable, $\Delta \alpha_k$, $\Delta \phi_i$ and $\Delta \bar{\phi}_i$ are the amount of change in ϕ_i and $\bar{\phi}_i$ as a result of $\Delta \alpha_k$, respectively.

Assuming that the system does not have any repeated eigenvalues, the method presented in Ref.

(1) can be used to find $\Delta \lambda_i$, $\Delta \phi_i$ and $\Delta \bar{\phi}_i$ as follows :

$$\Delta \lambda_i = \bar{\phi}_i^T (\Delta K - \lambda_i \Delta M) \phi_i \quad (19)$$

$$\Delta \phi_j = \sum_{i=1}^n \phi_i C_{ij}, \quad \Delta \bar{\phi}_j = \sum_{i=1}^n \bar{\phi}_i D_{ij}, \quad (20)$$

where

$$C_{ij} = \begin{cases} \frac{-1}{\lambda_i - \lambda_j} \bar{\phi}_i^T (\Delta K - \lambda_j \Delta M) \phi_j & \text{for } i \neq j \\ -\frac{1}{2} \bar{\phi}_i^T \Delta M \phi_j & \text{for } i = j \end{cases} \quad (21)$$

and

$$D_{ij} = \begin{cases} \frac{1}{\lambda_i - \lambda_j} \bar{\phi}_i^T (\Delta K - \lambda_j \Delta M) \phi_j & \text{for } i \neq j \\ -\frac{1}{2} \bar{\phi}_i^T \Delta M \phi_j & \text{for } i = j. \end{cases} \quad (22)$$

After calculating $\Delta \lambda_i$, $\Delta \phi_i$ and $\Delta \bar{\phi}_i$ using Eqs.(19) and (20), the values thus can be substituted into Eqs.(16) and (17) to obtain ΔU . Then, the frequency response sensitivity, s_k , can be found from $s_k = \Delta U / \Delta \alpha_k$.

4.2 Method based on direct differentiation of MFR

The formulation in Eq.(13) for analyzing the MFR can be written in matrix form as in Ref.(2).

$$U = \Phi \Omega^{-1} \bar{\Phi}^T F \quad (23)$$

Differentiating Eq.(23) yields

$$dU = (d\Phi \Omega^{-1} \bar{\Phi}^T + \Phi d\Omega^{-1} \bar{\Phi}^T + \Phi \Omega^{-1} d\bar{\Phi}^T) F. \quad (24)$$

Accordingly, by approximating the infinitely small amount of change, $d(\cdot)$, as a minute change, $\Delta(\cdot)$, we obtain

$$\Delta U = (\Delta \Phi \Omega^{-1} \bar{\Phi}^T + \Phi \Delta \Omega^{-1} \bar{\Phi}^T + \Phi \Omega^{-1} \Delta \bar{\Phi}^T) F, \quad (25)$$

where

$$\Delta \Phi = [\Delta \phi_1, \Delta \phi_2, \dots, \Delta \phi_n], \quad \text{and} \quad \Delta \bar{\Phi} = [\Delta \bar{\phi}_1, \Delta \bar{\phi}_2, \dots, \Delta \bar{\phi}_n].$$

Also

$$\Delta \Omega^{-1} = \text{diag} \left\{ \frac{-\Delta \lambda_i}{(\lambda_i - \omega^2)^2} \right\}. \quad (26)$$

Assuming that $\Delta \lambda_i$, $\Delta \phi_i$ and $\Delta \bar{\phi}_i$ have been found, their values can be substituted into Eq.(25) to calculate ΔU .

