Rider's Operation of a Motorcycle Running Straight at Low Speed*

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To maintain the stability of a motorcycle running straight, the rider steers the handlebars with his hands and leans his body to the right or left on the seat in dead time (time lag) to the motion of the disturbances of the motorcycle's frame. Since these three movements relate to one another and it is difficult to analyze the motion of the motorcycle-rider system, we treat the motorcycle and the rider as one rigid body. In this report, we studied the conditions necessary for stability and the rider's operation and control of the motorcycle running straight at low speed using the equation of motion, in which the movement of the center of gravity of the rider to the right or left, the reaction of the motorcycle to the rider's movement and the moment of inertia of the rider are considered.

Key Words: Stability, Motorcycle, Two Wheels, Rider, Control, Movement, Dead Time, Vibration

1. Introduction

The frame of a motorcycle in operation leans to the right or left due to disturbances in the road surface roughness, the wind and the rider's own movement. Therefore, natural oscillation (vibration) of the handlebars about the steering axis results and the rider maintains an upright position, running straight, by steering and by controlling his posture. Since these motions involving the motorcycle frame, the vibration of the handlebars and the movements of the rider, are related to one another, it is very complicated to analyze the motion of the motorcycle-rider system. In the studies of stability reported thus far, investigators have discussed the case of the lean-with posture in which the top and bottom axis of the rider leans right or left in the XZ plane of the motorcycle frame⁽¹⁾⁻⁽⁶⁾.

Ooya⁽⁷⁾ and Ooya et al.⁽⁸⁾ and Tukada and Ooya⁽⁹⁾ reported on the stability of a bicycle based on the assumption of the rider and the bicycle as one rigid body, as in the case of the motorcycle.

Since the study of Fu⁽¹⁰⁾ in 1977, studies have dealt with the characteristics of rider movement such as the lean-in (riding posture in which the rider's lean angle is larger than the bank angle of the frame) and the lean-out (riding posture in which the rider's lean angle is smaller than the bank angle of the frame) on the stability and control of the rider-motorcycle system. Nishimi et al.⁽¹¹⁾ and Katayama et al.⁽¹²⁾ expanded on the ideas of Kondo et al.⁽³⁾ or Sharp⁽⁵⁾ by investigating the vibration of weave and wobble modes by use of the running test and performing a theoretical analysis with regard to the mathematical model involving 12 degrees of freedom including the vibration characteristics of the rider's lower limbs and the upper body on the seat. Sharp reported results on the vibration characteristics of the rider's upper body, the influences on the weave mode (1-4 Hz) with respect to the rolling and yawing of the frame, and the influence of the rider's lower limbs on the wobble mode (6-10 Hz) of the steering system. Nagai^{(13),(14)} studied control theory

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of the rider's handling and the shift of rider weight in connection with motorcycle risk avoidance. He also tested a bicycle on a moving belt using a microcomputer.

These studies covered large movements involving changing course to the right or left for lane changes and steady circular turns or high-speed running at more than 100 km/h. The present report takes note of the postural changes of the rider and motorcycle in maintaining upright stability at low speed and discusses operational characteristics, such as rider stability in upright and running straight, by calculation of the influence of the transfer of the rider's center of gravity by inclination of the upper body, the reaction of the motorcycle to the rider and the moment of inertia of the rider. A motorcycle with a vehicle weight less than the weight of the rider, thus able to show the effects of rider movement under vehicle motion, was used, and the calculated and experimental results were compared regarding stability, especially at low speeds, when unstable operation is apt to occur.

2. Motorcycle Operation Model in a Straight Run

The outline of a motorcycle operation model using a rider for a straight run is shown in Fig. 1. We assumed that disturbances caused by the reaction to the road roughness, the wind and the rider's own movement occurred, so that the frame leans with the bank angle of frame θ_1 . The handlebars vibrate with the steering angle φ corresponding to these motions, and the rider operates the handlebars at will as needed. The rider simultaneously maintains upright stability and straight operation of the motorcycle by inclining right or left (lean angle θ_r), his upper torso on the seat in relation to the XZ plane of the frame.

3. Coordinates and Symbols

The rider of a motorcycle in operation not only steers but leans right or left and moves the upper body sideways on the seat by moving his lower limbs, leaning the frame to the right or left by pushing on the footrests and moving his upper body under reaction force. In this study, it is assumed that the rider retains

Fig. 1 The outline of a motorcycle operation model with a rider

stability, and upright and straight operation by handling, as show in Fig. 2, the supporting point of the seat (ground height, h_s), leaning (lean angle, θ_r) only his upper body right or left.

