Boundary Element Method Applied to Simulation of Active Noise Control in Ducts*

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This paper is concerned with a computer simulation of active noise control using the boundary element method for analyzing three-dimensional acoustic field problems. The active noise control under consideration is reduced to an optimum problem to determine the optimal set of parameters defining the sound pressure of a secondary source to be attached. The optimal set of parameters is selected through the conventional optimization procedure so that the sum of the sound pressure values measured at the duct exit is minimal. Computation is carried out for typical examples in which the duct is embodied in the infinite plane and the noise through the duct is radiated to the semi-infinite acoustic field, whereby the potential usefulness of the computer simulation system developed is demonstrated.

Key Words: Sound, Boundary Element Method, Noise, Inverse Problem, Vibration Control

1. Introduction

Control and reduction of noise are of much importance in engineering and a number of investigations are available in the literature. So called active noise control, which reduces noise by means of mutual interference of sound waves, has been attracting increasing attention⁽¹⁾⁻⁽⁷⁾. In particular, noise reduction in ducts is very important in conjunction with the increasing need for low noise in air-conditioning apparatuses. This subject has so far been studied through experimental approaches.

The authors previously treated active noise control as an acoustic inverse problem and proposed a procedure in which boundary element method was combined with the standard optimization technique. A general-purpose computer simulation system has been developed for the active control of noise⁽⁸⁾. In this study, the active control of duct noise is considered and the usefulness of the present computer simulation system is demonstrated by computing several examples.

Computer simulation is carried out for various values of frequency in pure sound, and finally the use of the results obtained for active cancelling of real duct noise is demonstrated. It is revealed that the property of active cancellation varies according to the frequency of the primary noise source. The optimal position of the secondary source to be attached is also discussed for efficient cancellation of duct noise.

2. Application of BEM to Acoustic Direct Problems

Under the conditions of steady-state vibration with small amplitude, the acoustic problem is governed by the Helmholtz equation. The corresponding boundary integral equation can be given as follows^{(9),(10)}:

$$\begin{aligned} c(y)p(y) + \int_S q^*(x, y)p(x)dS(x) \\ = -j\rho\omega \int_S p^*(x, y)v(x)dS(x), \end{aligned} \quad (1)$$

where $j = \sqrt{-1}$, ω is the angular frequency and ρ is the mass density of the medium. The $c(y)$ is the coefficient dependent only on the geometrical property of the boundary surface at point y . When the boundary is smooth, $c(y)$ is equal to 1/2. In addition, $p(x)$

* Received 28th November, 1991. Paper No. 90-0308A

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is the sound pressure, and $v(x)$ is the particle velocity, while $p^*(x, y)$ and $q^*(x, y)$ are the well-known fundamental solutions for the Helmholtz equation.

After calculating the sound pressure and the particle velocity on the boundary surface, we can compute the sound pressure at an internal point y by using the following equation:

$$p(y) = - \int_S q^*(x, y) p(x) dS(x) - j\rho\omega \int_S p^*(x, y) v(x) dS(x). \quad (2)$$

Discretization of the boundary integral equation (1) and the integral equation (2) by means of the standard boundary element method can lead to the following system of equations expressed in matrix form⁽¹¹⁾:

$$[H]\{p\}_S = [G]\{v\}_S \quad (3)$$

$$p_V = -[A]\{p\}_S + [B]\{v\}_S, \quad (4)$$

where the coefficient matrices $[H]$, $[G]$, $[A]$ and $[B]$ can be calculated from the fundamental solutions, and hence they are all known. The subscripts S and V are used to denote the values on the boundary and in the domain, respectively. Solving Eq. (3) under the given boundary conditions, we can obtain the nodal values of sound pressure p and particle velocity v on the boundary, and then from Eq. (4) we compute the sound pressure at an arbitrary internal point in the acoustic field.

