Optimum Structural Design Considering Vibration Control*

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This paper deals with a problem of optimum structural design in which the shape for a beamlike structure that minimizes the cost for vibration control under the condition of volume constancy is determined. This is an approach to the simultaneous optimization of the structure and control functions. The objective structure is modeled by the finite-element method, and the control law obeys the optimal regulator theory. Through numerical examples, the effects of the initial state and weighting matrices on the result are discussed. Moreover, a two-step method for obtaining the optimum shape is proposed.

Key Words: Optimal Design, Vibration Control, Continuum Mechanics, Sensitivity Analysis, Structural Shape Determination, FEM, Optimal Regulator Theory

1. Introduction

It is desirable that machines and structures should be designed so as to minimize weight from the viewpoint of material and transportation costs. As light structures, however, have comparatively low stiffness, large vibration is liable to be induced by even a small disturbance and problems due to vibration are apt to occur. Furthermore, as most machines are composed of components which have small damping factors, once the vibration begins, it is not easily stopped. The vibration causes not only deterioration in performance and reliability of machines but also has various ill effects on daily life. The suppression and control of

vibration become important in the fields of civil and architectural engineering, and in the development of space structures as well as in mechanical engineering.

One of the effective methods for solving problems of vibration involves an adaptive control, and it is important to make studies of design considering both the structural system and the control system because of the close relationship between them. In the simultaneous optimization of the design of a structure and its control system, the weighted sum of objective functions for the structure and for the control system is considered as a new objective function to be minimized⁽¹⁾⁻⁽⁴⁾.

This paper considers a shape determination problem in which the shape that minimizes a cost for the vibration control under the condition of constant structural volume is sought. A beam is the object of shape optimization and is discretized by the finite-element method. The control law obeys the optimal regulator theory and a weighted sum of quadratic quantities from the structural responses and control inputs is adopted as an objective function. The shape is optimized by a simple repetitive method based on the optimality conditions. Moreover, a two-step

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optimization method is proposed which uses a comparatively good initial approximation in the iterative method.

2. Control of Vibration and Simultaneous Optimization

2.1 Optimal regulator theory⁽⁵⁾

We consider that the object system is represented by a linear state equation as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad (1)$$

where $\mathbf{x}$ is the state vector of $(m \times 1)$, $\mathbf{u}$ is the vector of control variables of $(r \times 1)$, t is the time and a quantity with a dot denotes its derivative with respect to time t . When the equilibrium state of the system is represented by

$$\mathbf{x} = \mathbf{0} \quad \text{and} \quad \mathbf{u} = \mathbf{0}, \quad (2)$$

we assume that the system deviates from the equilibrium state at time $t=0$ to the state

$$\mathbf{x}(0) = \mathbf{x}_0. \quad (3)$$

In this case, the requirement of the control system is to return the system to the equilibrium state as quickly as possible with as little energy as possible. Therefore, the control which minimizes the following quadratic performance index

$$J(\mathbf{x}_0, \mathbf{u}) = (1/2) \int_0^\infty \{\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u}\} dt \quad (4)$$

is considered in the optimal regulator theory. In the above equation, $\mathbf{Q}$ and $\mathbf{R}$ are positive semidefinite and positive definite matrices, respectively. The control which minimizes the performance index of Eq. (4) is given by negative feedback as

$$\mathbf{u}^*(t) = -\mathbf{G}\mathbf{x}(t) \quad (5)$$

$$\mathbf{G} = \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P}, \quad (6)$$

where $\mathbf{P}$ is the positive semidefinite solution of the Riccati equation

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} - \mathbf{P} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q} = \mathbf{0}. \quad (7)$$

When this control is adopted, the minimum value of the performance index J is given by

$$J^*(\mathbf{x}_0) = \min_{\mathbf{u}} J(\mathbf{x}_0, \mathbf{u}) = J(\mathbf{x}_0, \mathbf{u}^*) = (1/2) \mathbf{x}_0^T \mathbf{P} \mathbf{x}_0. \quad (8)$$

2.2 Optimization of structural and control systems

2.2.1 Separated optimization In the conventional design of structures which may vibrate, the structural design and design of the control system are produced separately⁽⁶⁾. That is, they are constructed by use of the following two processes.

