Active Wave Control of a Flexible Beam*

(Fundamental Characteristics of an Active-Sink System and Its Verification)

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This paper deals with the active wave control of a flexible beam. It is the purpose of this paper to realize experimentally the active sink proposed in the previous paper, and to discuss the fundamental characteristics of the active-sink system. First, for the purpose of observing wave flow traveling along the beam, a visualization system is developed. Next, on the basis of the visualization system, an experiment is carried out to verify the existence of the active sink. Then, from an analytical point of view, this paper further investigates the characteristics of the active-sink system, showing that an active source also exists in addition to the active sink. It is also found that there are two suppression patterns in active wave control; that is, the progressive-wave type and the standing-wave type. Finally, by introducing both phase distribution and gain of the complex reflection coefficient over a flexible beam, the generation mechanism of these patterns is clarified.

Key Words: Wave, Flexible Beam, Active Sink, Active Source, Active Control, Complex Reflection Coefficient

1. Introduction

In the same way that there are two kinds of vibration analysis⁽¹⁾, that is, standing-wave analysis and progressive-wave analysis, there are also two kinds of vibration control methods, that is, the standing-wave type and the progressive-wave type. Summarizing the conventional vibration control methods reported so far, we find that almost all the vibration control methods belong to the standing-wave type^{(2)~(4)}, and the application^{(5)~(8)} of the progressive-wave type to vibration control problems is very limited. This is simply because progressive waves are rarely observed in mechanical structures in our daily life.

In vibration control of a huge and flexible structure such as an LSS (large space structure) which inherently possesses innumerable vibration modes with very light damping, a conventional modal-based control method (standing-wave type) is no longer

applicable, because the modal-based control method requires the same number of actuators as that of vibration modes. In such a case, it is advantageous to apply a wave control method, a progressive-wave type, to suppress the vibration of the structure, which attempts to make all the vibration modes inactive before they are excited. On the contrary, the modal-based control method has the premise that all the vibration modes of a structure are already excited.

In the previous paper⁽⁸⁾, the authors proposed an active-sink method which makes all the vibration modes inactive, and derived the optimal control law for an active-sink system. The purpose of this paper is to realize the active-sink system and verify the validity of the method. It is also the purpose of this paper to show the existence of another method which makes it possible to render all the vibration modes inactive: a standing-wave suppression method.

In the first half of this paper, emphasis is placed on the realization of the active-sink method. To do this, a wave visualization system is constructed to observe the realization of the active sink located at the left end of the beam. It is then demonstrated that the vibration mode excited at the modal frequency is

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never excited as long as the active sink is effective. Next, by changing the position of the active sink from the left end to the center of the beam, the effect on control is investigated. Two kinds of control laws are applied in this case; one is to suppress the reflection wave in the element in which an exciting force exists, and the other is for the element where an exciting force is nonexistent.

In the latter half of this paper, the fundamental characteristics of the active-sink method are discussed. In comparison with the conventional vibration control method, the position of the active-sink method in the category of active vibration control is clarified. Then, in addition to the active-sink method, a standing-wave-type method is presented; this can also make all the vibration modes inactive. Finally, generation mechanism of vibration patterns is discussed from the point of view of the complex reflection coefficient.

2. Realization of the Active Sink

2.1 Wave visualization system

In measuring the vibration of a distributed parameter system such as a flexible beam, the displacement distribution over the whole beam should be acquired. For this reason, a visualization system of waves is constructed as shown in Fig. 1. The specifications of the flexible beam used in the experiment are as follows: length of 2.77 meters, width of 5 cm, and thickness of 5.8 mm. The beam is suspended by two pieces of soft spring. In order to shake this beam, an electrodynamic exciter is installed at the right tip of the beam. Between the beam and the exciter, a force transducer capable of detecting an external force f_{in} to a maximum force of 4.9 N is mounted. At the left end or the center of the beam, where the active sink is supposed to be located, an exciter is installed to produce a control force f_d of the active sink.

