Position Control of a Flexible Arm Using a Dither Signal*

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In this paper, we present the feedback control of a one-link flexible arm having reduction gear friction. Tip position control was very difficult when large friction was treated as an equivalent viscous friction. Experiments showed that tip position feedback control was possible using the dither signal. We investigated the influence of the dither amplitude on linearizing the nonlinear friction and the response of feedback control. The acceleration signal from the accelerometer at the arm tip, like the signal from the strain gauge at the arm root, could use feedback control for the fundamental vibration mode.

Key Words: Positioning, Coulomb Friction, Nonlinear Vibration, Automatic Control, Transient Response, Robotics, Measurement, Dither, Flexible Arm

1. Introduction

Nonlinear friction containing Coulomb friction can be examined by, for example, regarding the friction as a stationary disturbance or estimating the friction using an adaptive observer⁽¹⁾. Another method involves linearizing the friction by use of the dither signal⁽²⁾ and an example of its application is the direct DC motor driving a flexible arm⁽³⁾.

In this paper, we showed that, in the feedback control of a one-link flexible arm having high nonlinear friction, positioning control was impossible using the equivalent viscous friction, but that such control was possible to linearize by applying a dither signal. Furthermore, we investigated the influence of the dither amplitude on linearizing and control. In addition, the acceleration signal at the arm tip was effective in feedback control for the fundamental vibration mode, like the strain gauge signal at the arm

root.

2. Control Model and Experimental Equipment

2.1 Control model

Figure 1 shows the model of the flexible arm. M_1 and m_1 are, respectively, the mass of the payload and elastic beam. J_1 , j_1 and J_0^* are the moments of the inertia of the payload, elastic beam and rotational

Fig. 1 Model of the one-link flexible arm

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joint. EI is the bending rigidity of the elastic beam. In addition, β is the angle of the rotational joint, ϕ is the angle of elastic bending deformation and ψ is the angle of payload rotation.

The difference between this model and the Shimoyama and Miura model⁽⁴⁾ is the consideration of the length r , a_1 in this figure. If the moment of inertia J_1 of the tip payload is negligible, the equation of motion which considers the rigid body mode and the fundamental mode of vibration is expressed by Eq. (1)⁽⁵⁾, where the drive torque is T_0 , friction torque is T_f , angle $\theta = [\beta \ \phi]^T$ and $f = [1 \ 0]^T$:

$$[M]\ddot{\theta} + [K]\theta = f(T_0 - T_f), \quad (1)$$

where

$$[M] = \begin{bmatrix} J_0^* + M_1^* L_1^2 & -M_1^* L_1^2 \\ -M_1^* L_1^2 & M_1^* L_1^2 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 0 & 0 \\ 0 & K_1^* \end{bmatrix}$$

and

$$K_1^* = \frac{3EI}{l_1} \left(1 + 2\frac{r}{l_1} - \frac{a_1}{l_1} \right)$$

$$M_1^* = M_1 + \frac{1}{3}m_1 \left(1 + \frac{r}{l_1} - 2\frac{a_1}{l_1} \right).$$

T_f is the nonlinear torque caused mainly by the reduction gear friction which consists of viscous friction T_v and Coulomb friction T_c and can be expressed by

$$T_f = T_v + T_c, \quad (2)$$

where

$$T_v = c_v \dot{\beta}, \quad T_c = c_c \operatorname{sgn}(\dot{\beta}).$$

With the substitution of Eq. (2) into Eq. (1), Eq. (3), the linear system which includes Coulomb friction compensation is obtained:

$$[M]\ddot{\theta} + [C]\dot{\theta} + [K]\theta = f(T_0 - T_c), \quad (3)$$

where

$$[C] = \begin{bmatrix} c_v & 0 \\ 0 & 0 \end{bmatrix}.$$

When the torque by reduction gear friction is the equivalent of viscous friction expressed by $T_f = c_0 \dot{\beta}$, Eq. (3) can be linearized without Coulomb friction compensation by using c_0 , instead of c_v , with $T_c = 0$. Equivalent viscous friction is obtained by averaging the values of the viscous friction coefficient, namely, the friction torque T_f divided by the angular velocity $\dot{\beta}$.

