Force Control of the Robot Manipulator with Collision Phenomena by Learning Control*

Hiroshi WADA**, Toshio FUKUDA*** Keigo WATANABE****,
Hideo MATSUURA*** and Fumihito ARAI***

Impact/collision is a fast and non-linear phenomenon, so it is difficult to control a robotic manipulator undergoing collision phenomena. Therefore, in the past, manipulators were moved slowly in order to avoid collision. But with the recent increase in the amount of high-speed tasks, control of the manipulator undergoing collision has become indispensable. In such a situation, it is effective to use learning control in the forward manner. In this paper, we propose a new learning control method to optimize the weighted least-squares criterion of learning errors. This method can be applied to obtain a unique control gain, and it is shown here that the convergence of learning error can be readily assured. Using this learning control method, we carried out experiments on force control with collision phenomena, and proved the convergence of the output error. The robotic manipulator was made of an air-driven rubber actuator with no reduction gears to avoid damage due to impact.

Key Words: Robotics, Mechatronics, Collision, Force Control, Learning Control, Weighted Least-Squares Method

1. Introduction

In controlling a robotic manipulator to make contact with objects, collision occurs between them, resulting in difficult control problems. This is because collision phenomena are nonlinear phenomena involving a transient change of system between contact and noncontact state, and are generally very fast phenomena compared with the velocity of manipulators being moved. Therefore, in the past, manipulators have been controlled to move slowly to avoid collision. However, with the growing necessity for high-speed execution of tasks, or for high speed grasp and insertion, it is indispensable to control manipulators with respect to collision phenomena.

To this end, recently, some research works have

been carried out with respect to the control problem of impact phenomena. In the modeling of impact, Wang and Mason⁽¹⁾ studied the impact of two objects, and Kahng and Anirouche⁽²⁾ derived the maximum impact force equation for two-body collision problems. Zheng and Hemami derived the mathematical model of a collision⁽³⁾, and proposed the assembly method of two arms using impulsive information⁽⁴⁾. Yousef-Toumi et al.⁽⁵⁾ validated an analytical model of impact experimentally. In stable control of a robotic manipulator undergoing collision phenomena, Khatib and Burdick⁽⁶⁾ adopted velocity damping to dissipate excessive impact energy arising between end-effector and objects. Kitagaki and Uchiyama⁽⁷⁾ and Nagata et al.⁽⁸⁾ derived an optimal approach velocity of a manipulator to its environment to carry out smooth contact. Mills⁽⁹⁾ developed a dynamic model including the effect of impact, and proposed a discontinuous control for the transition from noncontact to contact or vice versa in a robotic manipulator. Shoji et al.⁽¹⁰⁾⁽¹¹⁾ adopted a Hertz-type model with energy loss to collision dynamics, and proved the stability with respect to the control system of a robotic manipulator undergoing collision phenomena. In reducing

* Received 17th June 1992. Paper No. 90-0665 A

** Toyoda Automatic Loom Works, Ltd., 8 Cyaya, Kyowa-cho, Obu 474, Japan

*** Nagoya University, Dept. of Mechano-Informatics and Systems, Furo-cho, Chikusa-ku, Nagoya 464-01, Japan

**** Saga University, Dept. of Mechanical Engineering, 1 Honjo-machi, Saga 840, Japan

impact force, Walker⁽¹²⁾ and Gertz et al.⁽¹³⁾ proposed a strategy using the redundant manipulator.

It is difficult to control manipulators undergoing such fast collision phenomena in the on-line manner. Therefore, it is effective to use a learning control method in the forward manner. In this paper, we propose a learning control method optimizing the weighted least-squares method of learning errors. The methods of learning gain until now have been designed only to secure stability⁽¹⁴⁾⁽¹⁵⁾. However, when using these methods, the learning errors often become excessive during the learning process, and the learning gain cannot be fixed. Therefore, study of the optimal learning control method has increased, in order that a performance criterion be minimized (or maximized) as proposed here. Our proposed method⁽¹⁶⁾ has the characteristic of enabling determination of a unique learning control gain for discrete-time systems. The gain can be obtained by means of the Riccati equation which has a state dimension equal to the sampling number. It is shown that the convergence of the learning error can easily be assured, because the present learning rule consists of a steady-state Kalman filter. Using this learning method, we performed experiments on force control, and proved the convergence of the output error.

