Digital Control of Electrohydraulic Servo System Operated by Differential Pulse Width Modulation*

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In the preceding papers, the authors proposed a new method of differential PWM for a hydraulic actuator operated by two 3-way solenoid valves. An arbitrary pulse width of the pressure difference across both sides of the actuator piston was realized by adjusting the switching time of each valve. The actuator operated by differential PWM shows good linearity as a control element, achieving accurate positioning. This study, as an application, deals with a hydraulic servo system composed of a 4-way spool valve and a load cylinder, where the spool valve is driven directly by the differential PWM actuator. The servo system is designed based on the optimal regulator method of the state variable model. The system performance is investigated through simulations and experiments. Consequently, it is shown that the actuator, operated by differential PWM, plays the role of a linear control element, and that the servo system is well designed by the optimal regulator.

**Key Words**: Fluid Power Systems, Computer Control, Mechatronics, Electrohydraulic Servo Systems, Differential PWM, Optimal Control, Digital Control

1. Introduction

Since high-speed on/off solenoid valves are attracting attention as powerful interface equipment uniting a hydraulic control system and electronics, extensive research is being conducted in an attempt to develop intelligent hydraulic control systems(1)(6).

In the preceding papers, the authors proposed a new concept of differential PWM for a hydraulic actuator operated by two 3-way solenoid valves. In the papers, the following features of the differential PWM were confirmed.

1) The operating characteristic of the differential PWM actuator is represented by the duty difference \(\Delta D(=D_h-D_l)\) versus the average value of the pressure difference for driving the actuator \(\Delta p\) (which is the mean value of \(\Delta p(=p_h-p_l)\) over the time interval \(T_c\)). Although the characteristic of the conventional PWM method shows a serious nonlinearity in the vicinity of the origin, in the differential PWM method, the nonlinearity vanishes.

2) As a result of the good linearity, accurate positioning of the actuator can be achieved by this method.

This study, as an application, deals with a hydraulic servo system which is composed of a 4-way spool valve and a load cylinder, where the spool valve is driven directly by the differential PWM actuator. The servo system is designed based on the optimal regulator method of the state variable model. The system performance is investigated through simulations and experiments based on a mathematical model, in order to confirm the effectiveness of the system and validity of the control design.

As is well known, the nonlinearities which are present in a hydraulic control system make it difficult to apply the control theory. In this study we look also into this problem.
2. Nomenclature

$A_1$: cross-sectional area of actuator piston
$A_2$: cross-sectional area of load piston
$b_{r1}$: equivalent viscous damping coefficient of actuator
$b_{r2}$: equivalent viscous damping coefficient of load piston
$c_r$: flow coefficient of spool valve
$D_1, D_2$: duty \( = (\text{actuating time})/T_c \)
$\Delta D$: differential duty \( = D_1 - D_2 \)
$\Delta p_0$: differential pressure acting on actuator
$\Delta p_0':$ time average of $\Delta p_0$ \( = \int_0^{T_s} \Delta p_0 dt/T_s \)
$\Delta p_2$: differential pressure acting on load piston \( = p_a - p_b \)
$f_c$: frequency of PWM carrier wave \( f_c = 1/T_c \)
$f_s$: frequency of sampling \( f_s = 1/T_s \)
l, l: length of pipeline
$m_1$: mass of actuator
$m_2$: mass of load piston
$p_a, p_b$: pressures in two actuator cylinder chambers
$p_c, p_d$: pressures in two load cylinder chambers
$p_s$: supply pressure (constant)
r$: circumference of spool land
$t_a, t_b, t_c, t_d$: time parameters of valve switching
$t_0$: nominal value of time \( t_0 = 2.5 \times 10^{-3} \) sec
$u$: control input \( = \Delta D \)
$\rho$: mass density of fluid
$x$: displacement of actuator
$x_i$: nominal value of displacement $x$
$x$: state variable \( j = 1 \sim 4 \)
y: displacement of load piston
$y_o$: nominal value of displacement $y$
x: state variable vector
$y':$ output variable vector
$F, g, h$: matrix for continuous-time system
$A, b, c$: matrix for discrete-time system

