An Unsteady Flow Rate Measurement Method Utilizing Dynamic Characteristics between Pressure and Flow Rate along a Pipeline*

(A Method Using Pressures at Two Separated Points)

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This paper describes an approach for estimating unsteady flow rate through oil hydraulic pipelines and components in real time. In this method, unsteady flow rate is estimated by using the hydraulic pipeline dynamics and the measured pressures at two distant locations along the pipeline. Unsteady flow rate in the time domain is obtained by convolutions between the weighting functions and the measured values of pressure. The weighting functions are obtained from the transfer functions by application of the inverse fast Fourier transform (IFFT). By scheming the high-speed operations of convolutions, data processing in a microcomputer is achieved in real time, using exclusive precalculated tables. Under unsteady laminar oil flow conditions, the good agreement between the estimated and measured flow rate waveforms illustrates the validity of the method proposed here.

Key Words: Unsteady Flow Rate Measurement, Fluid Power Systems, Hydraulic Pipeline Dynamics, Real-Time Data Processing System

1. Introduction

In the field of hydraulic control systems, intelligent control can be comparatively easily realized according to the rapid development of microelectronics. From the standpoint of realizing intelligent control, and elucidating and improving the dynamic characteristics of hydraulic control systems, measurement of the unsteady flow rate is indispensable. There is also a demand for a new real-time flow sensor with improved performance in terms of high accuracy and fast response under high-pressure conditions. Several methods for estimating unsteady flow rate in real time have been proposed and developed for these purposes in previous research studies⁽¹⁾⁻⁽³⁾.

Most recently, we proposed a remote and a quasi-remote instantaneous flow rate measurement method^{(4),(5)}. In these methods, unsteady flow rate can be remotely estimated by using the distributed param-

eter models of the hydraulic pipeline between two distant cross sections along the pipeline. Using these methods, it is possible to measure unsteady flow rate through an arbitrary cross section of the pipeline where it is difficult to install an instantaneous flowmeter at the desired location. These methods require an instantaneous flowmeter. In the present situation, it is an obstacle to procure an instantaneous flowmeter with a fast response and high accuracy, but pressure-sensing instruments (pressure transducers) with a fast response and high accuracy may be available at reasonable cost. Lee and Srinivasan⁽⁶⁾ presented a transient hydraulic flow rate computational method which used two pressure measurements at the ends of a long straight pipeline and dynamic models of the hydraulic pipeline, and simulation results by finite difference method are indicated. However, they did few investigations involving experiments on unsteady flow rate measurement and dynamic response.

In this paper, we propose a new approach for estimating unsteady flow rate in real time under high-pressure hydraulic control systems. An instantaneous flow rate is estimated by making use of the dynamic characteristics between pressure and flow rate, and

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measuring pressures at two distant locations (upstream and downstream sections in Fig. 1(a)) along the pipeline. There is no need to install an instantaneous flowmeter for estimating unsteady flow rate in this method. Under unsteady laminar oil flow conditions, the estimated flow rate waveforms are compared with directly measured results by a cylindrical choke-type instantaneous flowmeter^{(2),(3)}. By scheming the high-speed numerical operations of discrete convolutions, data processing in a microcomputer system is also achieved in real time by utilizing only a software algorithm. The method proposed here is useful in estimating unsteady flow rate in real time through an arbitrary cross section of pipelines and components in hydraulic control systems without installing an instantaneous flowmeter.

2. Principle of Unsteady Flow Rate Estimation

2.1 Principle

Consider an axisymmetric, compressible and viscous laminar liquid flow for a circular straight pipeline with constant cross-sectional area as shown in Fig. 1(a). The dynamic characteristics between pressure and flow rate variables at two distant cross sections (upstream and downstream sections in Fig. 1(a)) along the pipeline are expressed by a four-terminal network⁽⁷⁾. In the transfer matrix description, they are given by