As shown in Fig. 1, the control force whose gain and phase are adjusted by a phase shifter is measured

Fig. 1 Schematic diagram of a wave visualization system

by the same force transducer as used for measuring the external force. Along the 2.77-meter-long beam, 22 gap sensors (effective stroke, 2.0 mm; resolution, 0.5 μm) are arrayed in steps of 13 cm to measure the displacement distribution. Signal outputs of these gap sensors are acquired simultaneously by a data logger, and then are input into a computer. After processing the data by spline interpolation, the computer outputs the waveforms travelling along the beam.

2.2 When the active sink is placed at the left end of the beam

Figure 2 shows a driving point compliance obtained experimentally by exciting the right end of the beam. As noted in the figure, there are seven resonant peaks in the frequency range up to 100 Hz. In addition, a rigid body mode which does not appear in the figure exists near 0.8 Hz. Figure 3 shows experimental results of the visualized waveforms at the 3rd vibration mode. The horizontal line is the distance x from the left end of the beam, while the vertical axis shows elapsed time. The maximum amplitude of the waveforms is 376 μm in this case. Moiré fringes produced by the intersection between the waveforms and the horizontal lines are observed to run vertically. This means that the standing waves are dominant over the vibration phenomenon. The top part of the figure shows the envelope of those waveforms, showing also the existence of standing waves whose nodes never shift their positions.

Next, taking into consideration the gain-phase properties of the controller for the active sink presented in the previous paper⁽⁸⁾, an experiment was con-

Fig. 2 Driving point compliance obtained by exciting the left end of a beam

ducted for the purpose of realizing the active sink. In this section, the experimental results pertaining to the 3rd vibration mode are demonstrated for comparison with the analytical results shown in the previous paper⁽⁸⁾. Figure 4 shows time historical responses of the displacement distribution suppressed by the active sink installed at the left end of the beam. As can be noted in the figure, we can observe waveforms traveling from the right tip to the left end of the beam, where the active sink is located. It is significant that the maximum displacement amplitude is suppressed to only 49 μm from 376 μm , corresponding to 2.6 per-

Fig. 3 Time historical responses of waveforms in the case of noncontrol (around the 3rd vibration modal frequency)

Fig. 4 Time historical responses of waveforms in the case where an active sink is placed at the left end of a beam (around the 3rd vibration modal frequency)

cent of the maximum amplitude without an active sink. Note that the amplitudes of the external force f_{in} vary in the case with and without the active sink, as shown in Figs. 3 and 4. Therefore, in the calculation of the reduction rate, the correction is accounted for.

As depicted in the top part of Figs. 3 and 4, the waveforms are transformed from standing waves into progressive waves. In the case of the progressive waves shown in Fig. 4, nodes do not exist in the waveforms, so the envelope appears to consist of two parallel lines. In this case, the influence of the near field is recognized at both ends of the envelope.

2.3 When the active sink is placed at the center of the beam

As mentioned in the previous paper⁽⁸⁾, two kinds of control laws are considered in the case where the active sink is placed at an arbitrary position on a beam; that is,

Case 1: the active sink suppresses the reflection waves in the region where an external force exists.

Case 2: the active sink eliminates the transmitted waves in the region where an external force is nonexistent.

Figure 5 shows the time historical responses of the waveforms suppressed by the active sink in terms of the control law, Case 1, placed at the center of the beam. In the right-hand side element of the beam, where the exciting force acts, the waves are seen travelling from the right end to the central point of the beam. In contrast, in the left part of the beam the standing waves are observed, because the active sink is not effective in this area. With respect to the control effect in this case, the maximum displacement ampli-

Fig. 5 Time historical responses of waveforms in the case where an active sink with Case 1 control law is placed at the center of a beam (around the 3rd vibration modal frequency)

tude is suppressed to $53.5 \mu\text{m}$, corresponding to 2.8 percent of the case without the active sink.