$k$: feedback coefficient vector

3. System Constitution and Mathematical Model

3.1 System constitution and control method

The schematic diagram of the hydraulic servo system employed in this study is shown in Fig. 1. Basically, the system consists of a hydraulic actuator driven by two 3-way solenoid valves (small cylinder), a load cylinder which is connected to a 4-way spool valve, and a microcomputer used as a control operator. The actuator, which is discussed in the preceding paper, is composed of two solenoid valves and a small cylinder. The present system is combined with a spool valve and a load piston. Both displacements of the actuator $x$ (displacement of the spool) and of the load cylinder $y$ are measured by differential transformers and input into the computer through A/D converters. The control input $u$ is calculated by the computer based on the control algorithm and converted into the duty signals $D_1$ and $D_2$. The on/off signal is supplied to the solenoid valves through the valve driving amplifier. The pressure difference, which is a result of the action of the solenoid valves, drives both the actuator and the spool valve connected to the actuator. The load piston is driven by the flow rate of oil according to the displacement of the spool valve.

3.2 Mathematical model of the control system (state equation)

The process for constructing the mathematical model for the control system is shown in Fig. 1. Nonlinear characteristics such as the frictional force acting on the piston, the flow rate characteristic of the spool valve (pressure versus flow rate) and the saturated characteristic of displacement of the spool valve should be taken into consideration. In order to derive the state equation, these nonlinearities are linearized in this section.

The following assumptions are made in order to

Fig. 1 PWM hydraulic servo system
obtain the mathematical model of the system.

(1) The spool valve is a zero lap (ideal valve)
and the flow coefficient is constant.

(2) The supply pressure from the pump \( p_s \) is constant.

(3) Compressibility of oil is neglected.

(4) Unsteady flows in the pipelines are neglected.

(5) The frictional forces (viscous damping force and Coulomb friction force) acting on the actuator and the load piston are regarded as equivalent viscous damping coefficients.

First, the equation of motion for the actuator is written as

\[
m \dot{z} + \frac{b_1}{t} \dot{z} = A_1 \Delta p_i
\]

where \( \Delta p_i \) is the time average of the differential pressure pulse given by the differential PWM method. Since the differential PWM has a linear property in the \( \Delta P = \Delta p_i \) characteristic, \( \Delta p_i \) is proportional to the control input \( u = AD \) and the following equation is obtained (where \( AD_i \) is a large region and the saturation zone is neglected).

\[
\Delta p_i = \Delta p_{\text{max}} \cdot u
\]

Second, the equation of motion for the load piston is written as

\[
m \dot{y} + \frac{b_2}{t} \dot{y} = A_2 \Delta p_2
\]

Volumetric flow through the control valve \( q_a \) (the load flow rate) is written as

\[
q_a = c_s r_x \sqrt{\frac{1}{\rho} (p_s - |\Delta p_2|)}
\]

In the above equation, \( p_s \) is given as

\[
p_s = p_c + p_d
\]

Furthermore, neglecting the compressibility of oil, the relationship between load flow rate \( q_a \) and displacement of the load piston \( y \) is

\[
q_a = A_2 \frac{d p_2}{d t}
\]

By rearranging Eqs. (1) ~ (6) and using the dot notation for the time derivative, the equations take on the nondimensional forms.

\[
\dot{X} + B_{11} X = H_1 u
\]

\[
\dot{Y} + B_{22} Y = H_2 \Delta P_2
\]

\[
Q_a = c_s X \sqrt{\frac{1}{\rho} (1 - |\Delta P_2|)}
\]

\[
q_a = F \dot{Y}
\]

To solve the above equations, we introduce the following quantities.