$$\begin{pmatrix} Q_u(s) \\ Q_d(s) \end{pmatrix} = \begin{bmatrix} \frac{1}{Z_0(s) \cdot \tanh\{\lambda(s) \cdot L\}} & -1 \\ \frac{1}{Z_0(s) \cdot \sinh\{\lambda(s) \cdot L\}} & -1 \end{bmatrix} \begin{pmatrix} P_u(s) \\ P_d(s) \end{pmatrix} \quad (1)$$

in which L is the length of the pipeline between

Fig. 1 (a) Schematic of hydraulic pipeline

Fig. 1 (b) Block diagram representation

upstream and downstream sections and s is the Laplace operator. $P(s)$ and $Q(s)$ denote Laplace transforms of the deviations of pressure $p(t)$ and flow rate $q(t)$ at a cross section of the pipeline, respectively. And subscripts u and d refer to upstream and downstream sections. Block diagram representation of transfer matrix Eq. (1) is shown in Fig. 1(b).

With consideration of the frequency-dependent viscous friction, the propagation constant $\lambda(s)$ and characteristic impedance $Z_0(s)$ of the pipeline are given as follows⁽⁸⁾;

$$\lambda(s) = \frac{s}{c} \cdot \left\{ 1 - \frac{2J_1(ja\sqrt{s/\nu})}{ja\sqrt{s/\nu} J_0(ja\sqrt{s/\nu})} \right\}^{-1/2} \quad (2)$$

$$Z_0(s) = \frac{\rho c}{\pi a^2} \cdot \left\{ 1 - \frac{2J_1(ja\sqrt{s/\nu})}{ja\sqrt{s/\nu} J_0(ja\sqrt{s/\nu})} \right\}^{-1/2} \quad (3)$$

where a is inner radius of the pipeline, c is sonic velocity in the oil-filled pipeline, $J_0(x)$ and $J_1(x)$ are Bessel functions of the first kind of orders zero and one, $j = \sqrt{-1}$, ν is kinematic viscosity of the oil, and ρ is density of the oil.

Judging from Eq. (1), it is possible that an instantaneous flow rate through the upstream or downstream section in Fig. 1(a) can be estimated from measurement of the upstream and downstream pressure $p_u(t)$ and $p_d(t)$. With this method, there is no necessity of inserting an instantaneous flowmeter at the desired location to obtain flow rate information.

Here, we investigate estimation of the unsteady flow rate through the upstream section in Fig. 1(a). From the transfer matrix Eq. (1), the upstream flow rate in the Laplace domain is given by

$$Q_u(s) = G_u(s) \cdot P_u(s) - G_d(s) \cdot P_d(s) \quad (4)$$

where

$$G_u(s) = \frac{1}{Z_0(s) \cdot \tanh\{\lambda(s) \cdot L\}} \quad (5)$$

$$G_d(s) = \frac{1}{Z_0(s) \cdot \sinh\{\lambda(s) \cdot L\}} \quad (6)$$

In Eqs. (5) and (6), $G_u(s)$ represents the transfer function relating upstream flow rate $Q_u(s)$ to upstream pressure input $P_u(s)$, and $G_d(s)$ is the transfer function relating upstream flow rate $Q_u(s)$ to downstream pressure input $P_d(s)$.

Figure 2(a) and (b) show the frequency characteristics (solid line) of transfer functions $G_u(s)$ and $G_d(s)$ for the length of the pipeline $L=170$ cm, inner radius of the pipeline $a=0.7$ cm, kinematic viscosity of the oil $\nu=0.4$ cm²/s at the oil temperature 35°C, and sonic velocity in the oil $c=1.35 \times 10^5$ cm/s. The ordinate represents gain and phase angle; the abscissa denotes frequency.

The upstream flow rate is theoretically given by transforming Eq. (4) into the time domain. By taking the inverse Laplace transform of Eq. (4), upstream flow rate $\bar{q}_u(t)$ is given by the following convolution

integrals.

$$\bar{q}_u(t) = \int_0^t g_u(\tau) \cdot p_u(t-\tau) d\tau - \int_0^t g_d(\tau) \cdot p_d(t-\tau) d\tau \quad (7)$$

$$g_u(t)=0, \quad g_d(t)=0 \quad \text{for } t < 0$$

where $g_u(t)$ and $g_d(t)$ are the weighting functions corresponding to transfer functions $G_u(s)$ and $G_d(s)$.

