Self-Excited Oscillations of a Circular Disk Rotating in Air*

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Self-excited oscillations are often observed in a circular disk when it is rotated in air. Theoretical as well as experimental studies are conducted on these oscillations. As a preparatory study, free vibrations of the disk are discussed theoretically without taking into account the effect of air. Then, utilizing the results of this preparatory study, and taking into account the effect of air, free vibrations of the disk are studied. It is shown that self-excited oscillations can occur at a rotating speed higher than a certain value in the form of a backward traveling wave. Experiments confirm the theoretical analysis.

Key Words: Self-Excited Oscillations, Vibration of Continuous System, Rotating Circular Disk, Traveling Wave, Effect of Air

1. Introduction

Rotating disks are used in various machines such as floppy disks, hard disks, turbines, gyroscopes and circular sawblades. The problems of vibrations of rotating disks are important in improving these machines, so many investigators have dealt with these problems. Lamb and Southwell⁽¹⁾, Southwell⁽²⁾, Eversman and Dodson⁽³⁾, and Barasch and Chen⁽⁴⁾ obtained theoretically the natural frequencies of a rotating disk. Nowinski⁽⁵⁾, Advani⁽⁶⁾, Advani and Bulkeley⁽⁷⁾, and Bulkeley⁽⁸⁾ treated nonlinear vibrations for large amplitudes. Tobias and Arnold⁽⁹⁾, Krauter and Bulkeley⁽¹⁰⁾, and Macbain et al.⁽¹¹⁾ conducted experimental analysis. Iwan and Moeller⁽¹²⁾, Benson and Bogy⁽¹³⁾, and Honda et al.⁽¹⁴⁾ treated rotating disks supported at, or excited by, a point fixed in space. Ono

et al.^{(16),(17)}, Ono and Maeno⁽¹⁸⁾, and Hosaka and Nishida⁽¹⁹⁾ treated vibrations of floppy disks, and Matsuhisa and Sato⁽²⁰⁾, Chonan et al.⁽²¹⁾, and Hutton et al.⁽²²⁾ treated vibrations of rotating circular saw blades.

Self-excited oscillations are often observed in a disk rotating in air at high speed. But, to the author's knowledge, no theoretical investigations treating such vibrations have been published. In this paper, theoretical as well as experimental studies are conducted to explain the occurrence of self-excited oscillations of a disk.

Firstly, as a preparatory study, free vibrations of a rotating disk, without taking into account the effect of air, are considered. Then, vibrations of a rotating disk taking into account the effect of air are analyzed. In the analysis, the effect of air is modeled by a simple mathematical model. It will be shown that this simplified model can well explain the occurrence of self-excited oscillations. It also explains the fact that the self-excited oscillations occur at a rotating speed higher than a certain value. Lastly, experiments are conducted using a steel circular disk, and the validity of the theoretical analysis is confirmed.

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2. Theoretical Analysis

2.1 Analysis neglecting the effect of air

A circular disk with thickness h , outer radius a , and inner radius b , rotates with angular velocity ω . Firstly, without taking into account the effect of air, analysis of vibrations of the disk will be conducted. To this end, we fix the polar coordinate system (r, θ) in the middle plane of the disk in its equilibrium state, with the origin coinciding with the center of the disk. Equations of motion expressed with respect to this coordinate system are as follows^{(3),(4)}:

$$\rho h \frac{\partial^2 w}{\partial t^2} + D \nabla^4 w - \omega^2 L(w) = 0, \quad (1)$$

where

$$\begin{aligned} D &= \frac{Eh^3}{12(1-\nu^2)} \\ L(w) &= \frac{1}{r} \frac{\partial}{\partial r} \left(r N_r \frac{\partial w}{\partial r} \right) \\ &\quad + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{1}{r} N_\theta \frac{\partial w}{\partial \theta} \right) \\ N_r &= \frac{1}{8} \rho h \left\{ -(3+\nu)r^2 + c_1 + c_2 \frac{1}{r^2} \right\} \\ N_\theta &= \frac{1}{8} \rho h \left\{ -(1+3\nu)r^2 + c_1 - c_2 \frac{1}{r^2} \right\} \\ c_1 &= \frac{(1+\nu)(3+\nu)a^4 + (1-\nu^2)b^4}{(1-\nu)b^2 + (1+\nu)a^2} \\ c_2 &= a^2 b^2 \frac{(1-\nu)(3+\nu)a^2 - (1-\nu^2)b^2}{(1-\nu)b^2 + (1+\nu)a^2} \end{aligned}$$