When the state vector is $x = [\beta \ \phi \ \dot{\beta} \ \dot{\phi}]^T$ and $b = [0 \ 0 \ b_1 \ b_1]^T$, the state equation is obtained from Eq. (3):

$$\frac{d}{dt}x = [A]x + b(T_0 - T_c), \quad (4)$$

where

$$[A] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_1^* & a_3^* & 0 \\ 0 & a_1^* + a_2^* & a_3^* & 0 \end{bmatrix}$$

and

$$a_1^* = -K_1^*/J_0^*, \quad a_2^* = -K_1^*/M_1^*L_1^2$$

$$a_3^* = -c_v/J_0^*, \quad b_1 = 1/J_0^*.$$

When the sampling period is applied and discretized, Eq. (5) is obtained as

$$x_{k+1} = [P]x_k + qu_k \quad (5)$$

$$y_k = cx_k = [1 \ -1 \ 0 \ 0]x_k$$

$$= \beta_k - \phi_k,$$

where the subscript $k(=0, 1, 2, \dots)$ indicates each timestep, y_k is the controlled variable and u_k is the manipulated signal of torque T_0 . When state feedback is given by the system using linear quadratic integral control⁽⁶⁾ containing integrating action as shown Fig. 2, u_k is expressed by Eq. (6).

$$u_k = k_1 \frac{Z}{Z-1} (r_k - y_k) - k_2 \beta_k - k_3 \phi_k - k_4 \dot{\beta}_k - k_5 \dot{\phi}_k \quad (6)$$

$k_1 \sim k_5$ are the optimal feedback gains obtained by solving the Riccati equation. Z is the operator which indicates the time lead corresponding to one sampling period, and the first term of Eq. (6) represents the integrating action.

2.2 Experimental equipment

Figure 3 shows the configuration of the experimental equipment. The angle β was detected by the potentiometer and the angular velocity $\dot{\beta}$ by the encoder. Either the strain gauge at the arm root or the accelerometer at the arm tip could be selected to measure the bending angle ϕ . The signal was supplied through a 30 Hz low-pass filter, so as to eliminate high-order harmonics other than the fundamental natural frequency. If the fundamental mode was taken as the elastic deformation mode when the tip payload was pressed in the transverse direction, the relationship between the bending angle ϕ and the strain ε at point A in Fig. 1 is expressed as

Fig. 2 LQI control system

$$\phi = \frac{2}{3} \cdot \frac{1+3(a_1/l_1)}{1+a_1/l_1} \cdot \frac{l_1^2}{tL_1} \varepsilon, \quad (7)$$

where t is the thickness of the elastic beam. The relationship with the acceleration a at the center of gravity G_1 of the payload is

$$\phi = \frac{M_1^* l_1^3}{3EI \cdot L_1} [1+3(a_1/l_1)] a. \quad (8)$$

Therefore, from Eq. (7) and Eq. (8), the strain ε and the acceleration a show a linear relationship with the angle ϕ and could be used as sensor output. It was difficult to measure the angular velocity $\dot{\phi}$ directly, hence it was obtained by the time difference of ϕ . The torque instruction value was calculated based on these signals, subjected to D/A conversion and supplied to the servodriver. Sampling time was 48 ms. However, when the dither signal was required, a 10 Hz low-pass filter and 4.8 ms sampling time were used.