Each point of the center of gravity of the rider (center of gravity, G_r), the motorcycle (G_m) and the motorcycle-rider system (G_o) transfers right or left and up or down with the lean angle of the upper body, but it is assumed that the displacement in the vertical direction is very small because the lean angle is small during straight operation. The respective symbols are shown in Table 1, and the other symbols and coordinates are as given in Fig. 2. Further, each physical value of the motorcycle and rider used in later calculation is also shown in Table 1.

4. Equations of Motion

The following equations are posited in accordance with the foregoing and Fig. 2.

$$\begin{aligned}\theta_2 &\doteq \theta_r \cdot h_{rs}/h_r \\ h_{rs} + (h_s - h_m) &\doteq h_r \\ h_t &\doteq h_r + h_m.\end{aligned}\quad (1)$$

The abscissas of the center of gravity for the motorcycle and the rider are represented, respectively, by the horizontal distance y_1, y_2 from the vertical axis on the contact point of the earth. Thus,

$$y_2 = y_1 - h_r(\theta_2 - \theta_1). \quad (2)$$

When the rider shifts right or left with his upper body, the horizontal component of the reaction force from the motorcycle to the rider is represented by F_y and the vertical component of the reaction force to the rider's weight by F_z ,

$$\begin{aligned}F_y &= M_r \ddot{y}_2 \\ F_z &\doteq M_r g.\end{aligned}\quad (3)$$

The horizontal reaction force S and the vertical reaction force N from the road surface to the tire at the contact point of the tire and the road may be expressed by:

$$S = F_y + M_m \ddot{y}_1 = M_r \ddot{y}_2 + M_m \ddot{y}_1 \quad (4)$$

Table 1 The respective symbols

VEHICLE MASS : $M_m = 49\text{kg}$
RIDER MASS : $M_r = 79.5\text{kg (A)}, = 56.0\text{kg (B)}$
MASS OF STEERING SYSTEM : $M_s = 9.93\text{kg}$
MOMENT OF INERTIA ON VEHICLE : $J_m = 1.9\text{kgm}^2$
MOMENT OF INERTIA ON RIDER : $J_r = 5.62\text{kgm}^2\text{ (A)}, = 4.76\text{kgm}^2\text{ (B)}$
MOMENT OF INERTIA ON WHEEL AXLE : $J_{rw}, J_{rw} = 0.1\text{kgm}^2$
CASTER ANGLE : $\alpha = 67^\circ 57'$, WHEELBASE : $\ell = 1.05\text{m}$
$\rho_2 = 0.417\text{m}$
SHORTEST DISTANCE BETWEEN GRAVITY CENTER ON STEERING SYSTEM AND STEERING AXLE : $\rho_4 = 0.011\text{m}$
RADIUS OF WHEEL : $r = 0.233\text{m}$, $h_r = 0.500\text{m (A)}, = 0.488\text{m (B)}$
$h_m = 0.445\text{m}$, $h_{rs} = 0.20\text{m (A)}, = 0.21\text{m (B)}$
$h_t = 0.939\text{m (A)}, = 0.927\text{m (B)}$
HEIGHT OF GRAVITY CENTER ON STEERING SYSTEM : $h_s = 0.511\text{m}$
CASTER LENGTH : $e_2 = 0.025\text{m}$, $q = \sin \alpha$
ACCELERATION OF GRAVITY : $g = 9.8\text{m/s}^2$

$$N \doteq M_m g + F_z = M_m g + M_r g = M g. \quad (5)$$

The camber thrust may be neglected since the cornering power is very large. When the rider inclines the motorcycle frame using his hands, feet and hips, the moment of reaction undergone by the rider from the motorcycle may be denoted as T . Thus, the equation of rotational motion for balance in the change $(\theta_r - \theta_1)$ in the rider's upper body inclination is

$$J_r(\ddot{\theta}_r - \ddot{\theta}_1) = T + M_r g h_{rs}(\theta_r - \theta_1) + F_y h_{rs} \quad (6)$$

and the equation of rotational motion for balance of the motorcycle inclination θ_1 is

$$J_m \ddot{\theta}_1 = T - (h_s - h_m) F_y + (h_s - h_m) M_r g \theta_1 + N h_m \theta_1 - S h_m + M_{Mw} + M_{Mf} + M_{MFN}. \quad (7)$$