(a) Optimum Structural Design: Under the constraints placed on the displacement, stress and/or eigenfrequency, structural variables $\mathbf{a}^*$ that minimize the structural objective function, J_s , such as the weight, are sought. That is,

$$\text{Minimize}_{\mathbf{a}} J_s(\mathbf{a}) \quad (9)$$

$$\text{subject to } g(\mathbf{a}) \leq 0. \quad (10)$$

(b) Optimum Design of Control System: For the structure described by the design variables $\mathbf{a}^*$ determined in (a), control variable $\mathbf{u}^*$ that minimizes the objective function for the evaluation of control performance, J_c , is determined; that is,

$$\text{Minimize}_{\mathbf{u}} J_c(\mathbf{a}^*, \mathbf{u}). \quad (11)$$

2.2.2 Simultaneous optimization Recently, especially in the field of space structure design, the necessity of simultaneous optimization for structural and control systems has been recognized considering the close relationship between structure and control, and many studies have been published. Simultaneous optimization is formulated as

$$\text{Minimize}_{\mathbf{a}, \mathbf{u}} \begin{cases} J_s(\mathbf{a}) \\ J_c(\mathbf{a}, \mathbf{u}) \end{cases} \quad (12)$$

$$\text{subject to } g(\mathbf{a}) \leq 0. \quad (13)$$

In some cases, the optimization in Eq. (12) is carried out also with respect to parameters such as the positions of sensors and actuators.

Equation (12) has the form of multicriteria optimization. As the objectives of designs of the structure and control systems are generally in conflict with each other, the Pareto-optima are considered in this problem.

3. Formulation of Optimization Problem

3.1 Problem statement

A discretized model of a continuum structure obtained using the finite-element method is treated as the object of design and control. The equation of motion is generally written as

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = \mathbf{B}_0 \mathbf{u}(t), \quad (14)$$

where $\mathbf{y}$ is the displacement vector of $(n \times 1)$, and $\mathbf{u}$ is the control force vector of $(r \times 1)$. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of $(n \times n)$. $\mathbf{B}_0$ is a matrix of $(n \times r)$, which represents the location of the control force.

Expressing Eq. (14) in the form of the state equation as in Eq. (1), we obtain

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{y}(t) \\ \dot{\mathbf{y}}(t) \end{bmatrix} \quad (15)$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \quad (16)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{B}_0 \end{bmatrix}. \quad (17)$$

The mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$ are both functions of structural variables $\mathbf{a} = (a_i : i = 1, \dots, N)$, where N is the number of design variables, and the damping matrix $\mathbf{C}$ is also expressed in terms of structural variables assuming proportional damping, that is,

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K}, \quad (18)$$

where α and β are constants properly assigned. Therefore, the coefficient matrices of the state equation in

Eq. (1), A and B , are functions of structural variables a .

This study considers the minimization problem of the quadratic performance index, J , which is used in the optimal regulator theory, with respect to structural variables a under the constraint that the volume of the model body is constant. If this performance index represents the control cost, our problem is regarded as that of determining the structural shape while minimizing the control cost under the condition of volume constancy. With the use of Eq. (8), the problem is written as

$$\text{Minimize } J^*(x_0; a) \quad (19)$$

$$\text{subject to } V(a) = V_c \quad (20)$$

$$a_l \leq a_i \leq a_u \quad (i=1, \dots, N), \quad (21)$$

where V is the volume of the structure and V_c is the specified value for the volume. Equation (21) is introduced as a constraint on the structural size, and a_l and a_u are the lower and upper bounds of the structural variables a_i ($i=1, \dots, N$).

3.2 Pareto-optima

For the multicriteria problem with s objectives,

$$\text{Minimize } f(a) = [f_1(a), \dots, f_s(a)]^T \quad (22)$$

$$\text{subject to } a \in A = \{a : g(a) \leq 0\}, \quad (23)$$

and methods for obtaining the Pareto-optima are as follows.

(a) Transformation to a weighted sum: The weighted sum of s objective functions is minimized as

$$\text{Minimize } \sum_{i=1}^s w_i f_i(a), \quad (24)$$

where $w_i > 0$ ($i=1, \dots, s$) is the weight.

(b) Setting of constraints: With the setting of constraints on $s-1$ functions excluding the r -th objective function, the r -th objective is minimized as

$$\text{Minimize } f_r(a) \quad (25)$$

$$\text{subject to } f_j(a) \leq e_j \quad (j=1, \dots, s, j \neq r), \quad (26)$$

where e_j is a constraint value for the j -th objective.