\[
X = \frac{x}{x_0}, \quad Y = \frac{y}{y_0}, \quad T = t/t_0
\]

\[
q_s = r_s x_0 \sqrt{\frac{2}{\rho}} p_s/\rho, \quad Q_a = q_a/q_0
\]

\[
\Delta P_i = \Delta p_i/\Delta p_{\text{max}} = (u)
\]

\[
\Delta P_2 = \Delta p_2/p_s, \quad B_{11} = b_1 R_0/\rho
\]

\[
B_{22} = b_2 R_0/\rho, \quad F = A_2 y_0/(q c_0)
\]

\[
H_1 = A_1 \Delta p_{\text{max}} \ell/(m_1 x_0)
\]

\[
H_2 = A_2 y_0 \ell/(m_2 y_0)
\]

Next, we derive the state equations. The state variable vector is defined as

\[
\mathbf{x}(T) = (X, X_0, X, X_0, X, X_0)^T = (X, \dot{X}, Y, \dot{Y})^T
\]

By rearranging the terms in Eqs. (7) ~ (10) and substituting the state variables, we obtain the following normalized set of equations.

\[
\dot{X}_1 = X_1
\]

\[
X_2 = -B_{11} X_2 + H_1 u
\]

\[
X_3 = X_4
\]

\[
X_4 = -B_{22} X_4 + H_2 \left(1 - \frac{2(FX_4)^2}{c_s X_4^2}\right)
\]

The state differential equations involve the nonlinear equation (14), so we linearize the equations around the nominal state values \( X_i = X_{i0} \) and \( X_i = X_{i0} \) (namely, the state for which the spool valve is maintained at a constant displacement \( X_{i0} \) and the load piston is driven at a constant velocity \( X_{i0} \)), and obtain the state equation as

\[
\dot{x}(T) = Fx(T) + gu(T)
\]

where

\[
F = \begin{bmatrix}
0 & 1 & 0 & 0
0 & -B_{11} & 0 & 0
0 & 0 & 0 & 1
L_1 & 0 & 0 & L_2
\end{bmatrix}, \quad g = \begin{bmatrix} H_1 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\[
L_1 = 4H_2(FX_{i0})^2/(c_s X_{i0}^2)
\]

\[
L_2 = -B_{22} - 4H_2F^2X_{i0}^2/(c_s X_{i0}^2)
\]

with the output equation written as

\[
y(T) = h x(T)
\]

where

\[
h=[0, 0, 1, 0]
\]

Namely, Eqs. (15) denote the state equations of this system.

3.3 Discrete state equation

According to the principle of the differential PWM method, a differential pressure acting on the actuator is given by two differential pressure pulses with an interval which is equivalent to the period of the PWM carrier wave, \( T_c \). As shown in Fig. 2, during the first half of the carrier wave period \( (0 < t < T_c/2) \), the difference in the timing of the OFF-to-ON switching of valves 1 and 2 generates the first differential pressure pulse. Similarly, during the latter half \( (T_c/2 < t < T_c) \), the difference in the timing of the ON-to-OFF switchings of valves 1 and 2 generates the second differential pressure pulse. Therefore, if we introduce \( T_s = T_c/2 \), the average pressure acting on the actuator is given by the time average of the differential pressure pulse which is generated at intervals of \( T_s \). Then the state equation can be transformed to discrete time by the sampling period \( T_s \). (Incidentally, by the conventional PWM method, only one differential pressure pulse is given for each sampling interval of \( T_s \), so we must set
\( T_s = T_c \). Therefore, it is an advantage of the differential PWM method that the sampling period \( T_s \) can be set to half that of the conventional method.

With the above matrix \( F, g, h \), the discrete-time system, transformed by the zero-th-order holder, is obtained as

\[
x(k+1) = Ax(k) + bu(k) \quad (16.1a)
\]

\[
y(k) = cx(k) \quad (16.1b)
\]

where

\[
A = e^{Ts}, \quad b = \int_0^{Ts} e^{r}qdr, \quad c = h \quad (17)
\]

4. Digital Simulation of the System

In the previous section in which the state equation of the system was derived, we made some assumptions as well as linearizations. In order to validate these assumptions, we discuss the mathematical model of the system.