However, substituting

$$\Delta \Phi = \Phi C, \quad \Delta \bar{\Phi} = \bar{\Phi} D \quad (27)$$

into Eq.(25) yields

$$\Delta U = \Phi A \bar{\Phi}^T F, \quad (28)$$

where

$$A = C \Omega^{-1} + \Omega^{-1} D + \Delta \Omega^{-1}, \quad (29)$$

and $C = (C_{ij})$ and $D = (D_{ij})$. By substituting Eqs.(21), (22) and (26) into Eq.(29) and rearranging the expression, we obtain

$$A = -\Omega^{-1} \bar{\Phi}^T (\Delta K - \omega^2 \Delta M) \Phi \Omega^{-1} = S \Omega^{-1}, \quad (30)$$

where

$$S = -\Omega^{-1} \bar{\Phi}^T (\Delta K - \omega^2 \Delta M) \Phi. \quad (31)$$

In addition, from Eq.(14), $P = \text{col}\{p_i\} = \Omega^{-1} \bar{\Phi}^T F$, and so the foregoing expressions can be summarized to yield the following equation :

$$\Delta U = \Phi S P. \quad (32)$$

4.3 Brief investigation

Equation (32) can be rewritten as

$$\Delta U = \sum_{i=1}^n \sum_{j=1}^n \phi_i S_{ij} p_j, \quad (33)$$

where

$$S_{ij} = \frac{-1}{\lambda_i - \omega^2} \bar{\phi}_i^T (\Delta K - \omega^2 \Delta M) \phi_j. \quad (34)$$

Dividing both sides of Eq.(33) by p_j yields

$$\begin{aligned} \frac{\Delta U}{p_j} &= \sum_{i=1}^n \sum_{k=1}^n \phi_i S_{ik} \left(\frac{p_k}{p_j} \right) \\ &= \sum_{i=1}^n \phi_i S_{ij} + \sum_{i=1}^n \sum_{k \neq j}^n \phi_i S_{ik} \left(\frac{p_k}{p_j} \right). \end{aligned} \quad (35)$$

When $\omega^2 \rightarrow \lambda_j$, the following expression is obtained from Eq.(14).

$$\frac{p_k}{p_j} = \frac{\lambda_j - \omega^2 \bar{\phi}_k^T F}{\lambda_k - \omega^2 \bar{\phi}_j^T F} \rightarrow 0 \quad (\text{for } k \neq j) \quad (36)$$

Hence, from Eq.(35) we obtain

$$\lim_{\omega^2 \rightarrow \lambda_j} \left(\frac{\Delta U}{p_j} - \phi_j S_{jj} \right) = \sum_{i \neq j}^n \phi_i S_{ij}^* = \Delta \phi_{1j}, \quad (37)$$

where

$$\begin{aligned} S_{ij}^* &= \lim_{\omega^2 \rightarrow \lambda_j} S_{ij} = \frac{-1}{\lambda_i - \lambda_j} \bar{\phi}_i^T (\Delta K - \lambda_j \Delta M) \phi_j \\ &= C_{ij} \quad (\text{for } i \neq j). \end{aligned} \quad (38)$$

On the other hand, from Eq.(20) we obtain

$$\Delta \phi_{1j} = \Delta \phi_j - \phi_j C_{jj}. \quad (39)$$

It is clear⁽¹⁾ that $\Delta \phi_{1j}$ is a component of $\Delta \phi_j$ that is orthogonal to $\bar{\phi}_j$. However, in this case, $\Delta \phi_{1j}$ was determined from the standpoint of a forced vibration mode.

Using the DFR sensitivity analysis method in Ref. (2), we obtain

$$\Delta U = -Z^{-1} \Delta Z U, \quad (40)$$

where $Z = K - \omega^2 M$ and $\Delta z = \Delta K - \omega^2 \Delta M$. Focusing on $U = -Z^{-1} F$, Z^{-1} can be expanded as explained in Ref.(2) to the eigenmode of a coupled system, denoted as $Z^{-1} = \Phi \Omega^{-1} \Phi^T$. We obtain

$$\Delta U = -\Phi \Omega^{-1} \bar{\Phi}^T \Delta Z \Phi \Omega^{-1} \bar{\Phi}^T F. \quad (41)$$

A comparison of Eqs.(32) and (41) reveals that they are identical expressions. This indicates that the solution presented here can also be derived from the DFR sensitivity analysis method, i.e., it is identical to the solution given in section 4.1 of Ref.(2).