Equation (7) is discretized with respect to time to estimate unsteady flow rate in real time by implementing the convolutions using a microcomputer system, so that the difference equation is described by the following discrete convolution summations.

$$\bar{q}_u(n\Delta T) = \sum_{i=0}^{N-1} g_u(i\Delta T) \cdot p_u(n\Delta T - i\Delta T) \cdot \Delta T - \sum_{i=0}^{N-1} g_d(i\Delta T) \cdot p_d(n\Delta T - i\Delta T) \cdot \Delta T \quad (8)$$

where ΔT is sampling time interval, N is the number of discrete convolution terms, and implementing time of discrete convolutions T_0 is given by $\Delta T \times N$.

2.2 Derivation of the weighting functions

To estimate the unsteady flow rate based on the above-mentioned principle, it is necessary to obtain the weighting functions $g_u(t)$ and $g_d(t)$ in the time domain. The weighting functions $g_u(t)$ and $g_d(t)$ corresponding to the transfer functions $G_u(s)$ and $G_d(s)$ can be theoretically obtained by inverse Laplace transforms. In practice, the evaluation of the inverse Laplace of Eqs. (5) and (6) is highly complex and time-consuming. In this paper, an efficient numerical

method applying the inverse fast Fourier transform (abbreviated as IFFT) is utilized to obtain the weighting functions. This method is relatively simple. The weighting functions $g_u(t)$ and $g_d(t)$ are obtained as follows:

$$g_u(k\Delta T) = \Delta f \cdot \sum_{n=0}^{N_s-1} G_u^*(j2\pi n\Delta f) \cdot \exp(-j2\pi nk/N_s) \quad (9)$$

$$g_d(k\Delta T) = \Delta f \cdot \sum_{n=0}^{N_s-1} G_d^*(j2\pi n\Delta f) \cdot \exp(-j2\pi nk/N_s) \quad (10)$$

$$k=0, 1, 2, 3, \dots, N_s-1$$

where $G_u^*(j2\pi n\Delta f)$, $G_d^*(j2\pi n\Delta f)$ are complex conjugates of $G_u(j2\pi n\Delta f)$ and $G_d(j2\pi n\Delta f)$, respectively. Here, ΔT is sampling time interval, and N_s is total number of samples. The angular frequencies in Eqs. (9) and (10) are determined from $\omega = 2\pi f$, where $f = n\Delta f$, $n=1, 2, 3, \dots, N_s-1$, and frequency step size (spacing between frequency samples) Δf is given by $1/(N_s \cdot \Delta T)$. In actual computations, the fast Fourier transform (abbreviated as FFT) procedure based on the Sande-Tukey algorithm is used⁽⁹⁾.

Figures 3(a) and (b) show typical examples of the weighting functions $g_u(t)$ and $g_d(t)$ for the length of pipeline $L=170$ cm, the number of samples $N_s=2048$ and sampling time interval $\Delta T=0.2$ ms. In this case, the high cut-off frequency is 2.5 kHz for the impulse input.

The validity of the weighting functions obtained

Fig. 2 (a) Frequency characteristics $G_u(j\omega)$

Fig. 2 (b) Frequency characteristics $G_d(j\omega)$

Fig. 3 (a) The weighting function $g_u(t)$

Fig. 3 (b) The weighting function $g_d(t)$

here is also investigated in comparing theoretical values of frequency response characteristics of $G_u(j\omega)$ and $G_d(j\omega)$ with the results of convolution between the weighting functions and sinusoidal input for various frequencies. The results are plotted in Figs. 2(a) and (b), where the solid lines represent the gain and phase angle calculated from Eqs. (5) and (6). The results of convolutions are indicated by the symbols $\bigcirc$, respectively. In comparison, experimental gains and phase angles are in good agreement with the theoretical values up to about 500 Hz in the frequency range, and show the validity and accuracy of weighting functions obtained here.