2. Modeling

2.1 Dynamic model

The dynamic model is shown in Fig. 1. We assume that the manipulator is 1 degree of freedom, and the objects consist of the mass-damper-spring system. Dynamic equations for the manipulator and the object are given as

$$I_G \ddot{\theta} = \tau - l \cdot f_w, \quad (1)$$

object :

$$m \ddot{x} + c \dot{x} + kx = f_w, \quad (2)$$

Fig. 1 Dynamic model

where l is the length of the manipulator, I_G is the moment of inertia around the center of gravity, θ is the angle, m is the equivalent mass, k is the coefficient of the spring, c is the coefficient of damper, τ is input, and f_w is impact force. Hence the impact force f_w is expressed in the spring system, as

$$f_w = H \cdot (l \cdot \theta - x) \quad (3)$$

2.2 State equation

From these kinetic equations, the following state equations can be obtained.

$$\dot{\mathbf{x}}(t) = \mathbf{A}' \mathbf{x}(t) + \mathbf{B}' \tau(t) \quad (4)$$

$$y(t) = \mathbf{C}' \mathbf{x}(t) \quad (5)$$

$$\mathbf{A}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a'_1 & -a'_2 & -a'_3 & -a'_4 \end{bmatrix} \quad (6)$$

$$a'_1 = l^2 H (k + H) / (m I_G)$$

$$a'_2 = l^2 H c / (m I_G)$$

$$a'_3 = (k + H) / m + l^2 H / I_G$$

$$a'_4 = c / m \quad (7)$$

$$\mathbf{B}' = [0 \ 0 \ 0 \ 1]^T \quad (8)$$

$$\mathbf{C}' = [b_1 \ b_2 \ b_3 \ b_4] \quad (9)$$

$$b_1 = H l k / (m I_G)$$

$$b_2 = H l c / (m I_G)$$

$$b_3 = H l / I_G$$

$$b_4 = 0 \quad (10)$$

where the dimensions are $\mathbf{x}(t) \in R^4$, $\tau(t) \in R^1$, and $y(t) \in R^1$. The discrete model of Eqs. (4) and (5) is expressed by

$$\mathbf{x}(i+1) = \mathbf{A} \mathbf{x}(i) + \mathbf{B} \tau(i) \quad (11)$$

$$y(i) = \mathbf{C} \mathbf{x}(i). \quad (12)$$

$\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are matrices on the discrete-time system corresponding to $\mathbf{A}'$, $\mathbf{B}'$ and $\mathbf{C}'$, where $\mathbf{C} = \mathbf{C}'$ in this case.

3. Learning Control Using the Weighted Least-Squares Method

3.1 Problem statement

First we derive the direct transmission term from Eqs.(11) and (12)⁽¹⁷⁾, so Eq.(12) is transformed into Eq.(13),

$$y(i+r) = \mathbf{C}^* \mathbf{x}(i) + \mathbf{D}^* \tau(i), \quad (13)$$

where r is relative degree and equal to 2 in this case, and $\mathbf{C}^*$ and $\mathbf{D}^*$ are shown in Eq.(14).

$$\mathbf{C}^* = \mathbf{C} \cdot \mathbf{A}^2$$

$$\mathbf{D}^* = \mathbf{C} \cdot \mathbf{A} \cdot \mathbf{B}. \quad (14)$$

In order to utilize the impulse response model, Eqs. (11) and (13) are transformed into Eq.(15).