The frequency response of the original structure is given by

$$U = \sum_{i=1}^n \phi_i p_i. \quad (42)$$

Hence, a comparison of Eqs.(33) and (42) shows that ΔU is given by a new combination of ϕ_i and p_i . Since the coefficient matrix of this combination is $S = (S_{ij})$, it is clear that the matrix S has a great deal of significance to a sensitivity analysis.

5. Sound Pressure Level Sensitivity in a Certain Frequency Region

In the previous section, an investigation was

made of two methods for conducting a frequency response sensitivity analysis. It was seen that the frequency response sensitivity of a specified input frequency, ω , could be found by either method. However, in dealing with actual noise problems, there are times when it is necessary to evaluate the overall rate of change in the frequency response in a certain frequency range in relation to a design parameter modification. For instance, reducing the noise level in the vehicle interior may require a noise reduction within a certain range of frequencies, as opposed to reducing the noise at any one particular frequency. In such cases, it is necessary to find the sensitivity coefficient that expresses the overall rate of change in the sound pressure level within the frequency range of interest. In the following discussion, definitions will be given for "sound pressure level integration" and "sound pressure level sensitivity integration" in a certain frequency range, and methods of calculating these integrations will be examined.

The sound pressure level can be expressed in terms of a dBA value as

$$p = 10 \log_{10} \left(\frac{U}{U_0} \right)^2, \quad (43)$$

where $U_0 = 2 \times 10^{-5}$ Pa. When $U \leq U_0$, the sound is not audible, so p in the above equation is zero in this case. Thus, p always has a positive value.

We will let $[\omega_a, \omega_b]$ represent the frequency range of interest, where $\omega_a \leq \omega_b$. The dBA sound pressure level p in Eq.(43) can be integrated throughout this frequency range as*

$$G_p = \int_{\omega_a}^{\omega_b} p \, d\omega. \quad (44)$$

where G_p is a quantity that represents the overall sound pressure level throughout the frequency range and is referred to here as the sound pressure level integration.

We will let Δp represent the amount of change in the dBA sound pressure level p in relation to the amount of change made to the design variable, $\Delta \alpha_k$. Accordingly, the sensitivity of the dBA sound pressure level is given by

$$s_p = \frac{\Delta p}{\Delta \alpha}. \quad (45)$$

The following expression is used to integrate s_p throughout the frequency range $[\omega_a, \omega_b]$:

$$G_s = \int_{\omega_a}^{\omega_b} s_p \, d\omega = \frac{1}{\Delta \alpha} \int_{\omega_a}^{\omega_b} \Delta p \, d\omega, \quad (46)$$

where G_s is a quantity that expresses the rate of

* The damping of the system is omitted from consideration here. In this case, when $\omega^2 = \lambda_i$, ($i = 1, 2, \dots, n$), p becomes infinitely large. Hence, to perform the integration in Eq.(44), the frequency range $[\omega_a, \omega_b]$ must not contain any eigenfrequencies of the system.

change in the overall sound pressure level in $[\omega_a, \omega_b]$ and is referred to here as the sound pressure level sensitivity integration.

Further, from Eq.(43) we obtain

$$\Delta p = \begin{cases} \frac{20}{\ln 10} \left(\frac{\Delta U}{U} \right) & \text{for } U > U_0 \\ 0 & \text{for } U \leq U_0 \end{cases} \quad (47)$$

Once ΔU has been found, the value can be substituted into Eq.(47) and then into Eq.(46) to calculate the sound pressure level sensitivity integration.