Figure 6 shows the time historical responses of the waveforms obtained by applying the control law, Case 2. As predicted in the numerical analysis⁽⁸⁾, the shape of displacement distribution in Case 2 is completely different from that in Case 1. In the left-side element of the beam, progressive waves should exist. However, since there is no external force in the region, both progressive waves and reflection waves are eliminated owing to the relationship of the duality. On the contrary, in the right-hand side element where the external force acts, standing waves appear with the maximum at the right end of the beam, corresponding to the suppression rate of 1.9 percent without the active sink.

3. Fundamental Characteristics of the Active Sink System

3.1 Position of the active sink in the category of active vibration control

In this section, the characteristics of the active-sink system are considered through the comparison between the control effects in terms of the conventional vibration control method and the active-sink method. Then the position of the active-sink method in the category of active vibration control is also clarified.

In order to make use of the analytical results presented in the previous paper⁽⁸⁾, the specifications for the active-sink system should be the same as those in the analysis. For this reason, the active sink is placed at the left end of the beam, while an exciting

force acts at the right end of the beam.

The control law⁽⁸⁾ Case 1, of the active-sink system in this case is characterized by a time lag property; that is, the gain is 1.0 throughout the entire frequency range, while the phase is delayed monotonously. Taking into consideration these characteristics of the control law, we describe the control force f_d by using the exciting force f_{in} in the following:

$$f_d = f_{in} \exp[j\theta]. \quad (1)$$

Then, the control effect is investigated by varying only the phase of the control force. Figure 7 shows the gain plot of a complex reflection coefficient⁽⁹⁾ at the active-sink point ($x=0$) obtained by changing the amount of phase from 0 to 360 degrees. The amplitude ratio w of the suppressed displacement by the active sink to the displacement without control at the exciting point ($x=1$) is also shown in Fig. 8.

$$\eta = \left| \frac{C_1 \exp[-jkx]}{C_3 \exp[jkx]} \right|, \quad (2)$$

$$w = \frac{\xi_{\max}}{\xi_0} \Big|_{x=1}, \quad (3)$$

where the numerator and the denominator of Eq. 2 represent the 1st term and the 3rd term of the general

Fig. 7 Gain η of a complex reflection coefficient versus phase difference θ

Fig. 8 Displacement amplitude ratio w versus phase difference θ

Fig. 6 Time historical responses of waveforms in the case where an active sink with Case 2 control law is placed at the center of a beam (around the 3rd vibration modal frequency)

solution to the Euler-Bernoulli equation, that is,

$$-\xi(x) = C_1 \exp[-jkx] + C_2 \exp[-kx] + C_3 \exp[jkx] + C_4 \exp[kx], \quad (4)$$

where ξ , x , k and j are the flexural displacement, distance from the left end of the beam, wave number and imaginary unit, respectively.

As shown in Fig. 7, the gain η of the complex reflection coefficient is 1.0 at almost all of the phase θ except at around $\theta=180.0$. This means that the same amount of reflection waves as incident waves occurs in this range. Therefore, due to the interference of the incident waves and the reflection waves, standing waves are generated, with the result that the 1st vibration mode is excited. Figure 9 shows the relationships between the vibration intensity ratio, the amplitude ratio, the reflection coefficient and the phase θ at around $\theta=180.0$. At $\theta=179.94$, extremely close to $\theta=180.0$, the gain η of the complex reflection coefficient becomes infinity. At $\theta=180.06$, the gain η becomes zero this time. The reflection coefficient becoming zero means that waves flow into the active-sink point completely. On the other hand, at $\eta=\infty$, the left-end point of the beam generates the waves, so we call this situation an active source. In this case, the point where the exciting force acts is regarded as the active sink.