The following new assumptions are added in order to obtain the mathematical model for the simulation in this section.

(1) The saturation of the flow rate through the spool valve, which is related to the spool valve displacement, is taken into consideration.

(2) The compressibility of oil is taken into consideration.

(3) Two fluid lines, that is, \( l_1 \) and \( l_2 \) in Fig. 1, are taken into consideration, and it is assumed that the equation of motion for each fluid line is approximately described by that of a one-degree-of-freedom system.

(4) As for the frictional force acting on a piston, empirical values which include nonlinear terms, such as static friction and Coulomb friction, are employed.

(5) Equation (4) which describes the load flow rate is used in its nonlinear form.

In addition to the above assumptions, the following statements are made concerning the simulation.

(1) The function \( y_0 = f(t) \) which represents the switching behavior of the solenoid valve (the switching of the valve poppet displacement \( y_0 \) is caused by on/off action of input voltage \( v \)) is described by a composite function of a proportional element and a time lag element, as described in the previous paper\(^{40}\). (The switching behavior of the on/off valve can be characterized in terms of four time parameters \( t_a, t_b, t_c \) and \( t_d \).

(2) The saturation property of the spool valve can be simulated based on the measurements shown in Fig. 3 (where only measurement data for \( x > 0 \) is shown). The figure represents the relationship between the flow rate \( q_\pi \) and spool valve displacement \( x \) (port area). As seen in Fig. 3, the flow rate \( q_\pi \) approaches saturation with increasing absolute displacement \( |x| \). This property can be equally treated as a change of the flow coefficient \( C_x \) with respect to \( x \). Namely, the relationship given in Fig. 3 can be converted into the curve of the flow coefficient \( C_x \) versus displacement \( x \), which is given in Fig. 4. Thus in the simulation, the change of flow coefficient is approximated by a linear function which, as shown in the figure, is divided into three zones.

\[ \begin{align*}
&1 \quad |x| \leq 0.5 \\
&2 \quad 0.5 < |x| \leq 1.0 \\
&3 \quad |x| > 1.0
\end{align*} \]

Further discussion of the mathematical model of the other elements is omitted here for the sake of brevity, and a block diagram representation for the total system is given in Fig. 5. A detailed diagram for
the D-PWM actuator is given separately in Fig. 7 of the previous paper.\(^\text{40}\)

The algorithm for the digital simulation can be carried out using signals and transfer functions, as designated in Fig. 5.

5. Design and Performance of the Control System

5.1 Design of the control system

In this section, we design the digital control system (state feedback control system) based on the state equation (16) derived in section 3. Based on the consideration of the following points, the optimal regulator method is chosen for the controller in this study.

1. Since the optimal regulator is generally superior in terms of robust stability, control appears to be feasible despite saturation of the flow rate (namely, parameter fluctuation exists in the system).

2. Although a system designed by the optimal regulator sometimes causes a steady-state error due to continuous disturbance, if we increase the maximum driving force \( \Delta p_{\text{max}} = p_s A_2 \) for the piston in comparison with Coulomb friction, consideration of the above problem is rendered unnecessary.

3. In order to follow any desired input, we regard the input as a control error and set the error as an initial value, then it can be treated as a regulator problem.

Here, we consider the performance index for the state equation (16) as

\[
J(u) = \sum_{k=0}^{\infty} [x^T(k)Qx(k) + u^T(k)Ru(k)].
\]  

(18)

The control input \( u(k) \) which minimizes \( J(u) \) is given by

\[
u(k) = hx(k).
\]  

(19)

Optimal feedback gain \( h \) (optimal gain, hereafter) in the above function is written as

\[
h = -(b^T P b + R)^{-1} b^T P A
\]  

(20)

where, matrix \( P \) is positive definite symmetric and is a solution of the following Riccati equation.

\[
P = A^T P A - A^T P b(b^T P b + R)^{-1} b^T P A + Q.
\]  

(21)

In this study, we chose the weighting matrices \( Q \) and \( R \) in the above function by using a trial-and-error method to obtain the desired response with respect to the simulated results.