6. Application Examples

6.1 Frequency response of coupled acoustic-box structure

Figure 1 shows the model of a coupled acoustic-structural system that was used in the analysis. Similar to the model employed in Ref.(2), it consisted of a hollow rectangular parallelepiped made of steel plates and measuring 200 cm in length, 160 cm in width and 150 cm in height. For the sake of simplicity, using 68 low-order eigenfrequency modes for the structure and 70 low-order eigenfrequency modes for the sound field respectively, the overall coupled system was reduced to only 138 degrees of freedom. The following discussion will consider a system having 138 degrees of freedom. The excitation point of the system was node 40 along the y-axis of the box structure and the measurement point was node 32 in the sound field.

Figure 2 shows the sound pressure level sensitivities in a frequency range of 10 Hz to 50 Hz for three different numbers of modes used: 10, 20 and 138. These results were obtained using the modal frequency response analysis method presented in section 3. The results in the figure show that the use of low-order modes alone yields good approximations in the low frequency region, just as in the case of conven-

Fig. 1 Analytical model of coupled acoustic-box structure system

tional modal analysis methods.

6.2 Frequency response sensitivity of coupled acoustic-box structure

As shown in Fig.1, the thicknesses of the 1st ~10th, 11th~20th and 21st~30th shell elements were used as the design variables, α_1 , α_2 and α_3 , respectively. Each of the design variables was changed by 0.005% (i.e., $\Delta\alpha_k/\alpha_k = 0.005\%$; $k=1, 2$, and 3) and the resulting sound pressure level sensitivities were found using the MFR sensitivity analysis method presented in section 4.2 above.

Figure 3 shows the sound pressure level sensitivities found in a frequency range of 22 Hz to 29 Hz using this method. The solid line indicates the calculated results obtained when all 138 modes were used. The dashed line and the dashed-dot line indicate the results obtained for 10 modes and 20 modes, respectively. The results indicate that good approximations were obtained even when only the low-order modes were used.

The MFR sensitivities in relation to three

Fig. 2 Modal frequency responses of coupled acoustic-structural system (for 10, 20 and 138 modes)

Fig. 3 Modal frequency response sensitivities (for 10, 20 and 138 modes)

different design variables, α_1 , α_2 and α_3 , are compared in Fig. 4. The change in the sound pressure level sensitivity as a function of frequency is clearly seen for each of the design variables. This indicates that it is possible to determine which design variable should be modified in order to reduce the sound source at a particular frequency. As a result, it will be possible to optimize the body structure design for the purpose of

reducing the interior noise level.

Table 1 presents a comparison of the frequency response sensitivities calculated with the DFR sensitivity analysis methods proposed in Ref.(2) and the MFR sensitivity analysis methods presented in this paper. The four methods compared here are 1) a conventional DFR sensitivity analysis method, 2) an iterative DFR sensitivity analysis method explained in section 3.2 of Ref.(2) and presented in the appendix, 3) the MFR sensitivity analysis method explained in section 4.1 above (method 1), and 4) the MFR sensitivity analysis method presented in section 4.2 (method 2). The calculation made using the MFR sensitivity analysis methods were for all 138 modes.

The data in the table indicate that all four methods yielded virtually the same results, although the calculation accuracy of method 1 was slightly lower.

The CPU time required for each method is compared in table 2. The figures in parentheses are the calculation times for 20 modes. As the data in the table indicate, the calculation time in the case of MFR sensitivity analysis is reduced by using a smaller number of modes.

6.3 MFR sensitivity analysis of a coupled acoustic-structural system

To verify the applicability of these analysis methods to large-scale FEM problems, calculations were performed for an actual vehicle model (Fig. 5). As indicated in the figure, the vehicle body model had a total of 2,982 nodes and 3,713 elements, including

Table 1 Comparison of MFR sensitivities obtained by different methods

Results for design variable α_1				
Method	15Hz	22Hz	50Hz	80Hz
1.	0.37279	1.1448	-2.4984	-1.0804
2.	0.37280	1.1448	-2.4984	-1.0804
3.	0.37402	1.1452	-2.4962	-1.0805
4.	0.37279	1.1448	-2.4984	-1.0804