A reduction gear equipped with an inscribed cycloidal gear was mounted on a 50 W AC servomotor with a perpendicular rotational axis. The flexible arm was a stainless steel plate of 2.5 mm thickness and 20 mm width with a steel cube of about 45 mm edge length at its tip. Two kinds of reduction gears were used: A (reduction ratio: 29) with low friction and equipped only with an antiseize device, without grease, and B (reduction ratio: 59) with high friction and with grease. Hereafter, the experiment using the former is called case A and that using the latter is called case B. Dimensions of the experimental

Table 1 Dimensions of experimental equipment

M_1	0.642 kg	r	0.045 m
m_1	0.224 kg	l_1	0.570 m
J_1	$2.1 \times 10^{-4} \text{ kg} \cdot \text{m}^2$	a_1	0.023 m
j_1	$6.1 \times 10^{-3} \text{ kg} \cdot \text{m}^2$	L_1	0.638 m
J_{0^*} (Case A)	0.066 kg $\cdot$ m ²	EI	5.36 N $\cdot$ m ²
J_{0^*} (Case B)	0.186 kg $\cdot$ m ²

Fig. 3 Experimental equipment

equipment are shown in Table 1.

3. Control Using the Equivalent Viscous Friction

3.1 Friction torque measurement

Friction torque was measured using only the rotational part of the experimental equipment without the flexible arm and the tip mass. As is clearly seen in Fig. 4, friction torque was nonlinear, consisting of viscous and Coulomb friction. The relationship between T_f [N $\cdot$ m] and $\dot{\beta}$ [rad/s] is expressed by Eq. (9).

$$\begin{aligned} \text{A: } T_f &= 0.06 \dot{\beta} + 0.35 \operatorname{sgn}(\dot{\beta}) \\ \text{B: } T_f &= 2 \dot{\beta} + 7 \operatorname{sgn}(\dot{\beta}) \quad [\text{Upper limit}] \\ T_f &= 1.25 \dot{\beta} + 5.2 \operatorname{sgn}(\dot{\beta}) \quad [\text{Lower limit}] \end{aligned} \quad (9)$$

As shown in Fig. 4, case B showed marked fluctuation of friction torque depending on conditions such as the temperature. In the open loop experiment, friction torque approached the maximum and in the feedback experiment, it approached the minimum. The open loop experiment was conducted by applying positive and negative drive torque alternately (1 N $\cdot$ m for case A, 10 N $\cdot$ m for case B) at an interval of 1.5 seconds. It was found that Eq. (10) of the equivalent viscous friction from the nonlinear calculation (using Eq. (1) and the uppermost of Eq. (9)) gave approximately the proper value and agreed well with the experimental results⁽⁵⁾.

$$\begin{aligned} \text{A: } T_f &= 0.2 \dot{\beta} \\ \text{B: } T_f &= 10 \dot{\beta} \end{aligned} \quad (10)$$

3.2 Feedback control using the equivalent viscous friction

The feedback gain was calculated by means of a linear model using the equivalent viscous friction of

Fig. 4 Reduction gear friction torque characteristics

Eq.(10), and the manipulated signal obtained by Eq. (6) was fed to the servomotor. The strain gauge or the accelerometer was used to detect the bending angle ϕ and the change of the tip angle $y = \beta - \phi$ with the state feedback. The initial position of the tip angle y was 1.8 rad and its target was 3.0 rad.

As clearly seen in Fig. 5, which shows the results of case A, the target value was reached in a short time, the equivalent viscous friction was properly set, and feedback control by the acceleration signal hardly differed from that by the strain gauge signal. Figure 6 shows the control results of case B. Equivalent viscous friction, shown by solid lines A and B, corresponds with the upper and lower values of nonlinear friction torque in Eq.(9). The dotted line C shows the case where the lowermost nonlinear Coulomb friction of Eq.(9) is compensated and controlled as a linear system of Eq.(3). In either case, the tip angle fluctuated without maintaining the target value. Therefore, it was found that when the friction was high, feedback control was impossible by the use of the equivalent viscous friction or compensating Coulomb friction.