In this equation, E , ρ and ν are Young's modulus, the density and Poisson's ratio, respectively. Boundary conditions for the disk fixed at the inner edge ($r=b$) and free at the outer edge ($r=a$) are expressed as⁽⁴⁾

$$w=0, \frac{\partial w}{\partial r}=0 \quad (2)$$

and as

$$\left\{ \begin{aligned} &\left\{ \frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r} \frac{\partial w}{\partial r} + \frac{\nu}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right\} = 0 \\ &\left\{ \frac{\partial}{\partial r} (\nabla^2 w) + \frac{1}{r} (1-\nu) \right. \\ &\quad \times \left. \frac{\partial}{\partial r} \left(\frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) \right\} = 0 \end{aligned} \right. \quad (3)$$

Introducing nondimensional quantities

$$\left\{ \begin{aligned} r' &= r/a, t' = t/a^2 \sqrt{\rho h/D} \\ w' &= w \sqrt{12(1-\nu^2)}/h \\ \omega' &= \omega a^2 \sqrt{\rho h/D}, x = b/a \end{aligned} \right. \quad (4)$$

and rewriting Eq.(1) in terms of these quantities, yields, after omitting primes, the following equation:

$$\frac{\partial^2 w}{\partial t^2} + \nabla^4 w - \omega^2 L(w) = 0. \quad (5)$$

The solution to Eq.(5) is sought in the following form:

$$w = \Phi(r) \cos m\theta e^{st}. \quad (6)$$

Substituting this into Eq.(5) yields

$$s^2 \Phi + \nabla^4 \Phi + \omega^2 L(\Phi) = 0. \quad (7)$$

Substituting Eq.(6) into boundary conditions (2) and (3) yields

$$\begin{aligned} &\Phi \Big|_{r=x} = 0, \Phi' \Big|_{r=x} = 0 \\ &\left\{ \Phi'' + \frac{\nu}{r} \Phi' - m^2 \frac{\nu}{r} \Phi \right\} \Big|_{r=1} = 0 \\ &\left\{ \frac{d}{dr} \left(\Phi'' + \frac{1}{r} \Phi' - \frac{m^2}{r^2} \Phi \right) \right. \\ &\quad \left. - (1-\nu) \frac{1}{r} \frac{d}{dr} \left(\frac{m^2}{r^2} \Phi \right) \right\} \Big|_{r=1} = 0, \end{aligned} \quad (8)$$

To solve Eq.(7) by Galerkin's method, the solution is put in the following form:

$$\Phi(r) = \sum_{k=1}^{\infty} a_k u_k(r), \quad (9)$$

where, as the testing functions $u_k(r)$, the eigenfunctions of a disk with the same boundary conditions as above but not rotating are used. Thus, $u_k(r)$ are given in terms of Bessel functions and modified Bessel functions of order m in the following form:

$$\begin{aligned} u_k(r) &= C_1 J_m(\mu_k r) + C_2 I_m(\mu_k r) \\ &\quad + C_3 Y_m(\mu_k r) + C_4 K_m(\mu_k r). \end{aligned} \quad (10)$$

In this expression, μ_k and the ratio of $C_1 : C_2 : C_3 : C_4$ are determined by boundary conditions (8). The magnitude of C_1 , C_2 , C_3 and C_4 is determined by a normalizing condition. Thus, u_k 's constitute a normalized orthogonal set of functions that satisfy

$$\mu_k^4 u_k = \nabla^4 u_k. \quad (11)$$

Substituting Eq.(10) into Eq.(7), multiplying ru_n , and integrating over the region of the disk, yields, due to the orthogonality of u_n , the following equation:

$$s^2 a_n + \mu_n^4 a_n - \omega^2 \sum_{k=1}^{\infty} a_k \int_x^1 L(u_k) r u_n dr = 0. \quad (12)$$

Retaining the first N terms in the summation of Eq.(12) and expressing the resulting equation in matrix form, yields

$$[A]\{a\} = s^2 \{a\}, \quad (13)$$

where

$$\{a\} = \{a_1, a_2, a_3, \dots, a_N\}^T$$

$$A_{ij} = -\delta_{ij} \mu_i^4 + \omega^2 \int_x^1 L(u_j) r u_i dr.$$

Solving the eigenvalue problem of Eq.(13) determines the natural frequencies from its eigenvalues and the modal function $\Phi(r)$ from its modal vector $\{a\}$.