Results for design variable α_2				
Method	15Hz	22Hz	50Hz	80Hz
1.	0.43563	-0.12643	-5.4992	-2.0409
2.	0.43564	-0.12643	-5.4992	-2.0409
3.	0.43606	-0.12626	-5.4940	-2.0407
4.	0.43563	-0.12643	-5.4992	-2.0409

1. Direct method for DFR sensitivity analysis (section 3.1 in ref.(2))
2. Iteration method for DFR sensitivity analysis (section 3.2 in ref.(2))
3. Method 1 for MFR sensitivity analysis (section 4.1 of present paper)
4. Method 2 for MFR sensitivity analysis (section 4.2 of present paper)

Fig. 4 Comparison of MFR sensitivities for three design variables (α_1 , α_2 , and α_3)

Table 2 Comparison of CPU times

Method	1.	2.	3.	4.
CPU	3.075	1.714	3.197	3.075
		(1.22)	(1.21)

Fig. 5 Finite-element models of coupled acoustic-structural system

Fig. 6 Sound pressure level sensitivities of coupled acoustic-structural system

2,100 CQUAD4 elements, 694 CTRIA3 elements, 732 CBAR elements and 187 RBAR elements⁽⁷⁾. The sound field model contained a total of 1,374 nodes and 1,060 elements, including 794 CHEXA elements and 266 CPENTA elements⁽⁷⁾.

The first- to the fifth-order eigenmodes of the vehicle body and the sound field were first determined and used to calculate the eigenpairs of the coupled system. The computation time was 662.7 seconds using a Cray XMP/EA432 computer. Then, MFR analyses were conducted using the methods presented in this paper. The thicknesses of the CQUAD4 elements and CTRIA3 elements of the vehicle body model were used as the design variables in the analysis.

Figure 6 shows the sound pressure level sensitivity distribution found for $\omega=80$ Hz using the MFR analysis method discussed in section 4. 2. The calculation time needed to perform the sensitivity analysis proposed here was 1,414 seconds for 2,794 design variables. In other words, it took only 0.5 second to calculate the MFR sensitivity in relation to a change in one design variable. This represents a substantial reduction in computer time compared with the 662.7 seconds that were required when sensitivity analysis methods were not used. This result verifies that the methods presented here are effective tools for obtaining the optimum design for a large-scale coupled acoustic-structural system.

7. Conclusions

As a continuation of the work done in Ref.(1) and (2), methods were investigated for analyzing the modal frequency response and modal frequency response sensitivity of coupled acoustic-structural systems. Fundamental equations were proposed for

conducting MFR analyses of coupled systems, based on the orthogonality and normalization conditions proposed in Ref.(1) for the eigenvectors of such systems. Those equations were derived because conventional MFR analysis methods are not applicable to the asymmetrical coefficient matrices of the dynamic equations of coupled acoustic-structural systems.

Based on those equations, two methods were derived for conducting frequency response analyses and their validity was examined. One method is based on the eigenpair sensitivity of a coupled system and the other method is based on the direct differentiation of the modal frequency response. It was shown that the latter method can also be derived from the DFR sensitivity procedure proposed in Ref.(2) and that it is the more effective method of the two in terms of calculation accuracy and efficiency.

Further, definitions were given for "sound pressure level integration" and "sound pressure level sensitivity integration". An investigation was made of methods for evaluating the sound pressure level in a certain given frequency range and for calculating the corresponding sensitivity coefficients.

The methods proposed here were used to analyze a coupled acoustic-box structure system and a coupled acoustic-structural system formed by the interior sound field and the vehicle body. The results verified the validity of the methods.

In future work, the authors intend to investigate a sensitivity analysis method for treating systems having repeated eigenvalues and examine the possible application of the method to problems involving the optimization of coupled acoustic-structural systems.