3. Experiments

3.1 Experimental apparatus

Figure 4 shows a schematic diagram of the experimental apparatus used in this study. In Fig. 4, the experimental apparatus consists of the oil hydraulic pipeline, data acquisition and a real-time numerical operating system. Straight circular copper pipelines 58 cm and 170 cm long with an inner diameter of 1.4 cm are used for the test pipeline between upstream and downstream sections.

In the experiment, the upstream pressure $p_u(t)$ and downstream pressure $p_d(t)$ as input data are directly measured by the semiconductor type pressure transducers (the resonant frequency is about 100 kHz). The cylindrical choke-type instantaneous flowmeter (abbreviated as CCFM) is used for measuring unsteady flow rate through the upstream (CCFM1) and downstream sections (CCFM2) in order to compare with the estimated results by this method. It has been experimentally confirmed that the CCFM can be measured by the change of unsteady flow rate in high frequencies above 200 Hz in high pressure oil hydraulic systems^{(2),(3)}. In CCFM, the values for the pressure difference dp_u and dp_d , caused by flow rates $q_u(t)$ and $q_d(t)$ through cylindrical choke #1 and choke #2 are

Fig. 4 Experimental apparatus

measured by pressure difference pickups which are made up of two semiconductor-type pressure transducers, and fed into a microcomputer system through low drift differential amplifiers and an A/D converter (8 channels). An instantaneous flow rate $q(t)$ through the upstream or downstream section in Fig. 4 is computed in real time by making use of dynamic characteristics between the pressure difference dp and flow rate $q(t)$ ^{(2),(3)}. Cylindrical choke #1 and choke #2 are used for the same dimensions: inner diameter of the choke $d=2.55$ mm and length $l=15$ mm.

The hydraulic pipeline is connected to a power source of constant pressure at the upstream end, and a servo valve is set at the downstream end in order to generate unsteady flow conditions in the hydraulic pipeline system. Oil temperature is measured by a thermistor-type thermometer.

In the experiment, oil temperature is maintained at about $35 \pm 1^\circ\text{C}$, and the kinematic viscosity of oil is $0.4 \text{ cm}^2/\text{s}$ at 35°C .

3.2 Experimental results and comparisons

The accuracy and response of the estimated flow rate $\hat{q}(t)$ are compared with the directly measured one $q(t)$ by CCFM under unsteady laminar oil flow conditions which are generated by a servo valve.

Figure 5 depicts typical examples of recorded pressure and flow rate waveforms for a rectangular wave input 10 Hz and the length of the pipeline $L=170$ cm. The ordinate represents pressure and flow rate; the abscissa denotes time. The top and 2nd waveforms indicate the measured upstream and downstream pressures $p_u(t)$ and $p_d(t)$, which are input data in the method proposed here. The 3rd one represents the

Fig. 5 Comparison of the estimated and measured upstream flow rate waveforms

estimated upstream flow rate waveform $\hat{q}_u(t)$ for the number of convolution terms $N=1250$. In this case, the weighting functions are utilized for sampling time interval $\Delta T=0.2$ ms shown in Figs. 3(a) and (b). The bottom one is the upstream flow rate $q_u(t)$ directly measured by CCFM1. The 4th one shows the estimated and measured flow rate waveforms at the same coordinate. In comparison, the estimated flow rate waveform $\hat{q}_u(t)$ is in good agreement with the directly measured one $q_u(t)$.

Similarly, it is possible that the downstream flow rate $q_d(t)$ is estimated from the pressure measurements $p_u(t)$ and $p_d(t)$. From transfer matrix Eq. (1), the transfer function relating downstream flow rate $Q_d(s)$ to upstream pressure input $P_u(s)$ is given by $G_u(s)=1/[Z_0(s)\cdot\sinh\{\lambda(s)\cdot L\}]$, and the transfer function relating the downstream flow rate $Q_d(s)$ to the downstream pressure input $P_d(s)$ is also given by

Fig. 6 Comparison of the estimated and measured downstream flow rate waveforms

Fig. 7 Comparison of the estimated and measured upstream flow rate waveforms

$G_d(s)=1/[Z_0\cdot\tanh\{\lambda(s)\cdot L\}]$. The weighting functions $g_u(t)$ and $g_d(t)$ can be obtained from Eqs. (9) and (10).