3. Application of Optimization Technique

In this study, the active control of duct noise is treated as an optimum problem in which the boundary element method is used as the central computation technique. The objective function is defined as the sum of absolute values of sound pressures at selected measuring points arranged at the duct exit. Then, the optimum set of parameters defining the sound pressure of a secondary noise source is sought so that the

value of the objective function should be minimal. The nondimensional objective function R is defined as

$$R = \frac{1}{N} \sum_{i=1}^N \left\{ \frac{|p_i|}{|p_{0i}|} \right\}, \quad (5)$$

where N is the number of measuring points of sound pressure at the duct exit, and p_{0i} is the sound pressure value computed at the i th measuring point when only the primary noise source vibrates.

The objective function defined as the power emitted from the duct exit may be suitable for noise cancellation. In this paper, however, Eq. (5) is used from the viewpoint of sound pressure measurement.

As the convergence criterion of iterative computation, we use either of the following two expressions:

$$|R^k - R^{k-1}| < \varepsilon, \quad \sum_{i=1}^M |x_i^k - x_i^{k-1}| < \eta, \quad (6)$$

where k is the number of iterations, and M the number of parameters, while ε and η are the given tolerance limits.

In this study, the conjugate gradient method^{(12),(13)} is employed as an optimization technique. The main flow of the present solution procedure is illustrated in Fig. 1.

4. Numerical Results and Discussion

We now show the numerical results obtained by carrying out computer simulation of active noise control in a duct by using the computational software developed in this study on the basis of the proposed solution procedure discussed in the previous sections. It is assumed that a duct with a square cross section is embodied in the infinite plane and the noise through the duct is radiated to the semi-infinite three-dimensional acoustic field, as shown in Fig. 2. We attempt to cancel this noise at the duct exit by attaching a secondary noise source along the center line of the duct bottom plate.

It is assumed that both the primary and secondary noise sources vibrate with the same frequency and are in a steady state. We further assume that all information on the primary noise source is known, and of those concerning the secondary noise source, only the vibrating state is unknown. Under these assumptions

Fig. 1 Main flow of solution procedure

Fig. 2 Duct model

the present example can be reduced into an optimum problem to determine the optimal set of parameters defining the real and imaginary parts of sound pressure at the secondary noise source.

It is assumed that the cross section of the duct is square, and that the secondary noise source is circular with the diameter of 0.1 m and is attached to the center of the duct bottom plate. Because of symmetry, half of the duct surface is discretized into 8-node isoparametric boundary elements as shown in Fig. 3, where 56 elements with 199 nodal points in total are used. Fifteen measuring points are arranged at the duct exit, as shown in Fig. 4.

The material constants and parameters used in the computation are as follows:

velocity of sound: $C_0 = 340$ m/s

mass density of air: $\rho = 1.2$ kg/m³

convergence criterion: $\varepsilon = \eta = 10^{-5}$.

Boundary element analysis of the acoustic field is carried out by means of the method of subregions. Namely, the acoustic field is modelled such that a subregion of the duct is connected with the semi-infinite field at the interface located at the duct exit.

4.1 Application to pure sound

First, to examine the validity of the proposed method, we consider the case where pure sound with arbitrary frequency is radiated from the primary noise source. We show the results obtained by simulation, for example, for the primary source of pure sound of 200 Hz and strength 75 dB of the sound pressure level. In Fig. 5 is shown the convergence

Fig. 3 Boundary element discretization for internal surface of duct with secondary noise source

Fig. 4 Arrangement of measuring points

property with respect to the nondimensional objective function. In this case, convergence is realized after 4 iterations. It can be seen that the sound pressure values at the measuring points can be reduced approximately to 1/1 000 of those obtained without any secondary noise source. This corresponds to the noise reduction of about 60 dB of the sound pressure level.

In Fig. 6, comparison is made with respect to the contours of sound pressure level on the symmetric plane of the duct for the case without any secondary source and also the case with an optimal secondary source. It can be seen from these figures that the secondary source sound cancels out the primary source sound near the duct exit. Figure 7 shows the contours of sound pressure level on the symmetric plane in the semi-infinite acoustic field. It is demonstrated in these figures that once cancellation is achieved at the duct exit, the semi-infinite space is also calm. It can be recognized from these simulation results that the optimal oscillation state of the secondary sound source, which performs effective cancellation, can be obtained in a few iterations using the present system.