As is obvious from Fig. 8, the amplitude ratio w is 2.0 at $\theta=0$. This is because the control force f_d in this case plays the role of augmenting the vibration level instead of suppressing the vibration. In such a case, standing waves causing excitation of the vibration mode also become maximum. As the phase θ increases, the amplitude w decreases because the amplitude of the standing wave also lessens. However, in the range from $\theta=0$ to 120 and from 120 to 240, the

vibration level is larger than that of the case without control. In addition, since the gain η of the complex reflection coefficient is still 1.0 in this region, the amounts of incident waves and reflection waves are still the same, and hence the vibration levels decrease while the envelope shapes retain the mode shape.

The range from $\theta=120$ to 240 is what we call the suppression area; that is, w is less than 1.0. However, the gain η retains the value of 1.0 except at $\theta=180.0$; therefore, the envelope shape of the waveforms is still almost the same as that of the mode shape. As the left end of the beam tends to possess the characteristics of an active source, the gain η of the complex reflection coefficient eventually become more than 1.0, and an imbalance between the incident waves and the reflection waves occurs. As a result, the shape of the standing waves causing the excitation of the vibration mode changes, and the excitation rate of the vibration mode decreases. In this case, the reflection waves begin to dominate the beam, and the amplitude ratio w decreases suddenly. Furthermore, in the vicinity of the active source, only one-way waves exist, with the result that the vibration mode becomes inactive.

As shown in Fig. 9, it is evident that the gain η becomes infinity at the phase θ giving the active source. If θ becomes larger than that, η begins to decrease and becomes 1.0 at $\theta=180.0$. This means that the amounts of reflection waves and incident waves become equal, and the standing waves become dominant again. However, the vibration mode is not excited this time, and the amplitude ratio w reaches the minimum. This phenomenon seems to contradict the previous explanation of the generation mechanism of the vibration modes. The reason for this is to be shown in the following section.

If the phase θ further increases, the phenomenon becomes completely opposite to the former one. In the case of an active sink, incident waves prevail over the beam and flow toward the left end from the right end of the beam this time.

Figure 10 shows the envelopes of waveforms in the vicinity of $\theta=180.0$. In the case of no control, the maximum amplitude is 58.4 mm. At $\theta=179.5$, the maximum amplitude is suppressed to only 513 μm , and the nodes become swollen. This is the moment when the standing waves change to progressive waves. If we increase the amount of the phase by only 0.44 degrees, the left end of the beam comes to possess all of the characteristics of the active source. In this case, the progressive waves thoroughly dominate the beam, so that the maximum amplitude is suppressed surprisingly, to only 82.5 μm by such a slight change of the phase. Time historical responses shown in Fig. 11 clearly denote the transition of the vibration patterns.

Fig. 9 Vibration ratio, amplitude ratio and gain of complex reflection coefficient versus phase difference at around $\theta=180$

In the case of no control, the Moire fringe runs vertically, meaning that the standing waves are dominant over the beam. In the case of an active source, however, the Moire fringe moves slantwise from left to right and vice versa in the case of an active sink.

3.2 Progressive-wave suppression type and standing-wave suppression type

As shown in Fig. 9, the best control effect for suppressing the vibration level is obtained at $\theta=180$ degrees, which is the exact midpoint between the active source and active sink. Although the standing waves are dominant, the amplitude ratio w is 30 percent smaller than that in the case of the active sink or active source. The vibration intensity ratio is also plotted in the figure. Even from the value of the vibration intensity ratio at $\theta=180$, which is null, we can tell that the standing waves are completely dominant at $\theta=180$. The readers must think it strange that vibration modes never become excited in spite of the existence of the standing waves.

Before we address this problem, we should recall the conditions for vibration excitation. It was reported in a previous paper⁽⁹⁾ that the vibration mode is

excited when the standing waves consisting of incident waves and reflected waves are completely tuned to the vibration mode. In order for complete tuning to hold true, the following two conditions must be satisfied.

- (1) A beam must be excited at the modal frequency.
- (2) Mode shape must be one and the same with the standing wave shape.