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Appendix : Direct Modal Frequency Response Sensitivity

A.1 Direct method using Z^{-1}

We will differentiate the equation, $ZU = F$, which is used to calculate the direct frequency response, and approximate the infinitely small quantity $d(.)$ as a minute quantity $\Delta(.)$. This yields

$$\Delta U = -Z^{-1} \Delta Z U. \quad (48)$$

Equation (48) is a conventional sensitivity analysis method for determining the direct frequency response. In the calculation done using Eq.(48), it is necessary to compute the inverse matrix, Z^{-1} (or the triangular resolution) of the impedance matrix. To avoid performing this calculation of Z^{-1} , a sensitivity analysis method was investigated in Ref.(2) based on a kind of iteration procedure.

A.2 Iteration method that does not use Z^{-1}

The impedance matrix Z is first divided into two matrices as shown below, (Z_1 is a triangular matrix with a lower block and Z_2 contains the remaining components.):

$$Z_1 = \begin{bmatrix} K_{ss} - \omega^2 M_{ss} & 0 \\ -\omega^2 M_{as} & K_{aa} - \omega^2 M_{aa} \end{bmatrix}, \quad (49)$$

$$Z_2 = \begin{bmatrix} 0 & K_{sa} \\ 0 & 0 \end{bmatrix},$$

where

$$Z = Z_1 + Z_2. \quad (50)$$

Equation (48) can then be rewritten as

$$Z_1 \Delta U = -\Delta Z U - Z_2 \Delta U. \quad (51)$$

Assuming that $\Delta U^{(0)} = -Z_1^{-1} \Delta Z U$, the following iteration expression is obtained from Eq.(51):

$$\Delta U^{(n)} = \Delta U^{(0)} + B \Delta U^{(n-1)} \quad (n=1, 2, \dots), \quad (52)$$

where

$$B = -Z_1^{-1} Z_2. \quad (53)$$

Equation (52) can be divided and rewritten as two expressions, one for the response of the structure and one for the response of the sound field. This yields the following equations.

(a) When $\Delta Z_{ss} = (\Delta K_{ss} - \omega^2 \Delta M_{ss})$ alone has no zero values,

$$\begin{aligned} \Delta U_s^{(0)} &= D_{ss} U_s, \quad \Delta U_a^{(0)} = B_{as} \Delta U_s^{(0)}; \\ \Delta U_s^{(n)} &= \Delta U_s^{(0)} + B_{sa} \Delta U_a^{(n-1)}, \\ \Delta U_a^{(n)} &= B_{as} \Delta U_s^{(n)} \quad (n=1, 2, \dots), \end{aligned} \quad (54)$$

where

$$\begin{aligned} D_{ss} &= -Z_{ss}^{-1} \Delta Z_{ss}; \\ B_{as} - Z_{aa}^{-1} Z_{as}, \quad B_{sa} &= -Z_{ss}^{-1} Z_{sa}. \end{aligned} \quad (55)$$

(b) When $\Delta Z_{aa} = (\Delta K_{aa} - \omega^2 \Delta M_{aa})$ alone has no zero values,

$$\begin{aligned} \Delta U_s^{(0)} &= 0, \quad \Delta U_a^{(0)} = D_{aa} U_a; \\ \Delta U_s^{(n)} &= B_{sa} \Delta U_a^{(n-1)}, \\ \Delta U_a^{(n)} &= \Delta U_a^{(0)} + B_{as} \Delta U_s^{(n)}, \quad (n=1, 2, \dots), \end{aligned} \quad (56)$$

where

$$D_{aa} = -Z_{aa}^{-1} \Delta Z_{aa}. \quad (57)$$

The use of either Eq.(54) or Eq.(56) makes it possible to calculate in one iteration the inverse matrices (or triangular resolution) of the impedance matrices (Z_{ss} and Z_{aa}) of an uncoupled system. Therefore, either method substantially reduces the calculation volume.