2.2 Analysis taking into account the effect of air

As will be expected, the previous analysis cannot explain the occurrence of self-excited oscillations. Here the influence of air is taken into account. For this purpose, it is assumed that the force produced by air can be expressed by two terms, one being an ordinary viscous damping force and the other a lift force whose magnitude is proportional to the speed as well as the angular displacement of the disk. Denoting the coefficients of these terms by C_D and C_L , respectively, and adding those terms into Eq.(5), yields

$$\frac{\partial^2 w}{\partial t^2} + c_D \frac{\partial w}{\partial t} + \nabla^4 w - \omega^2 L(w) - c_L m \omega \frac{\partial w}{\partial \theta} = 0. \quad (14)$$

The solution of Eq.(14) is sought in the following form :

$$w = \{\Phi(r)\cos m\theta + \tilde{\Phi}(r)\sin m\theta\}e^{st}. \quad (15)$$

Substituting this into Eq.(14) yields

$$\begin{aligned} s^2 \Phi + c_{DS} \Phi + \nabla^4 \Phi - \omega^2 L(\Phi) - c_L m \omega \tilde{\Phi} &= 0 \\ s^2 \tilde{\Phi} + c_{DS} \tilde{\Phi} + \nabla^4 \tilde{\Phi} - \omega^2 L(\tilde{\Phi}) + c_L m \omega \Phi &= 0. \end{aligned} \quad (16)$$

Putting the solution of Eq.(16) in the form

$$\Phi(r) = \sum_{k=1}^{\infty} \alpha_k u_k(r), \quad \tilde{\Phi}(r) = \sum_{k=1}^{\infty} \tilde{\alpha}_k u_k(r), \quad (17)$$

and using Galerkin's method yields

$$\left. \begin{aligned} s^2 \alpha_n + \mu_n^4 \alpha_n - \omega^2 \sum_{k=1}^{\infty} \alpha_k \int_K L(u_k) r u_n dr \\ + c_{DS} \alpha_n - c_L m \omega \tilde{\alpha}_n &= 0 \\ s^2 \tilde{\alpha}_n + \mu_n^4 \tilde{\alpha}_n - \omega^2 \sum_{k=1}^{\infty} \tilde{\alpha}_k \int_K L(u_k) r u_n dr \\ + c_{DS} \tilde{\alpha}_n + c_L m \omega \alpha_n &= 0 \end{aligned} \right\}. \quad (18)$$

Retaining the first N terms in the summation of Eq. (18), and expressing the resulting equations in matrix form, yields

$$\begin{aligned} [A]\{\alpha\} + c_L m \omega \{\tilde{\alpha}\} &= (s^2 + c_{DS})\{\alpha\} \\ - c_L m \omega \{\alpha\} + [A]\{\tilde{\alpha}\} &= (s^2 + c_{DS})\{\tilde{\alpha}\}, \end{aligned} \quad (19)$$

where

$$\begin{aligned} \{\alpha\} &= \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_N\}^T \\ \{\tilde{\alpha}\} &= \{\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \dots, \tilde{\alpha}_N\}^T \end{aligned}$$

and matrix [A] is the same as in Eq.(13).

Solving the eigenvalue problem of Eq.(19) determines the natural frequencies and the modal functions for the case in which the effect of air is taken into account. However, to elucidate the characteristics of occurring vibrations, instead of solving Eq.(18) directly, another method is pursued. It is noted that the eigenvalue s_0 and the modal vector $\{\alpha_0\}$ without the effect of air, satisfy the following equation:

$$[A]\{\alpha_0\} = s_0^2 \{\alpha_0\}. \quad (20)$$

For a chosen solution s_0 , we determine s which satisfies the equation

$$s^2 + c_{DS} - s_0^2 = \pm i c_L m \omega, \quad (21)$$

where both the positive and negative signs ($\pm$) are used. It is easy to show that the obtained s as well as

$$\begin{aligned} \{\alpha\} &= \{\alpha_0\} \\ \{\tilde{\alpha}\} &= \pm i \{\alpha_0\} \end{aligned} \quad (22)$$

satisfies Eq.(19), due to Eqs.(20) and (21). Thus the eigenvalues of Eq.(19) are given by Eq.(21) and the modal vectors by Eq.(22).