4.2 Application to real noise

Figure 8 shows experimental data on the noise emanating from an air blower⁽²⁾. Real noise, in general, consists of a wide frequency range. In order to apply the present computer simulation system to the active cancellation of such real noise, we carry out computer simulation of pure sound in a wide frequency range.

Figure 9 shows the results obtained using the duct model shown in Fig. 3 for the sound pressure levels at the measuring points without secondary source and those with a secondary source under optimal cancellation. It has also been assumed that the secondary source is in a steady state and vibrates with a uniform sound pressure. It can be seen that all the noise considered in this simulation is reduced by 40 dB, and that very effective noise reduction can be obtained

Fig. 5 Convergence with respect to nondimensional objective function

Fig. 6 Contours of sound pressure level

Fig. 7 Contours of sound pressure level on duct symmetric plane in semi-infinite field

through active noise control.

In Fig. 10 is shown the power ratio of noise radiat-

Fig. 8 Typical noise emanating from fan blower

Fig. 9 Results obtained for noise with various values of frequency

ed from the secondary source to that from the primary source in optimal noise cancellation. The power of the sound source is the apparent power which is defined as the scalar product of the averaged sound pressure and particle velocity of the sound source

multiplied by the sound source area. It is clear from Fig. 10 that, for noise with relatively low frequency up to 250 Hz, effective noise cancellation can be realized by supplying the power of the secondary noise source which corresponds to 1/10 of the primary noise source. However, in the range near 340 Hz, power much larger than that of the primary noise source is required to realize optimal noise cancellation. Naturally, it is very reasonable and useful in practical applications if the noise radiated from the primary source can be reduced by minimal power of the secondary source. In this respect, the results obtained in the vicinity of 340 Hz cannot be recommended for practical use.

Figure 11 shows the phase difference between the sound pressures of primary and secondary sources in optimal noise cancellation. The phase difference is 180° in the relatively low range of frequency, while it becomes 0° at 340 Hz.

Now, we consider the reason as to why the results shown in Figs. 10 and 11 are obtained near 340 Hz. Suppose that the noise in the duct consists of a plane wave and the distribution of particle velocity is as shown in Fig. 12. As the wavelength of sound with 340 Hz corresponds to the length of the duct used in the present example, the node of particle velocity coin-

Fig. 10 Power ratio of secondary (P2) to primary (P1) noise sources

Fig. 11 Phase difference between primary and secondary noise sources

cides with the secondary source. In this case, active noise cancellation requires a large amount of power supplied at this secondary source. To verify this fact, computation is also carried out for the case of 340 Hz where the secondary source is displaced to near the primary source.

Figure 13 shows the contours of sound pressure level on the symmetric plane of the duct, obtained for two different positions of the secondary noise source, when optimal noise cancellation is realized at the duct exit. By displacing the secondary source to near the primary source, more effective noise cancellation can be realized; the power of the secondary source is only 30% of the primary source power. It is concluded that active noise cancellation should be performed after a suitable location of the secondary noise source is established through the use of the computer simulation system.

It is interesting to note that by using the present computer simulation system the optimal location as well as the optimal shape of the secondary source can be estimated. Such an application is left for future research work, since it requires further study and also consumes much computation time.

5. Conclusions

In the present paper, the computer simulation system developed for active noise control using the boundary element method together with the optimiza-

Fig. 12 Schematic illustrations of particle velocity distribution

(a) Results obtained using duct model with secondary noise source attached at duct base center

(b) Results obtained using duct model with secondary noise source attached near primary noise source

Fig. 13 Contours of sound pressure level in optimal noise cancellation at 340 Hz

tion technique has been applied to active cancellation of noise propagating in a duct. The system was first applied to pure sound with a certain value of frequency, whereby its potential usefulness was demonstrated. Then, the system was applied to actual noise consisting of a wide frequency range. It was revealed that the duct noise can be dramatically reduced by active cancellation using a secondary noise source at the optimal position supplying a small amount of power. Although this study is restricted to the frequency range 20 Hz to 580 Hz, it can be concluded that the duct noise is completely cancelled out by active noise control.

Parts of this study were financially supported by the Sound Technology Promotion Foundation. The first author wishes to express his cordial thanks to the Foundation.

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