Let M_{Mw} , M_{Mf} and M_{MFN} be the moments for the following. The moment M_{Mw} for improving the stability of the motorcycle frame due to the gyroscopic effect on the front and rear wheels is

$$M_{Mw} = -\frac{v^2}{r l} (J_{fw} + J_{rw}) q \varphi - \frac{v}{r} J_{fw} q \dot{\varphi}. \quad (8)$$

The moment M_{Mf} of the inertial force due to the vibration of handlebars in the mass of the front wheel system is

$$M_{Mf} = M_f l_4 (h_m - h_f) \ddot{\varphi}. \quad (9)$$

The moment M_{MFN} (the trail effect) due to the vertical reaction force which acts on the contact point

of the tire and the road surface and by the distance (trail) between the contact point of the front wheel and the intersection of the steering axis on the road surface is

$$M_{MFN} = M g \frac{l_2}{l} e_2 \varphi. \quad (10)$$

Moreover, in the case where the motorcycle runs at the constant speed $v^{(7),(9)}$,

$$\dot{y}_1 = \frac{v^2}{l} q \varphi + v \frac{l_2}{l} q \dot{\varphi} + h_m \ddot{\theta}_1. \quad (11)$$

In considering that the steering angle φ changes in proportion (the proportional coefficient x_1) to the bank angle θ_1 of the frame with the time lag τ_1 (the dead time),

$$\varphi(t) = x_1 \theta_1(t - \tau_1). \quad (12)$$

Moreover, in considering that the lean angle θ_r of the rider's upper body changes in proportion (the proportional coefficient x_2) to the bank angle θ_1 of the frame with the time lag τ_2 (the dead time),

$$\theta_r(t) = x_2 \theta_1(t - \tau_2). \quad (13)$$

If we expand Eqs. (12) and (13) in the series and take the approximate values to the third order, we obtain

$$\begin{aligned} \varphi &\doteq x_1 \theta_1 - x_1 \tau_1 \dot{\theta}_1 \\ \theta_r &\doteq x_2 \theta_1 - x_2 \tau_2 \dot{\theta}_1. \end{aligned} \quad (14)$$

5. Stability Conditions

The equation of motion for the bank angle of the frame from Eqs. (1)-(14) is shown in Eq. (15).

$$a_1 \ddot{\theta}_1 + a_2 \dot{\theta}_1 + a_3 \theta_1 + a_4 \theta_1 = 0 \quad (15)$$

Further, a_1 , a_2 , a_3 and a_4 in Eq. (15) are as indicated in Eqs. (16)-(19).

$$a_1 = M_f l_4 (h_m - h_f) x_1 \tau_1 + (M_r h_{rs} h_t + J_r) x_2 \tau_2 \quad (16)$$

$$\begin{aligned} a_2 = & J_m + J_r + M_r h_t^2 + M_m h_m^2 - M_f l_4 (h_m - h_f) x_1 \\ & - \left\{ (M_r h_t + M_m h_m) v \frac{l_2}{l} + \frac{v}{r} J_{fw} \right\} q x_1 \tau_1 \\ & - (J_r + M_r h_{rs} h_t) x_2 \end{aligned} \quad (17)$$

$$\begin{aligned} a_3 = & \left\{ M g \frac{l_2}{l} e_2 - (M_r h_t + M_m h_m) \frac{v^2}{l} q \right. \\ & \left. - \frac{v^2}{r l} (J_{fw} + J_{rw}) q \right\} x_1 \tau_1 - M_r g h_{rs} x_2 \tau_2 \\ & + \left\{ (M_r h_t + M_m h_m) \frac{l_2}{l} + \frac{J_{fw}}{r} \right\} q v x_1 \end{aligned} \quad (18)$$

$$\begin{aligned} a_4 = & \left\{ (M_r h_t + M_m h_m) \frac{v^2}{l} q + \frac{v^2}{r l} (J_{fw} + J_{rw}) q \right. \\ & \left. - M g \frac{l_2}{l} e^2 \right\} x_1 + M_r g h_{rs} x_2 \\ & - M_r g h_t - M_m g h_m \end{aligned} \quad (19)$$

The present study is the first to consider the motion of the rider's upper body to the right or left in the process leading up to Eq. (15).