To observe the effect of air, the solutions of Eqs. (21) and (22) are examined. The solutions s_1, s_2 of Eq. (21) for the positive sign can be put in the following form :

$$\begin{aligned} s_1 &= \lambda_b + ip \\ s_2 &= \lambda_f - ip \end{aligned} \quad (23)$$

It is to be noted that the absolute values of imaginary parts of s_1 and s_2 are equal. Denoting by $\Phi_1, \tilde{\Phi}_1, \Phi_2$ and $\tilde{\Phi}_2$ the modal functions corresponding to the eigenvalues s_1 , and s_2 , it follows from Eq.(22) that

$$\begin{aligned} \Phi_1 &= \Phi_0, \quad \tilde{\Phi}_1 = i\Phi_0 \\ \Phi_2 &= \Phi_0, \quad \tilde{\Phi}_2 = i\Phi_0 \end{aligned} \quad (24)$$

where Φ_0 is the modal function without the effect of air. Similarly, the solutions s_3, s_4 of Eq.(21) for the negative sign can be put in the form

$$\begin{aligned} s_3 &= \lambda_b - ip \\ s_4 &= \lambda_f + ip \end{aligned} \quad (25)$$

and the modal functions $\Phi_3, \tilde{\Phi}_3, \Phi_4$ and $\tilde{\Phi}_4$ corresponding to the eigenvalues s_3, s_4 are as follows:

$$\begin{aligned} \Phi_3 &= \Phi_0, \quad \tilde{\Phi}_3 = -i\Phi_0 \\ \Phi_4 &= \Phi_0, \quad \tilde{\Phi}_4 = -i\Phi_0 \end{aligned} \quad (26)$$

The deflection of the disk obtained from the first and the third eigenvalues and modal functions becomes

$$\begin{aligned} w &= (\Phi_1 \cos m\theta + \tilde{\Phi}_1 \sin m\theta) e^{s_1 t} \\ &+ (\Phi_3 \cos m\theta + \tilde{\Phi}_3 \sin m\theta) e^{s_3 t} \\ &= \Phi_0 e^{\lambda_b t} \cos(pt + m\theta). \end{aligned} \quad (27)$$

This implies the occurrence of the backward traveling wave with damping rate λ_b and frequency p . On other hand, the deflection of the disk obtained from the second and fourth eigenvalues and modal functions becomes

$$\begin{aligned} w &= (\Phi_2 \cos m\theta + \tilde{\Phi}_2 \sin m\theta) e^{s_2 t} \\ &+ (\Phi_4 \cos m\theta + \tilde{\Phi}_4 \sin m\theta) e^{s_4 t} \\ &= \Phi_0 e^{\lambda_f t} \cos(pt - m\theta). \end{aligned} \quad (28)$$

This implies the occurrence of the forward traveling wave with damping rate λ_f and frequency p . Consequently, the oscillations occurring in the disk appear in the form of forward and backward traveling waves.

To investigate whether these traveling waves converge or diverge, the signs of damping rate λ_f and λ_b are examined. To this end, Eq.(23) is substituted into Eq.(21) and the real and imaginary parts are separated. The result is

$$\begin{aligned} \lambda_b &= + \frac{1}{2} c_L m \frac{\omega}{p} - \frac{1}{2} c_D \\ \lambda_f &= - \frac{1}{2} c_L m \frac{\omega}{p} - \frac{1}{2} c_D \end{aligned} \quad (29)$$

From this it is seen that λ_f is always negative and that the forward traveling wave converges with time. On other hand, λ_b can be positive depending on values of c_D, c_L and ω . If λ_b is positive, the backward traveling wave diverges with time, and a self-excited oscillation occurs. To obtain the critical value between convergence and divergence, put $\lambda_b = 0$ in the first equation of Eq.(29). It follows that

$$p = m \frac{c_L}{c_D} \omega. \quad (30)$$

Thus, the backward traveling wave can be of self-excited nature in the region where the curve of the natural frequency versus angular velocity ω falls below the line given by Eq.(30).

2.3 Numerical examples

Here, numerical examples are given.

Firstly, the natural frequencies without the effect of air are shown in Fig.1. In the figure, p denotes the natural frequency of the primary mode with two nodal diameters. It is seen that the natural frequency increases hyperbolically with increase of angular velocity ω . This tendency agrees with that given in refs. 1 and 2.

Secondly, the natural frequencies p and damping rate λ_b with the effect of air, obtained from Eq.(21), are shown in Fig. 2 and Fig. 3. In the figures, the line determined by Eq.(30) is denoted a broken line.