If we regard f_1 as the control cost J^* and f_2 as the volume V , and they are conflicting, the present problem given by Eqs. (19)–(21) has the form of Eqs. (25) and (26) with $s=2$. Therefore, the present problem is one of seeking a Pareto-optimum for the two criteria, the control cost and the volume.

3.3 Initial state

As shown by Eq. (8), the optimum control cost is dependent on the initial state of the system, x_0 . Therefore, the solution of the simultaneous optimization problem is also dependent on this initial state. In applications of the optimal regulator theory, several states are adopted as the initial state. We suggest that the initial state is the disturbed state of the system due to various external loads, and x_0 is a function of the structural variables a . In this paper, the static displacement which is caused by a static load p is adopted

as the initial state. That is, as the initial velocity is regarded as zero, then

$$x_0 = \begin{bmatrix} y_0 \\ 0 \end{bmatrix}, \quad (27)$$

where the initial displacement y_0 is given by

$$K(a)y_0 = p. \quad (28)$$

3.4 Weighting matrices

As the matrices in the performance index [Eq. (4)] naturally affect the optimization problem, it is important to consider how they are determined. In this paper, the matrices Q and R are determined according to the following assumptions.

(Assumption 1) The energy to be supplied to the actuator is proportional to the square of the driving force, and the cost incurred is proportional to the supplied energy.

(Assumption 2) The influence of vibrations on the outside is evaluated by the squares of the displacement and velocity.

(Assumption 3) The life of the structure is dependent on the strain energy and the kinetic energy, and the evaluation of the cost required for maintenance is based on the strain and kinetic energies.

[Case I] From Assumptions 1 and 3,

$$Q = \begin{bmatrix} p_1 K & 0 \\ 0 & p_2 M \end{bmatrix}, \quad p_1, p_2: \text{constants} \quad (29)$$

$$R = qI, \quad q: \text{constant}. \quad (30)$$

[Case II] From Assumptions 1 and 2,

$$Q = \begin{bmatrix} p_1 & 0 \\ p_2 & \\ 0 & p_{2n} \end{bmatrix}, \quad p_i \quad (i=1, \dots, 2n): \text{constants} \quad (31)$$

$$R = qI, \quad q: \text{constant}. \quad (32)$$

In the above, I is the identity matrix. In Case I, the weighting matrix Q varies with the changes of the structure, and in Case II, it is independent of the structural changes.

4. Method for Shape Determination

4.1 Finite-element model

In this paper, the example structure is a beam, which is divided into $N-1$ elements. In an element, the cross-sectional area varies linearly; thus, the structural variables a are cross-sectional areas a_i ($i=1, \dots, N$) at N nodes. Moreover, cubic equations are used as the interpolation function of displacement for the approximation of the lateral deflection.

4.2 Scheme for optimization⁽⁷⁾

In order to solve the system of Eqs. (19)–(20), we propose a simple algorithm which invokes the condition of optimality.

The original problem is transformed into a sta-

tionary problem with a Lagrange multiplier λ ,

$$L(\mathbf{a}, \lambda) = J^*(\mathbf{a}) - \lambda \{V(\mathbf{a}) - V_c\} \rightarrow \text{stationary}(\mathbf{a}, \lambda). \quad (33)$$

The stationary conditions of the Lagrange function L can be obtained by differentiating it with respect to every variable, that is, the equation for shape determination

$$\{\partial J^*(\mathbf{a})/\partial a_i\} - \lambda \{\partial V(\mathbf{a})/\partial a_i\} = 0 \quad (i=1, \dots, N) \quad (34)$$

and the condition of volume constancy [Eq. (20)].

With the introduction of new variables λ_i ,

$$\lambda_i = \{\partial J^*/\partial a_i\} / \{\partial V/\partial a_i\} \quad (i=1, \dots, N), \quad (35)$$

the equation for shape determination is rewritten as

$$\lambda_i - \lambda = 0 \quad (i=1, \dots, N). \quad (36)$$

Therefore, shape determination which equalizes all λ_i is needed. This paper adopts the following scheme for shape reformation.