In designing the system, it is necessary to decide the equivalent viscous damping coefficients \( b_1 \) and \( b_2 \) of Eqs. (1) and (2), and the nominal state values \( X_{10} \) and \( X_{20} \) for the linearization of Eq. (14). The method by which these parameters are decided is explained in the following section.

The actuator operated by the differential PWM method is driven by a constant differential pressure \( \Delta p_{\text{max}} \). When the actuator is driven at a constant velocity under a constant driving force, Eq. (1) can be written as

\[
b_1 \dot{x} = A_1 \Delta p_{\text{max}}.
\]  

(22)

By substituting the maximum differential pressure \( \Delta p_{\text{max}} = 5.34 \times 10^{-2} \text{ MPa} \) and the velocity of the piston \( \dot{x} = 7.56 \times 10^{-2} \text{ m/s} \) into the above equation, \( b_1 = 2.81 \times 10^6 \text{ Ns/m} \) is obtained.

Secondly, for deciding the nominal state values for linearizing equation (14), that is, port area of spool \( x_0 \) and velocity of load piston \( \dot{y}_0 \) which correspond to \( X_{10} \) and \( X_{20} \), the following issues are considered.

To achieve accurate positioning of the load piston, it is important to maintain accurate positioning around a minute displacement of the spool valve. On the other hand, according to the characteristics of the spool displacement versus flow rate, as shown in Fig. 3, the limit of \( x \), which is considered linear, is approximately \( x < 0.2 \text{ mm} \). Hence we choose \( x_0 = 0.1 \text{ mm} \) and \( \dot{y}_0 = 4.4 \times 10^{-5} \text{ m/s} \) for the velocity of the piston. The flow coefficient corresponding to the above \( x_0 \) is \( C_s = 0.785 \).

Similarly, the equivalent viscous damping

Fig. 5 The block diagram of control system

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coefficients $b_{r2}$ for the load piston can be decided. Based on the standard velocity $\tilde{y}$ and the differential pressure $\Delta p_d=2.81 \times 10^{-2}$ MPa, we obtained $b_{r2}=1.36 \times 10^{3}$ Ns/m.

Other system parameters are given in Table 1. Moreover, the lap of the spool valve can be regarded as zero lap because the measured maximum value of the processing error of both the overlap and the underlap was less than 2 µm. Furthermore, the solenoid valve which is used in this study is the same as the one used in the previous paper. As for the switching characteristics, please refer to Ref. (6).

5.2 Performance of the control system

In this section, we compose the experimental system based on the design created in the previous section and compare the performance of position control, that is, step response, with the simulated result. We further discuss a system in which the actuator is operated by the conventional PWM method and compare the results.

First, we examine the system when it is driven by the differential PWM method, shown in Fig. 6. The figures describe the system response to a reference step input of $r=2$. For the conditions for the experiment, the supply pressure was set at $p_{s}=0.98$ MPa, the carrier wave period for the PWM was set at $T_c=5 \times 10^{-2}$ s (=2T_n, $f_c=20$ Hz), and the sampling period $t_s$, necessary for computation of the control algorithm, was set at $t_s=5 \times 10^{-2}$ s. Furthermore, the velocities $\dot{x}$ and $\dot{y}$ are computed by the difference approximation from the displacements $x$ and $y$ which are measured in each sampling period. As explained in section 5.1 (point (3)), since a reference signal exists in this case, displacement $y$ which is used as a feedback component, is combined with the reference signal $r$ to produce the feedback variable $y-r$. Response curves for the actuator displacement $X(=x_{20})$ and the load piston displacement $Y(=y_{p0})$ are shown in Fig. 6(a). The control input $u$, the pressure difference $\Delta \dot{p}(=\Delta p_{d}/p_{s})$, the duty signals $D_1, D_2$ and $\Delta D$ applied to the valves, and the input duty $\Delta D$ are shown in Figs. 6(b), (c) and (d), respectively.