If c_L is small as compared with c_D , as in the case

Fig. 1 Theoretical natural frequency p of the rotating disk obtained theoretically

Fig. 2 Theoretical natural frequency p and damping rate λ_b of the disk rotating in air

shown in Fig. 2, the curve of p falls for any ω above the broken line, and λ_b is negative. Therefore, self-excited oscillations do not occur. But if c_L is large enough, as in the case shown in Fig. 3, the curve of p intersects with the broken line at a certain ω , and λ_b becomes positive for ω higher than the value of the point of intersection. Thus, self-excited oscillations occur for a rotating velocity higher than a certain value.

It is noted in passing, that the natural frequencies p with and without the effect of air differ only slightly.

3. Experimental Analysis

3.1 Experimental method

To check the validity of the theoretical analysis, an experimental analysis was conducted using a thin steel circular disk. A schematic diagram of the experimental apparatus used in this study is shown in Fig. 4. A circular disk with thickness h and radius a was clamped at its center by a chuck with radius b , and was rotated by a motor. To measure its vibrations, four sensors were placed at intervals of 90 degrees apart in the circumferential direction. To extract data concerning circumstantial modes, the signals obtained

Fig. 3 Theoretical natural frequency p and damping rate λ_b of the disk rotating in air

Fig. 4 Schematic diagram of the experimental apparatus

Fig. 5 Spectrum of the free vibration of the rotating disk obtained experimentally ($\omega=12$ cycle/s)

Fig. 6 Experimental natural frequency of the rotating disk

from the sensors were operated by operating amplifiers.

The dimensions of the disk are as follows :

$$a=225 \text{ mm}$$

$$b=49 \text{ mm}$$

$$h=0.5 \text{ mm}$$

3.2 Experimental results

In the above-mentioned experimental device, to measure the natural frequencies of the disk, a shock was given to the disk when it was rotating. An example of the spectrum of the vibrations measured by a FFT analyzer is shown in Fig.5. It is seen that, because the vibrations of the rotating disk were measured by sensors fixed in space, the observed frequencies were shifted by $\pm m\omega$ (m is the number of nodal diameter). The vibration with the frequency shifted by $+m\omega$ indicates a forward traveling wave, and the vibration with the frequency shifted by $-m\omega$ is a backward traveling wave. The vibrations appearing with integral multiples of frequency ω are apparent vibrations brought about by the rotation of an inevitable initial deformation of the disk.

By varying the speed of rotation the natural frequencies were measured experimentally. The

Fig. 7 Spectrum of the self-excited vibration of the rotating disk obtained experimentally ($\omega=35$ cycle/s)

results are shown in Fig. 6. In this figure, the symbol $\circ$ denotes the frequencies obtained directly from the sensors, and the symbol $\times$ the reduced frequencies with respect to the rotating coordinate system, so that a comparison can be made with the theoretical results. It is seen in Fig. 6 that the natural frequency increases with increase in the rotating speed. The higher the rotating speed is, the more slowly the backward traveling waves dampen. At a speed higher than a certain value, a self-excited oscillation occurs even when no shock is given. In Fig. 6, the case in which the self-excited oscillation occurs is denoted by the symbol $\bullet$. In the experiment the amplitude of the vibration did not become infinite, as is predicted by the theory. This may be due to nonlinearity of the disk and/or of air. In Fig.7 is given an example of the spectrum of the self-excited oscillation occurring in the disk. From the results above, it can be concluded that the experimental results qualitatively confirm the validity of the theory.

In the experiment, two slightly different frequencies were observed at a relatively low rotating speed. This may be because the degenerate modes with the same natural frequency are separated into two different ones due to an initial deformation or an anisotropy of the disk.

4. Conclusions

Vibrations of a circular disk rotating in air have been considered. The obtained results can be summarized as follows :

(1) As a preparatory analysis, the free vibrations of the disk without the effect of air were analyzed.

(2) The vibrations of the rotating disk were analyzed, taking into account the effect of air. The forces produced by air were considered to consist of two terms, one being an ordinary viscous damping, and the other a lift force whose magnitude is proportional to the speed as well as the angular displacement

of the disk.

(3) The vibrations occurring in the disk appear, due to the effect of air, in the form of forward and backward traveling waves.

(4) The forward traveling wave always decays, but the backward traveling wave can become of a self-excited nature at a rotating speed higher than a certain value.

Finally, an experimental analysis has been conducted and the validity of the theoretical analysis has been confirmed.

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