In the case of $\theta=180$ degrees, condition (2) is not satisfied. As is obvious from Fig. 10, the mode shape shown at the top of the left-hand-side figure is inconsistent with the standing-wave shape depicted at the top of the right-hand-side figure. Then, even if perfect standing waves dominate the beam, the vibration modes remain inactive. Therefore, the existence of the standing waves, i.e., the existence of the reflected waves, is a necessary condition for vibration modes to be excited, and not a sufficient condition.

As explained in the preceding section, there are two types of vibration suppression. One is performed through use of the active-sink or active-source method, and the other is performed by making the standing waves dominant, which is inconsistent with the vibration mode shapes. In this paper, we call the former type progressive-wave suppression and the latter, standing-wave suppression.

Figure 12 shows the comparison of the progressive-wave suppression with the standing-wave suppression type at around the 1st mode. At 3 Hz, for instance, the phases for generation of these patterns differ by 36.6 degrees. In the case of the maximum amplitude, the standing-wave type is slightly superior to the progressive type. However, in either case the envelope shapes at around the modal frequency look alike. No nodes are recognized in the progressive patterns, of course, while one node is seen at the center of the beam in the case of the standing-wave patterns.

As seen from Fig. 12, standing-wave suppression

Fig. 10 Envelopes of waveforms under various conditions

Fig. 11 Time historical responses of waveforms in terms of progressive-wave suppression and standing-wave suppression at around the 1st vibration modal frequency

Fig. 12 Envelopes of waveforms in terms of progressive-wave suppression and standing-wave suppression at around the 1st vibration modal frequency

is achieved by setting the phase at 180 degrees even if the exciting frequency is different from the modal frequency. So, let us further investigate the characteristics of standing-wave suppression. Figure 13 shows envelopes of waveforms obtained by changing the exciting frequency from 3.3 Hz, less than the 1st mode, to 13 Hz, larger than the 2nd modal frequency. Solid lines and dotted lines show the difference in the half-period of the exciting force acting at the right end of the beam. The control force is assumed to act on the left-end point of the beam. Then, in order to maintain the out-of-phase relationship between the control force and the exciting force, this figure shows that the phase θ must be changed from 180 degrees to 0 at 9 Hz, the notch frequency in the compliance diagram.

By adjusting the phase of the control force, we obtain the driving point compliance as depicted in an alternate-long-and-short-dash line shown in Fig. 14. A broken line shows the gain of the compliance obtained by the active-sink method. The gain

Fig. 13 Envelopes of waveforms whose modal frequencies are varied from the 1st mode to the 2nd mode

Fig. 14 Driving point compliance in terms of progressive suppression and standing-wave suppression

decreases monotonously with the slope of 30 dB/dec without exciting any vibration modes. When we compare these gain characteristics with those of standing-wave suppression, it is evident that standing-wave suppression is superior to the progressive-wave one at around the notch frequencies which never appear in the progressive type.

Figure 15 shows the comparison between the envelopes of waves in the case of vibration modes and standing-wave suppression. Note that the scale of the vertical axis changes from mm to μm . It is significant that the wave envelope obtained by standing-wave suppression at the n -th mode is quite similar to that of no control at the $(n-1)$ -th vibration mode. Hence, the envelope of the standing-wave type at the 1st mode corresponds to the rigid mode (conical mode).

3.3 Phase distribution of complex reflection coefficient

In the case of progressive-wave suppression, either incident waves or reflected waves appear, and hence we cannot define the phase difference ϕ between these waves. In other cases, however, the phase difference is defined in the following.

$$\phi = \angle \frac{C_1 \exp[-jkx]}{C_3 \exp[jkx]} \quad (5)$$

When we compare Eq. (5) with Eq. (2), the phase difference turns out to be the phase of the complex reflection coefficient. Figure 16 shows the phase distribution over the beam at each vibration modal frequency. From these figures, we can see the

Fig. 15 Wave envelopes of waveforms in case of non-control and standing-wave suppression

significant characteristics; that is, there are only two patterns in the phase distribution at each modal frequency. Namely, the first category is seen in the region between the active source and the active-sink, including standing-wave suppression. For the sake of convenience, we call this phase distribution pattern Case I. Case II is then the remainder of the Case I region.