For the shape at the k -th iteration, $\mathbf{a}^k$, λ_i ($i=1, \dots, N$) are calculated and the average is denoted by $\bar{\lambda}$, that is,

$$\bar{\lambda} = \sum_{i=1}^N \lambda_i / N, \quad (37)$$

which is used as an estimated value of the true value of the Lagrange multiplier λ . Then, the cross-sectional areas are modified by

$$a_i^* = a_i^k + r\Lambda_i \quad (i=1, \dots, N) \quad (38)$$

$$\Lambda_i = (\lambda_i - \bar{\lambda}) / \bar{\lambda} \quad (i=1, \dots, N), \quad (39)$$

where Λ_i represents the degree of deviation of λ_i from the mean $\bar{\lambda}$, and r is a positive constant for the stability of computation. The area $\mathbf{a}^*$ obtained by Eq. (38) violates the volume constraint and the volume is

$$V(\mathbf{a}^*) = V_2 \neq V_c. \quad (40)$$

Then, the areas are reformed further by

$$a_i^{k+1} = a_i^* V_c / V_2 \quad (i=1, \dots, N) \quad (41)$$

so that they may satisfy the volume constraint.

With repetition of the above procedures, if the deviation Λ_i becomes small enough, that is, if

$$\text{Max}_i |\Lambda_i| \leq \varepsilon, \quad (42)$$

then the iterative process is stopped.

4.3 Calculation of sensitivity⁽²⁾

The above procedure requires the sensitivity $\partial J^*(\mathbf{a})/\partial a_i$. As J^* is expressed by

$$J^*(\mathbf{a}) = (1/2) \mathbf{x}_0(\mathbf{a})^T \mathbf{P} \mathbf{x}_0(\mathbf{a}) \quad (43)$$

differentiation by a_i yields

$$J^*_{,i} = (1/2) \mathbf{x}_0^T \mathbf{P}_{,i} \mathbf{x}_0 + \mathbf{x}_0^T \mathbf{P} \mathbf{x}_{0,i}, \quad (44)$$

where “ $,i$ ” indicates the partial differentiation with respect to a_i . As $\mathbf{P}$ is the positive semidefinite solution of Riccati's equation [Eq. (7)], the differentiation of Eq. (7) with respect to a_i gives the Lyapunov equation

$$\tilde{\mathbf{A}}^T \mathbf{P}_{,i} + \mathbf{P}_{,i} \tilde{\mathbf{A}} = -\tilde{\mathbf{Q}}, \quad (45)$$

where

$$\tilde{\mathbf{A}} = \mathbf{A} - \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} \quad (46)$$

$$\tilde{\mathbf{Q}} = \mathbf{A}^T \mathbf{P}_{,i} \mathbf{P} + \mathbf{P} \mathbf{A}_{,i} - \mathbf{P} (\mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T)_{,i} \mathbf{P} + \mathbf{Q}_{,i}. \quad (47)$$

As Eq. (45) is the system of linear equations for $\mathbf{P}_{,i}$, the sensitivity $\mathbf{P}_{,i}$ is calculated by solving the system. Next, the sensitivity of the initial state $\mathbf{x}_0$ is obtained by differentiating the static equilibrium equation⁽⁶⁾ [Eq. (28)]. That is, from

$$\mathbf{K}_{,i} \mathbf{y}_0 + \mathbf{K} \mathbf{y}_{0,i} = \mathbf{p}_{,i}, \quad (48)$$

we obtain

$$\mathbf{y}_{0,i} = \mathbf{K}^{-1} (\mathbf{p}_{,i} - \mathbf{K}_{,i} \mathbf{y}_0). \quad (49)$$

5. Example of Design

5.1 Object of design

The beam which is the object of the numerical example has the following characteristics:

The total length is 3.0 [unit length], the density is 1.0 [unit density], Young's modulus is 1.0 [unit force/unit area] and the cross section varies similarly in the longitudinal direction. In the numerical example, a clamped-free beam and a clamped-clamped beam are modeled with three elements, as shown in Fig. 1. In the case of the clamped-clamped beam, only the left half is considered because of the beam's symmetry.

As for the control system, actuators and sensors are located at all nodes except for the fixed point, as shown in Fig. 1. It is assumed that the sensors can detect all deflections and deflection angles and their velocities at every node, and that the actuators generate control forces in the vertical direction on the beam.