The stability conditions in Eq. (15) are positive for each coefficient a_1 , a_2 , a_3 , a_4 and $(a_2 a_3 - a_1 a_4)$. In this study, a_1 is small in comparison with a_2 , a_3 and

Fig. 2 The coordinates and the symbols of a motorcycle and rider

a_4 , so the latter ($a_2a_3 - a_1a_4$) is neglected.

6. Calculated Examples and Experimental Results

6.1 Calculated examples

As one example, the calculated results of the stability conditions are shown in the following section for two riders (mass M_r of rider A is 79.5 kg and rider B is 56.0 kg) on a motorcycle in which the engine displacement is 49 cc ($M_m=45.0$ kg), the so-called "family bike".

Figure 3 indicates only the proportional coefficient κ_1 of the steering angle and the proportional coefficient κ_2 of the rider's lean angle to the bank angle of the frame, the stable regions of the proportional coefficient κ_1 at each vehicle speed under the assumption of κ_2 . The proportional coefficient κ_1 is large when the vehicle speed is low and the steering angle must be kept large in relation to the bank angle of the frame to maintain stability for upright and straight operation, and under these conditions, the

Fig. 3 The stable regions of the proportional coefficient κ_1 at each vehicle speed

Fig. 4 The stable regions of the proportional coefficient κ_1 the time lag τ_1 during the so-called "lean-with"

rate of the lean angle of the rider to the bank angle of the frame becomes large. Thus, it is shown that the proportional coefficient κ_1 is small when κ_2 is large and a small steering angle is sufficient for stability in upright and straight operation. Differences in the lower limits of κ_1 due to the mass of each rider are minimal.

When the upper body of the rider does not move independently but instead moves with the frame of the motorcycle (the so-called "lean-with"), where the proportional coefficient κ_2 is zero ($\tau_2=0$), the stable regions of the proportional coefficient and the time lag τ_1 of the steering angle to the bank angle of the frame are as shown in Fig 4. In the case of low speed, the lower limit of the proportional coefficient κ_1 of the steering angle in relation to the bank angle of the frame is high and the rider must steer carefully, whereby the upper limits of the time lag τ_1 become high and the stable regions of τ_1 are extended. In the case of high speed, on the other hand, the lower limit of the proportional coefficient κ_1 is low and little steering by the rider is required, the upper limits of the time lag τ_1 become low and the stable regions τ_1 narrow. The upper limit of τ_1 becomes slightly smaller as the mass of the rider decreases (in the case of rider B), but the lower limit of κ_1 does not change.

Next, as shown by the straight lines in Figs. 5 (a), (b) and (c), in the case of $\kappa_2 \neq 0$ ($\tau_2 \neq 0$), there are stable regions (upper limits) of the time lag τ_1 , τ_2 , when the rider moves right or left with his upper body with the banking of the frame. This calculated result is obtained in the case of extremely low speed (about 5.0 km/h) and at 10 and 40 km/h, using suitable proportional coefficients, κ_1 and κ_2 , of the experimental values following (Figs. 6 and 7).

In the case of a positive time lag, τ_1 and τ_2 , passive control is indicated in which the changes in the steering angle and the rider's lean angle appear with a lag in relation to the bank angle of the frame. In the case of a negative time lag, with the changes in the steering angle and the rider's lean angle ahead of the bank angle of the frame, the rider has active control of the motorcycle.

When the vehicle speed is low, there is no limitation on the time lag τ_2 ; only the upper limit of the time lag τ_1 is restricted. With increases in the vehicle speed, the upper limits of the time lag τ_1 and τ_2 become small, and at 40 km/h, the region of passive control in which the bank angle of the frame proceeds the other angles becomes narrow. When the rider's mass is small (rider B), the stable region (upper limit) at low speed is slightly smaller than that with large-mass rider (rider A) but does not differ as the vehicle speed increases.

6.2 Experiments and results

We performed a straight operation test at low speed on a flat asphalt road using a small motorcycle, the same as used in the calculation. The steering angle was measured with a potentiometer and the bank angle of the frame and the rider's lean angle, a rategyroscope. The measurement distance was 200 m, and an approach and braking segments were used before and after the course. The running speed used was the lowest possible (extremely low speed, mean vehicle speed 5.0 km/h) and the instructed speeds were 10, 20, 30 and 40 km/h. As for the steering angle, the bank angle of the frame and the rider's lean

Fig. 5 The stable regions of the time lag τ_1 and τ_2 were calculated with the experimental proportional coefficients κ_1 and κ_2

angle, which change in close connection with time, we obtained values corresponding to the effective value at the start and end of measurement, taking the respective mean steering angle, mean banking angle and rider's lean angle for each run⁽¹⁵⁾⁻⁽¹⁷⁾. Further, for the bank angle and rider's lean angle, the results at the angular velocity were calculated with the peak frequency at the time.