From these results, we can make the following statements.

After the step reference is input, by observing the continuous alteration of every response wave from the control input $u$ to the differential duty $\Delta D$ to the differential pressure $\Delta \dot{p}$ to the displacement of actuator $X$ and to the displacement of the load piston $Y$ in sequence, it is confirmed that the servo system is controlled rationally.

According to the waveform of the load piston's displacement $Y$, the response is determined through a moderate transient. Therefore the validity of the system design can be confirmed.

Corresponding to the experimental result, a simulated result is shown in Fig. 7. By comparing the curves, it can be confirmed that the results wholly agree with each other. Therefore the validity of the mathematical model derived in this study has been confirmed.

From the above discussion, the validity of the design based on linearization and the other assumptions that were made for a system with the saturation property of the spool and Coulomb friction, is confirmed.

Next, in the same manner, we discuss the dynamic performance of the system when it is driven

![Figure 6](image_url)
(a) Displacement of actuator and load piston

(b) Control input $u$

(c) Differential pressure $\Delta p_h$

(d) Input signals to valves

Fig. 7 Simulated results ($Q = \text{diag}[1, 0, 1, 0], R = 1, f_s = 40$ Hz)

(a) Displacement of actuator and load piston

(b) Control input $u$

(c) Differential pressure $\Delta p_h$

(d) Input signals to valves

Fig. 8 Experimental results by conventional PWM method ($Q = \text{diag}[1, 0, 1, 0], R = 100, f_s = 20$ Hz)

by the conventional PWM method. The results of the experiment corresponding to those in Fig. 6 are shown in Figs. 8 (a), (b), (c) and (d). (Although the carrier wave period $T_c$ is equal to the previous one, the sampling period $T_s$ is twice that of the previous one due to the reason explained previously.) It is observed in Fig. 8 (a) that the $y$ response oscillates around the desired value. The main cause for such an undesirable motion is probably the nonlinearity in the $\Delta D - \Delta p_h$ characteristic of the conventional PWM method. Such a steep inclination in the range of small $\Delta D(=u)$ renders precise position control difficult because a differential pressure pulse cannot be supplied in proportion to minute duty. Therefore, it is difficult to achieve accurate positioning by means of the conventional PWM method when there is serious nonlinearity in the control element.

A further experimental study with a 1-type servo system, combining state feedback and an integrator, was performed. The experimental result for the differential PWM method showed an overshoot around the desired value, and the response was not improved. Through simulation, it is clarified that the saturation property of the spool is the principal cause.

(The simulated result without saturation did not have any overshoot.) From the above result, it is clear that for a system with parameter fluctuations such as variation in the flow coefficient followed by spool displacement, it is recommended that the design of the controller be based on the optimal regulator, which is robust, rather than on the 1-type servo system.

6. Conclusions

In this paper, as an application of the differential PWM method, we dealt with a hydraulic servo system composed of a 4-way spool valve and a load cylinder where the spool valve is driven directly by the differential PWM actuator, and the system design is based on the optimal regulator method of the state variable model. Dynamic response of the system was investigated through experiments and digital simulation. The results obtained are summarized below.

(1) Through the results of the step response experiment, the stability and good response of the system were confirmed, and the effectiveness of the optimal regulator for a system with parameter fluctuations was established.

(2) A system driven by the conventional PWM
method showed a worse response and therefore, the merits of the differential PWM method can be stated: ① it has good linearity as a control element and ② the sampling period can be reduced to half that of the conventional method.

(3) Simulated results of the system dynamics were in agreement with the experimental results, and thus the validity of the mathematical model proposed in this study was confirmed.

References