4. Control Using a Dither Signal

4.1 Friction linearized by a dither signal

The method of applying a dither signal to linearize the nonlinear friction was used⁽²⁾. Using a dither

Fig. 5 Changes of tip angle (with reduction gear A)

Fig. 6 Changes of tip angle (with reduction gear B)

signal prevents the vibration of the flexible arm because the feedback torque acts effectively without being absorbed by Coulomb friction. Calculations and experiments were performed for the high friction reduction gear B (lowermost). As in the previous section, using only the rotational part of the experimental equipment, we attempted to linearize the reduction gear friction by the use of a dither signal. The equation of motion considering the drive torque T_0 and the dither torque T_d is expressed as follows.

$$J_0^* \ddot{\beta} = T_0 + T_d - T_f \quad (11)$$

The dither torque T_d is a triangular wave,

$$T_d = \begin{cases} \frac{4a}{T}t & (0 \leq t \leq \frac{T}{4}) \\ -\frac{4a}{T}t + 2a & (\frac{T}{4} \leq t \leq \frac{3T}{4}) \\ \frac{4a}{T}t - 4a & (\frac{3T}{4} \leq t \leq T) \end{cases} \quad (12)$$

where a is the amplitude of dither torque and T is the period of dither torque. The state equation of Eq.(9) is expressed as follows.

$$\frac{d}{dt} \begin{bmatrix} \beta \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J_0^* \end{bmatrix} (T_0 + T_d - T_f) \quad (13)$$

After discretization at the sampling period of 4.8 ms, linearization was conducted using the dither torque at a frequency of 26.0 Hz. In both the simulation and the experiment, inputting the drive torque T_0 yielded an average value of angular velocity $\dot{\beta}$ which was constant. Following this procedure, the drive torque T_0 was equal to the friction torque T_f (the average value), and the relationship between the average values of T_f and $\dot{\beta}$ was obtained.

Figure 7 shows the simulated example of linearization using the dither signal and shows the friction

Fig. 7 Simulation of linearizing by dither torque (An output example of T_f and $\dot{\beta}$, $T_0=3 \text{ N}\cdot\text{m}$, $a=15 \text{ N}\cdot\text{m}$)

torque T_f and the angular velocity $\dot{\beta}$ when the drive torque was $T_0=3\text{ N}\cdot\text{m}$ and the amplitude of the dither torque T_d was $a=15\text{ N}\cdot\text{m}$. When the average value of T_f was equal to the drive torque T_0 , it was found that the average value of $\dot{\beta}$ converged to 0.319 rad/s . Figure 8 shows the results of the experiment and the simulation of the friction torque using the dither signal. The amplitude of the dither torque changed from 10 to $25\text{ N}\cdot\text{m}$. When the amplitude of the dither torque was larger, the inclination of T_f was smaller and linearization was easy.

Experimental results showed superior linearity to the simulation results and were expressed as Eq.(14), with $T_f[\text{N}\cdot\text{m}]$ and $\dot{\beta}[\text{rad/s}]$.

$$T_f = \begin{cases} 15\dot{\beta} (|\dot{\beta}| \leq 0.38) & \text{for } a=10\text{ N}\cdot\text{m} \\ 10\dot{\beta} (|\dot{\beta}| \leq 0.59) & \text{for } a=15\text{ N}\cdot\text{m} \\ 8.2\dot{\beta} (|\dot{\beta}| \leq 0.75) & \text{for } a=20\text{ N}\cdot\text{m} \\ 6.6\dot{\beta} (|\dot{\beta}| \leq 0.97) & \text{for } a=25\text{ N}\cdot\text{m} \end{cases} \quad (14)$$

In the outer range of $\dot{\beta}$ of Eq.(14), the friction torque was the same as the lowermost value of the original case B, whether there was a dither signal or not.

$$T_f = 1.25\dot{\beta} + 5.2\text{sgn}(\dot{\beta}) \quad (15)$$

4.2 Feedback control using linearized friction by dither signal

The nonlinear friction of case B was linearized by a dither signal as Eq.(14); then, the feedback gain was obtained from the linear system without compensating Coulomb friction. The friction torque was equal to Eq.(15) in the outer range of $\dot{\beta}$ of Eq.(14). Next, three cases were considered for the treatment

Fig. 8 Friction torque linearized by dither torque (Amplitude of dither torque 10~25 N·m)

of Eq.(15).