$$\mathbf{y} = \mathbf{L} \cdot \boldsymbol{\tau}, \quad (15)$$

where

$$\boldsymbol{\tau}^T = [\tau(0), \tau(1), \dots, \tau(N)] \quad (16)$$

$$\mathbf{y}^T = [y(r), y(r+1), \dots, y(r+N)]. \quad (17)$$

L is the lower triangular block, a Toeplitz matrix, and is shown in Eq.(18)

$$L = \begin{bmatrix} D^* & & & & \\ C^*B & D^* & & & 0 \\ C^*AB & C^*B & D^* & & \\ C^*A^2B & C^*AB & C^*B & \cdot & \\ \vdots & \vdots & & & \\ \vdots & \vdots & & & D^* \\ C^*A^{N-1}B & C^*A^{N-2}B & \cdot & \cdot & D^* \end{bmatrix}, \quad (18)$$

where N is the time of the last sampling.

In Eqs.(15)–(18), when the a priori desired values are given for unknown τ as shown in Eq.(19),

$$\mathbf{y}_d^T = [y_d(r), y_d(r+1), \dots, y_d(r+N)], \quad (19)$$

if we can obtain the accurate value of L , as shown in Eq.(20), we can determine a unique input to make $\mathbf{y} = \mathbf{y}_d$.

$$\tau = L^{-1} \mathbf{y}_d, \quad L^{-1} \triangleq [L^T L]^{-1} L^T \quad (20)$$

However, L is generally unknown, so that we cannot determine it uniquely. Therefore we use the weighted least-squares method in this paper.

3.2 Employment of the weighted least-squares method

In order to use the weighted least-squares method, we assume that the unknown input is constant at the learning stage, so we can introduce a identification model as shown in Eqs.(21) and (22) :

$$\tau_{k+1} = \tau_k \quad (21)$$

$$\mathbf{y}_d = L \tau_k, \quad (22)$$

where k is the number of trials. Then the optimal input to minimize the weighted least-squares criterion

$$J = \|\mathbf{y}_d - L \tau_k\|_V^2 \quad (23)$$

is given by

$$\hat{\tau}_k = \hat{\tau}_{k-1} + K_k \mathbf{e}_{k-1} \quad (24)$$

$$\hat{\tau}_{-1} = 0 \quad (25)$$

$$\mathbf{e}_{k-1} = \mathbf{y}_d - \mathbf{y}_{k-1}, \quad (25)$$

where $\mathbf{y}_{k-1}$ is a real input. Then $K_k \in R^{(N+1) \times (N+1)}$ is a matrix of gain, and is obtained iteratively by learning as shown in Eqs.(26) and (27) :

$$K_k = P_{k-1} L^T [L P_{k-1} L^T + V^{-1}]^{-1} \quad (26)$$

$$P_{-1} = \alpha I \quad (27)$$

$$P_k = [I - K_k L] P_{k-1}. \quad (27)$$

This algorithm is a recurrent type of the weighted least-squares criterion of time-invariant parameter in the case where k represents normal time. In general, because P_k in Eq.(27) does not have a nonzero unique stationary solution, the gain K_k must be calculated every time. As a result, when the sampling number is larger, the calculating overload of Eqs. (26) and (27) increases, possibly causing a problem of numerical instability.

Therefore, we propose a method to utilize the stationary gain. The system expressed by Eqs.(21) and (22) is time-invariant, but because of the exis-

tence of the uncontrollable mode on the unit circle, its system is not stabilizable. Hence we add virtual noise processes with zero mean value and positive definite covariances to Eq.(21) :

$$\tau_{k+1} = \tau_k + w_k. \quad (28)$$

Then

$$W = GG^T \\ L^T V L = F^T F, \quad (29)$$

and if (I, G) is reachable and (F, I) is detectable, as shown in Eq.(30), a unique positive definite stabilizable solution to the algebraic Riccati equation exists (18) :

$$P_s = P_s - P_s F^T \{L^T P_s L + V^{-1}\}^{-1} F P_s + G G^T. \quad (30)$$

Therefore, from this solution of P_s , the stationary gain is given by

$$K = P_s L^T \{L P_s L^T + V^{-1}\}^{-1}. \quad (31)$$

As a result, as shown in Eq.(32), the learning rule is introduced as follows :

$$\hat{\tau}_k = \hat{\tau}_{k-1} + K \mathbf{e}_{k-1} \\ \hat{\tau}_{-1} = 0. \quad (32)$$

This is a steady-state Kalman filter which has a state dimension and observation dimension equal to $(N+1)$, where N is a sampling number. The learning control system is shown in Fig. 2.