Moreover, the deformation which is caused by three concentrated loads of magnitude 1.0 [unit force] at the 2nd, 3rd and 4th nodes is adopted as the initial state.

5.2 Responses of beam with uniform cross section

In order to investigate the relationship between the structural and control systems, the variation of the optimal control cost J^* is calculated when the volume of the beam with uniform cross section is increased. The results for the weighting matrices

$$\mathbf{Q} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} \text{ and } \mathbf{R} = \mathbf{I} \text{ or } \mathbf{R} = 1000 \mathbf{I}$$

are shown in Fig. 2. In Fig. 2, the abscissa is the cross-sectional area of the uniform beam. It is found from Fig. 2 that for every case, as the cross-sectional area

(a) clamped-free beam (b) clamped-clamped beam

Fig. 1 Object of design

of the beam increases, the control cost decreases. In particular, when the cross-sectional area is small, a small decrease in weight elicits a great increase of the control cost. This means that if the weight of the structure is decreased without careful consideration, low reliability could result. Thus, as the optimization (lightening) of the structure conflicts with minimization of the control cost, simultaneous optimization is the issue of concern.

5.3 Examples

The optimization is carried out adopting the uniform beam with cross-sectional area of 10 [unit

area] as the initial design, the lower bound for the cross-sectional area is 3 [unit area] and weighting matrices of the following two types are used:

$$(I) \quad Q = \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix} \quad \text{and} \quad R = 100I$$

$$(II) \quad Q = I \quad \text{and} \quad R = 100I$$

In the following sections, we refer to the optimum shapes for the respective cases as optimum shape I and optimum shape II.

5.3.1 Design of clamped-free beam (cantilever)

The results of the design calculations for clamped-free beams are shown in Fig. 3. The obtained beams are similar in shape irrespective of weighting matrices. Figure 4 shows the change of the object function J^* in the optimization process normalized by the control cost necessary for the initial uniform beam, J_0^* . It is observed from Fig. 4 that the optimization process is stable in both cases. In Table 1, initial displacements y_0 , that is, nodal displacements for the static load, are listed for each beam. It is found that with increasing stiffness compared with the uniform beam (that is, by making the displacement for the same load small), the initial displacement becomes small, and as a result, the control cost can be de-

(a) clamped-free beam

(b) clamped-clamped beam

Fig. 2 Relationship between area and control cost of beam with uniform cross section

Table 1 Comparison of initial displacement (clamped-free beam)

Nodal No.	2	3	4
initial shape	0.3000	0.9780	1.8000
optimum shape I	0.1050	0.4385	1.0638
optimum shape II	0.1100	0.4442	1.0623

(a) case I

(b) case II

Fig. 4 Optimization history of clamped-free beam

(a) optimum shape I

(b) optimum shape II

Fig. 3 Initial and optimum shapes of clamped free beam

creased.

5.3.2 Design of clamped-clamped beam The results for the left half of a clamped-clamped beam are shown in Figs. 5 and 6 and in Table 2, and they are similar to those in the case for the clamped-free beam.

5.4 Case of initial state with specified displacement

In contrast to section 5.3, this section considers the case in which the initial state is not dependent on the structural shape, but is given by the value of displacement. That is, the displacement of the initial uniform beam when it is subject to the static load, $y_{0/uni}$, is used as a common initial state for every simulation. Figure 7 shows the results of design calculations when the weighting matrices are those of set (I). The obtained shapes are very different from

those in section 5.3 for both clamped-free and clamped-clamped beams. It seems that the beams become flexible to external loads so that they may revert from the given deformed state to their undeformed states with applied control force. However, it is practically impossible for the beam of arbitrary shape to achieve the same deformed state as the beam with uniform cross section. Therefore, "the initial state with specified displacement" is an unreasonable assumption.