Figure 6 depicts the comparison of the estimated results with the measured downstream flow rate waveforms $q_d(t)$. In this case, the input pressure data $p_u(t)$ and $p_d(t)$ are used for rectangular wave input 10 Hz and the length of the pipeline $L=170$ cm, as shown in Fig. 5.

Figure 7 shows the estimated results $\hat{q}_u(t)$ with measured upstream flow rate $q_u(t)$ for the length of the pipeline $L=170$ cm and rectangular wave input 30 Hz.

The estimated result for the length of the pipeline $L=58$ cm, subject to rectangular wave input 40 Hz, is shown in Fig. 8.

3.3 Real-time processing of unsteady flow rate estimation

In order to estimate unsteady flow rate in real time by using a microcomputer system, it is necessary to determine the sampling time interval ΔT , the number of the discrete convolution terms N and the implementing time of the discrete convolutions T_0 , to discretize the weighting functions, and to digitize the measured pressure input data.

To carry out high-speed operations in Eq. (8), it is necessary to use the values of the weighting functions within a finite time interval. Consider the determination of the implementing time of discrete convolutions T_0 , that is, the truncation time of the weighting functions obtained here. The value of T_0 should be as large as possible from the viewpoint of the accuracy of the operations.

As shown in Figs. 3(a) and (b), the consecu-

Fig. 8 Comparison of the estimated and measured upstream flow rate waveforms

tive amplitudes of the weighting functions become sufficiently small over the time period $t=250$ ms. In addition, from the result of the estimated upstream flow rate waveforms $\hat{q}_u(t)$ for $N=1250$ and $\Delta T=0.2$ ms (equivalent to $T_0=250$ ms) in Fig. 5, it has already been confirmed that the proper value of the implementing time of convolutions T_0 is approximately equal to 250 ms in this case. The proper value of T_0 , however, differed with the change of system parameters such as the length of the pipeline L , kinematic viscosity of the oil ν , sonic velocity in the oil-filled pipeline c , and inner radius of the pipeline a . Here, the

Fig. 9 (a) Logarithmic decrement of the envelope of the weighting functions with elapsed time

Fig. 9 (b) Truncation time of the weighting functions

Fig. 10 Comparison of the estimated and measured flow rate waveforms

two factors with time dimensions T_c and T_v , i.e., the ratio of the length of the pipeline L to sonic velocity c and the ratio of the square value of the inner radius a^2 to kinematic viscosity ν , are used to determine a measure for the implementation time of the discrete convolutions.

Figure 9(a) shows the logarithmic decrement of the envelope of weighting function $g_u(t)$ for various values of T_v against $T_c=1.26$ ms. In Fig. 3 ($T_c=1.26$ ms, $T_v=1.23$ s), the logarithmic value of the amplitude of the weighting function at $t=250$ ms is approximately equal to 0.8. This value is used as a measure for determining the implementing time of discrete convolutions T_0 . Fig. 9(b) is a plot of T_0 versus T_v for the cases of $T_c=1.26$ ms and $T_c=0.43$ ms.

In Fig. 9(b), symbol $\square$ represents $T_c=1.26$ ms ($L=170$ cm, $c=1.35 \times 10^5$ cm/s) and symbol $\circ$ represents $T_c=0.43$ ms ($L=58$ cm, $c=1.35 \times 10^5$ cm/s).

Figure 10 shows the influence of the estimated upstream flow rate waveforms for sampling time $\Delta T=0.8$ ms. In Fig. 10, the estimated flow rate waveforms are scarcely affected within this sampling time range in comparison with the result of Fig. 5.

Next, consider the implementation of numerical operations in real time. In digital high-speed operations, the number of multiplication and addition processes amounts to N times both the first and the second term in Eq. (8). As the operation time of discrete convolutions in Eq. (8) for estimating the flow rate at one discrete instant of time becomes very long, it is difficult to estimate unsteady flow rate within the sampling time interval ΔT .