Figure 6 shows the proportional coefficient κ_1 of the mean steering angle to the mean bank angle which was obtained from the bank angle of the frame. The proportional coefficient κ_1 is large at low speed, when stability is poor, and the rate of the steering angle to the bank angle of the frame is large; thus, stability is achieved with increasing speed, and the steering angle and κ_1 become small. As compared with the stability regions ($\kappa_2=2$, for example) of κ_1 in the calculation in Fig. 3, each value of 1 in the experiment was above the lower limit of the stable region in the calculation.

The proportional coefficient κ_2 of the mean lean angle of the rider to the mean bank angle of the frame, which is obtained by the angular velocity of frame banking, is shown in Fig. 7. The case of rider A, a rider running at a κ_1 of about 1 at any vehicle speed is shown at the angle at which the rider's lean angle is as large as the bank angle of the frame, that is, at a consistently upright posture. For the case of rider B, the level was about 1-2 at extremely low speed and

Fig. 6 The proportional coefficients κ_1 at each vehicle speed

Fig. 7 The proportional coefficients κ_2 at each vehicle speed

the indicated speed 10 km/h, and the rider sought to maintain upright stability and straight operation by large movement of the upper torso. However, the value of κ_1 was about 1 over the indicated speed of 20 km/h and showed the same tendency as for rider A. Further, each κ_1 and κ_2 (Figs. 6 and 7) of the experimental values satisfied Eq. (19) for the stability conditions.

Next, upon obtaining the crossspectral, in which the bank angle of the frame is the input and the steering angle or the rider's lean angle is the output, the phase lag at the frequency indicated at the maximum gain is obtained as the time lag (dead time) τ_1 and τ_2 . τ_1 and τ_2 are shown for each indicated speed for rider A(○) and B(●) in Figs. 5 (a) and (b) shown earlier.

At an extremely low speed (the mean vehicle speed is 5.0 km/h), the time lag τ_1 of the steering angle and τ_2 of the rider's lean angle to the bank angle of the frame were positive. The rider is shown to maintain passive control by which the bank angle of the frame precedes the maintenance of stability in upright and straight operation. It was expected that several experimental values would exceed the upper limit obtained by the theoretical calculation, since, in the equation of motion used in the present study, we neglected the large steering angle and very large frictional moment of the tire on the contact surface in steering at extremely low speeds, as shown in Fig. 6.

Even at the indicated speed of 10 km/h, just as at an extremely low speed, the time lags τ_1 and τ_2 are both positive, and the steering angle and rider's lean angle followed the change in the bank angle of the frame, while some experimental values exceeded the calculated values. However, at the indicated speed of 40 km/h, the time lag τ_1 was below the upper limit obtained by calculation and almost negative. This is probably because operation is very stable due to the small steering angle and bank angle of the frame at high speed, yet the steering continues to maintain the course. In this case, the steering operation precedes the bank angle of the frame.

7. Conclusions

We studied the operation characteristics using calculations which take into account the influence of the motorcycle on the reaction of the rider with respect to the leaning of the rider's upper body and the moment of inertia of the rider. From the results of the theoretical calculation and the experimental values of a motorcycle of light mass at low running speed, it was possible to elucidate the relationships between the bank angle of the frame, the rider's steering operation and the rider's upper body.

The main conclusions obtained were as follows:

(1) The lower limit in the stable region of the proportional coefficient κ_1 of the steering angle to the bank angle of the frame is large at low speeds and small as the speed increases. The lower limit of κ_1 is small when the proportional coefficient κ_2 of the rider's lean angle to the bank angle of the frame is large.

(2) In the case of the proportional coefficient $\kappa_2 = 0$, the upper limit of the time lag τ_1 is large at low speeds, and with increasing speed, the upper limit of τ_1 becomes small.

(3) In the case of the proportional coefficient $\kappa_2 = 0$ at low speed, the upper limit of the time lag τ_1 is defined but τ_2 is not limited. As the speed increases, the upper limits of τ_1 and τ_2 are defined and these values are small.

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