6. Two-step Method Using Initial Strain Energy

6.1 Two-step design

As shown by the examples in section 5.3, the design which can reduce the control cost J^* is one whose displacement by the external static load, y_0 , is small when the state deformed by the load is adopted

Fig. 5 Initial and optimum shapes of clamped-clamped beam

Fig. 6 Optimization history of clamped-clamped beam

Table 2 Comparison of initial displacement (clamped-clamped beam)

Nodal No.	2	3	4
initial shape	0.1600	0.4200	0.5400
optimum shape I	0.0676	0.2867	0.3952
optimum shape II	0.0703	0.2962	0.3832

Fig. 7 Optimum shape in the case by initial state with specified displacement

Fig. 8 Optimum shape minimizing initial strain energy

(a) case I

(b) case II

Fig. 9 Optimization history from shape of Fig. 8(a) in clamped-free beam

(a) case I

(b) case II

Fig. 10 Optimization history from shape of Fig. 8(b) in clamped-clamped beam

as the initial state. Then, it is assumed that a more effective method is to obtain the stiffest structure before solving the original problem [Eqs. (19) and (20)]. That is, as a preparatory problem, the shape which minimizes the strain energy exerted by the given load is determined under the condition of volume constancy. Secondly, the original optimization problem is solved using the solution of the preparatory problem as the initial design.

6.2 Minimization of initial strain energy

The problem of the determination of the shape which minimizes the strain energy induced by a static load $\mathbf{p}$ under the condition of volume constancy is written as

$$\text{Minimize } S(\mathbf{a}) = (1/2) \mathbf{y}_0(\mathbf{a})^T \mathbf{K}(\mathbf{a}) \mathbf{y}_0(\mathbf{a}) \quad (50)$$

$$\text{subject to } V(\mathbf{a}) = V_c. \quad (51)$$

The solution of this problem can be obtained with the same algorithm as that in section 4.2 by replacing $J^*(\mathbf{a})$ by $S(\mathbf{a})$.

6.3 Example of design

The proposed two-step method is applied to the same model as that in Chap. 5. Figure 8 shows the obtained shapes in the first step, that is, in the minimization of the strain energy. Figures 9 and 10 show the optimization histories obtained in the second step (original problem) for both clamped-free and clamped-clamped beams when the solution of the first step is used as the initial design for the second step. In Figs. 9 and 10, the control cost J^* is normalized by

that needed for the uniform beam with the same volume, J^* , similar to Figs. 4 and 6. As expected from the close resemblance between the shape in Fig. 8 and that in Fig. 3 or in Fig. 5, the control cost converges immediately to its minimum value.

For the original problem, the Riccati equation [Eq. (7)] and Lyapunov equation [Eq. (45)] must be solved at each iteration, that is, every time the shape is changed. The two-step method can reduce the number of iterations in the second (original) problem compared with the method which addresses the original problem from the outset starting from the beam with uniform cross section. Further, as the first step requires only the inversion of the stiffness matrix $\mathbf{K}$, the total computational time can be greatly reduced.

7. Conclusions

This study determined the shape of the beam that minimizes the quadratic performance index in the optimal regulator theory under the constraint that the volume of the beam is constant. Through numerical examples, the effects of the weighting matrices in the performance index and those of the initial state on the results were discussed. Moreover, through this process, a method for optimization was proposed which was deemed effective from the viewpoint of computational time and stability.

References

- (1) Onoda, J. and Haftka, R. T., An Approach to Structure/Control Simultaneous Optimization for Large Flexible Spacecraft, *AIAA J.*, Vol. 25, No. 8 (1987), p. 1133.
 - (2) Salama, M., Garba, J., Demsetz, L. and Udwadia, F., Simultaneous Optimization of Controlled Structures, *Computational Mechanics*, Vol. 3 (1988), p. 275.
 - (3) Grandhi, R. V., Structural and Control Optimization of Space Structures, *Computers & Structures*, Vol. 31, No. 2(1989), p. 139.
 - (4) Iwatsubo, T., Ikeda, M., Kawamura, S. and Adachi, K., Development of Simultaneous Optimum Design of Structural and Control System of Flexible Structures, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 57, No. 534, C(1991), p. 407.
 - (5) Athans, M. and Falb, P., *Optimal Control*, (1966), McGraw-Hill.
 - (6) Tada, Y. and Minami, E., Optimum Design and Control of 3-D Truss Structure, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 58, No. 545, A (1992), p. 146.
 - (7) Atrek, E., Gallagher, R. H., Ragsdell, K. M. and Zienkiewicz, O. C.(Ed.), *New Direction in Optimum Structural Design*, (1984), John Wiley & Sons, p. 197.
 - (8) *Jpn. Soc. Mech. Eng.(Ed.)*, *Optimum Design of Structures and Materials*, (in Japanese), (1989), Gihodo, p. 140.
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