Next, let us investigate the phase difference ϕ between the incident wave and the reflected wave at the 1st modal frequency. In Case I, at the left end of the beam a jumping phenomenon⁽¹⁰⁾ of -90 degrees in phase difference appears. At the right end of the beam, the same phenomenon occurs. The phase difference at the center of the beam is 0.0 degree, corresponding to the loop in the vibration shape. By contrast, at the points 46 cm from both ends of the beam, the phase difference is out of phase, which is thought to coincide with the nodes in the envelope of waves. However, there is a difference of 15 cm in the actual position of the nodes, and this is due to the influence of near fields produced at the boundary conditions, free-free ends in this case. If we calculate the influence rate of the near field, we find that it is 45.5 percent and 9.4 percent at the position 46 cm from the beam end and the center of the beam, respectively. Therefore, even if the phase difference between the incident waves and the reflected waves is 180 degrees, perfect cancellation does not occur at the point 46 cm from the beam end due to the influence of the near fields. On the other hand, although the influence rate is 9.4 percent at the

center of the beam, the loop position occurs at the aforementioned point. In this case, the reason is simple; namely the influence rates from both ends of the beam are exactly the same.

In Case II, the phase distribution completely differs from that of Case I by 180 degrees. In this case, at the center of the beam, the phase is 180 , and hence the node is generated. Unlike Case I, the gain η of the complex reflection coefficient in Case II varies violently, as shown in Fig. 7, so the vibration shape is greatly influenced by its fluctuation. As the gain η approaches 1.0 from infinity, a notch becomes deep, and finally, the standing-wave suppression pattern is produced at $\phi=180$ degrees.

4. Conclusions

The purpose of this paper was to realize experimentally the active sink, which makes all the vibration modes inactive. It was also the purpose of this paper to clarify the fundamental characteristics of the active-sink system. The results of this investigation can be summarized as follows:

1. In order to verify the existence of the active sink, first of all, waveforms propagating along the beam must be observed. For this purpose, a wave visualization system was constructed.

2. The realization of the active sink at the left end of the beam was successful, and waves travelling from the right end of the beam, where an exciting force acts, to the active-sink point were observed. In this case, even if the beam was excited at its modal frequency, the vibration mode was never excited. The maximum amplitude of the beam displacement was suppressed to only 2.6 percent of that in the case without control.

3. The position of the active sink was changed to the center of the beam, and two kinds of control laws were applied. In the first case, it was attempted to make the progressive waves dominant over the right-hand-side element of the beam, where an exciting force existed. The displacement amplitude at the exciting point was suppressed to 2.8 percent that of the one without control. In the second case, the active sink worked on the left-hand element of the beam, where no exciting force existed. In this case, we have succeeded in realizing the vibration-free state in that area. In other words, there were no reflected waves or incident waves in this element.

4. By changing the phase of the control force, the control effect of the active sink was further investigated. As a result, it turned out that there was another specific point other than the active sink, and we call this an active source. The control effect of the active source was exactly the same as that of the

Fig. 16 Phase distribution of complex reflected waves at each modal frequency

active sink except for the flow direction of the vibration intensity.

5. In addition to progressive-wave suppression methods such as the active-sink method and active-source method, standing-wave suppression was also presented. Unlike the progressive type, the reflection coefficient is 1.0 in this case. This method has its principle in the generation of standing waves, which is inconsistent with the vibration modes.

6. There are only two kinds of phase distribution patterns of the complex reflection coefficient. On the basis of these patterns, the generation mechanism of the vibration patterns was clarified.

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