Case I : As in Eq.(3), the system was treated as a linear system with compensating Coulomb friction. Namely, the feedback gain was obtained by the system considering the viscous friction T_v . In the control system, Coulomb friction T_c was compensated.

Case II : The above-mentioned gain was used, but Coulomb friction T_c was not compensated.

Case III : Linearized gain obtained by a dither signal was used as is. Namely, we assumed that the range $\dot{\beta}$ of Eq.(14) was not considered.

Case II was proposed to estimate the response difference between conditions with and without compensating Coulomb friction, and case III was proposed to estimate the response using constant gain instead of changing gain values. Figure 9 shows the results of case I. When the amplitude of the dither torque was small (10 N·m), the tip angle fluctuated without maintaining the target value, as in Fig. 6. But with increasing amplitude of the dither torque, the tip angle converged on the target value. Figure 10 shows the results of case II. The tip angle had some overshoot, but converged on the target value without large fluctuation. Overshoot tended to be small with increasing amplitude of the dither torque, but had little effect on the response. Figure 11 shows the results of case

Fig. 9 Changes of tip angle (Case I)

Fig. 10 Changes of tip angle (Case II)

III. Overshoot was relatively small, but the fluctuation was large. With increasing amplitude of the dither torque, the fluctuation tended to be small, but the overshoot was large. From these results, it was found that case II, showing a stable response, was appropriate for control. Namely, the gain obtained in the range of linearization of Eq.(14) using the dither torque and the gain obtained considering only the viscous friction component in the nonlinear range of Eq.(9) were used. Feedback control was achieved by using these gains and dithers, selecting the range of β .

The following occurred for case II with the amplitude 20 N·m of the dither torque. Figure 12 shows the results, similar to Fig. 5 of case A in which friction was small, using the strain gauge and the accelerometer as a feedback sensor. There was some difference between the responses, but the accelerometer was usable, as was the strain gauge. Figure 13 shows the measured value of β . It was confirmed that position control was achieved without the dither torque when β was large in rotating conditions, and with the dither torque when β was small near the target position. Figure 14 shows the bending angle ϕ measured by the strain gauge. As is clearly seen in this figure, feedback control was performed properly with the dither signal, and showed superior performance to that using the equivalent viscous friction or compensating Coulomb friction.

Fig. 11 Changes of tip angle (Case III)

Fig. 12 Difference between strain gauge and accelerometer (Case II, $a=20$ N·m)

sating Coulomb friction.

5. Conclusions

For the nonlinear friction of the reduction gear of a one-link flexible arm, treatment as an equivalent viscous friction and linearization of the friction by the dither signal were investigated, and feedback control was performed. As a result, the following conclusions were reached.

1) Experiments were performed using two kinds of reduction gears: one without grease and with low friction, the other with grease and high friction. When the friction was low, feedback control was possible using the equivalent viscous friction, but when the friction was high, position control was impossible using the equivalent viscous friction and compensating Coulomb friction.

2) To linearize the nonlinear friction of reduction gears with high friction, simulation and experiment were performed using the dither torque which was a discretized triangular wave. With the increase in the amplitude of the dither signal, linearization was easy.

3) Feedback control was possible using linearized friction by applying a dither signal. With this control, only the viscous friction component of nonlinear friction was considered, without compensating Coulomb friction in the outer range of linearization by a dither signal. Namely, the gain obtained from the

Fig. 13 Changes of β (Case II, $a=20$ N·m)

Fig. 14 Comparison of bending angle ϕ

linearized system in the range of linearization by a dither signal and the gain obtained from the system considering only the viscous friction component of nonlinear friction in the outer range of linearization were used. From the results of the experiment varying these gains, it was found that stable position control with small fluctuation was possible without being affected by the dither amplitude.

4) It was found that the acceleration signal from the accelerometer at the arm tip, like the signal from the strain gauge at the arm root, could be used to effect feedback control for the fundamental natural frequency mode.

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