We now consider the convergence of error. In the learning rule in Eq.(32), P_s is a stabilizable equation; therefore, its closed-loop system is asymptotically stable and it secures Eq.(33).

$$\|I - KL\|_s < 1, \quad (33)$$

where $\|\cdot\|_s$ is a spectrum norm (the maximum of absolute values in eigenvalues). Therefore the optimal input in stationary states at and the output for this input is defined in Eqs. (34) and (35).

$$\lim_{k \rightarrow \infty} \hat{\tau}_k = \tau_s \quad (34)$$

$$\lim_{k \rightarrow \infty} \mathbf{y}_k = \mathbf{y}_s \\ = L \tau_s. \quad (35)$$

If learning errors are defined in Eq. (36) as

Fig. 2 Learning control system

$$\lim_{k \rightarrow \infty} e_k = \lim_{k \rightarrow \infty} (y_d - y_k)$$

$$y = y_d - y_k \triangleq e_s, \quad (36)$$

Eq. (37) is derived from Eqs.(32), (34)-(36)

$$K \cdot \lim_{k \rightarrow \infty} e_{k-1} = K e_s = 0. \quad (37)$$

Fig. 3 Experimental equipment

Fig. 4 Sketch of experimental equipment

Fig. 5 Pneumatic actuators

Fig. 6 Control system

Finally, in order to make K a full-rank (rank $K=(N+1)$), Eq.(38) is derived:

$$e_s = 0. \quad (38)$$

Therefore, when the learning law given by Eqs.(30)-(32) is applied to the systems in Eqs.(11) and (13), required and sufficient conditions to realize Eq.(39) are such that (I, G) is reachable, and (F, I) is detectable, yielding

$$\lim_{k \rightarrow \infty} e_k = e_s = 0. \quad (39)$$

Moreover, the proofs described above are carried out from the deterministic standpoint because the objective system Eqs.(4) and (5), or Eqs.(11) and (12), is nonstochastic.

4. Experimental Equipment

The experimental equipment is shown in Fig. 3, and its outline is shown in Fig. 4. We used actuators made of rubber, driven by air supplied from compressors. These actuators are fiber woven around a rubber tube, and they expand in diameter and contracts vertically due to air pressure. A photograph is shown in Fig. 5. In this system, when we examined frequency response during the removal of the manipulator, in the case that frequency of the input sine wave exceeded about 40 Hz, the gain became zero. As shown in Fig. 4, the two pneumatic actuators are used to drive the arm; that is, the difference in the output torques generated by these actuators through the pulley drives the arm. We assume that the torque generated by an actuator is proportional to the input voltage to the controller of the actuator.

The control system is shown in Fig. 6. We use strain gauges in force sensors, which are pasted on the flat surface of the manipulator. For in the position sensor and the velocity sensor, we use the rotary encoder (resolution 1024) and the F/V converter respectively.

In this experiment, we must compute the Kalman gain which has a state dimension equal to the sampling number. For this reason, when a long period of time is needed for one trial, we must limit sampling time by introducing a waiting time. Then the sampling becomes erratic, and outputs obtained from strain gauges are degraded by a large number of errors. Therefore, we measure correct outputs from sensors using a transient memory for observation.

5. Experimental Methods and Results

The aim of the experiments is to obtain force control with collision phenomena using the learning law in Eqs.(25)-(27), to prove its convergence and to examine the value of its convergence for various weights.

The experimental procedure is as follows: [1] a manipulator is set up in an initial position as shown in Fig. 4, [2] from this position, it is moved to make contact with the level surface of the object and [3] it is controlled by a constant force.