In order to make the operation time shorter than the sampling time interval ΔT , the following operation technique is used⁽⁵⁾. In advance, the results of multiplication of 8 bit (weighting functions) $\times$ 8 bit (measured pressure data) for all possible discrete input and output data are made into a table and stored on the memory of the microcomputer system. By

Fig. 11 Table for high-speed multiplication

using this table, the results of multiplication in discrete convolutions are obtained by searching and referring to the table corresponding to the input data, instead of performing direct multiplication $g_u(t) \times p_u(t)$ and $g_d(t) \times p_d(t)$ in Eq. (8). A schematic of the table for high-speed multiplication is shown in Fig. 11.

The results of discrete convolution operations in Eq. (8) are obtained by carrying out only the movement of input/output data and summations by using this operation technique; therefore, data processing in the microcomputer system is achieved in real time by use of software only.

In Fig. 10, the estimated flow rate waveform at the bottom represents the results of real-time operations at sampling time $\Delta T = 0.8$ ms. Although the resolution decreased slightly at the steady state, the response of the estimated flow rate $\hat{q}_u(t)$ is in good agreement with the measured one $\hat{q}_u(t)$ by CCFM1.

4. Conclusions

A new approach for estimating unsteady flow rate in real time is proposed and experimentally investigated in this paper. The estimated flow rate waveforms are compared with directly measured ones at the same instant by a cylindrical choke-type instantaneous flowmeter. The results show a good agreement between the estimated and measured flow rate waveforms under unsteady laminar oil flow conditions.

By improving and scheming the high-speed numerical operations of discrete convolution summations, data processing of the flow rate measurement system is realized in real time by the use of software only.

The method proposed here is useful in estimating unsteady flow rate through an arbitrary cross section in the hydraulic pipelines and components without installing an instantaneous flowmeter at the desired location to obtain instantaneous flow rate information.

Finally, it is expected that this method would be

suitable for measuring the dynamic response under high-pressure oil hydraulic control systems.

References

- (1) Nakano, K., Yokota, S. and Miyamoto, T., A Study of Instantaneous Flowrate Measurement in a High-Frequency Region, Based on Hydraulic Pipeline Dynamics, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 52, No. 473, B(1984), p. 421.
- (2) Nakano, K. and Yokota, S., Unsteady Flowrate Sensor by Making Use of Dynamic Characteristics of Cylindrical Chokes, *Proc. of 11th Triennial World Congress of the IMEKO, Houston, Vol. Sensors*, (1988), p. 39.
- (3) Yokota, S. and Nakano, K., On Extending the Intelligent Potential of a Cylindrical Choke-Type Instantaneous Flowmeter, *Proc. of 2nd International Conference on Flow Measurement, London*, (1988), p. 35.
- (4) Yokota, S., Kim, D. T. and Nakano, K., An Approach for Remote Measurement of Instantaneous Flowrate by Making Use of Hydraulic Pipeline Dynamics, *J. Japan Hydraulics and Pneumatics Soc.*, (in Japanese), Vol. 21, No. 2 (1989), p. 187.
- (5) Kim, D. T., Yokota, S. and Nakano, K., An Approach for Remote Instantaneous Flowrate Measurement by Making Use of Hydraulic Pipeline Dynamics (The Quasi-Remote Instantaneous Flowrate Measurement Method in Real Time), *J. of Japan Hydraulics and Pneumatics Soc.*, (in Japanese), Vol. 22, No. 2 (1990), p. 169.
- (6) Lee, S. R. and Srinivasan, K., A Method for the Determination of Transient Hydraulic Flow Rate from Dynamic Pressure Measurements, *ASME Paper*, No. 87-WA/DSC-36, Boston, Massachusetts, December, (1987).
- (7) D'souza, A. F. and Oldenburger, R., Dynamic Response of Fluid Line, *Trans. ASME*, Vol. 86, No. 3, D (1964), p. 589.
- (8) Brown, F. T., The Transient Response of Fluid Line, *Trans. ASME*, Vol. 84, No. 4, D(1962), p. 547.
- (9) Brigham, E. O., *The Fast Fourier Transform*, (1974), Prentice-Hall.