The matrices A , B , C in Eqs.(11) and (12) are obtained by identification with the least squares method. However, A and A' have the same structure with the parameter $a_i (i=1, 2, 3, 4)$, and B is the same as B' . The results are shown as follows :

$$\begin{aligned} a_1 &= 0.1669 & b_1 &= -3.05 \times 10^{-3} \\ a_2 &= 0.1737 & b_2 &= 6.13 \times 10^{-3} \\ a_3 &= 0.1798 & b_3 &= 4.04 \times 10^{-3} \\ a_4 &= 0.1798 & b_4 &= 7.06 \times 10^{-3}. \end{aligned} \quad (40)$$

We computed the matrix L of these matrices and solved the algebraic Riccati equation. The matrices of weights W and V are defined as $W = \text{diag}(1, 1, \dots, 1)$ and $V = \text{diag}(v, v, \dots, v)$.

Fig. 7 Experimental result (weight=1.0)

Fig. 10 Experimental result by transient memory (weight=5.0)

Fig. 8 Experimental result by transient memory (weight=1.0)

Fig. 11 Experimental result (weight=10.0)

Fig. 9 Experimental result (weight=5.0)

Fig. 12 Experimental result by transient memory (weight=10.0)

Fig. 13 Experimental result (weight=50.0)

Fig. 15 Experimental result (weight=100.0)

Fig. 14 Experimental result by transient memory (weight=50.0)

Fig. 16 Experimental result by transient memory (weight=100.0)

Experimental results are shown in Figs. 7-16. As shown in section 4, odd-numbered figures are experimental results for outputs directly connected using the learning law, while even-numbered figures are for measurements using transient memory. The weights 1.0, 5.0, 10.0, 50.0 and 100.0 are each represented in two figures. In Fig. 17, the mean squared error to outputs is shown for all weights. As shown in this figure, the output error decreases as the number of trials increases.

6. Conclusions

We proposed a learning control algorithm for the control of collision between the manipulator and its environment. This proposed algorithm is based on learning control using the weighted least-squares method. Using this learning control, we carried out experiments on force control with collision phenomena, and proved the convergence of the output error.

References

- (1) Wang, Y. and Mason, M.T., Modeling Impact Dynamics for Robotic Operations, Proc. of the

Fig. 17 Mean squared errors for various weights

IEEE International Conference on Robotics and Automation, (1987), p. 678.

- (2) Kahng, J. and Anirouche, F.M.L., Impact Force Analysis in Mechanical Hand Design - Part I, International Journal of Robotics and Automation, Vol. 3, No. 3, (1988), p. 158.

- (3) Zheng, Y.F. and Hemami, H., Mathematical Modeling of a Robot Collision with its Environment *Journal of Robotic Systems*, Vol. 2, No. 3, (1985), p. 289.
- (4) Zheng, Y.F., Two Robot Arms in Assembly by Impulsive Information, *Journal of Robotic Systems*, Vol. 4, No. 5, (1987), p. 585.
- (5) Yousef-Toumi, K. and Gutz, D.A., Impact and Force Control, *Proc. of the IEEE International Conference on Robotics and Automation*, (1989), p. 410.
- (6) Khatib, O. and Burdick, J., Motion and Force Control of Robotic Manipulators, *Proc. of the IEEE International Conference on Robotics and Automation*, (1986), p. 1381.
- (7) Kitagaki, K. and Uchiyama, M., Optimal Approach Velocity of End-effector to the Environment, *Proc. of the IEEE International Conference on Robotics and Automation*, (1990), p. 1928.
- (8) Nagata, K., Ogasawara, T. and Omata, T., Optimal Velocity Vector of Articulated Robot for Soft Bumping, *Trans. of the Society of Instrument and Control Engineers*, (in Japanese), Vol. 26, No. 4, (1990), p. 435.
- (9) Mills, J.K., Manipulator Transition to and from Contact Tasks : A Discontinuous Control Approach, *Proc. of the IEEE International Conference on Robotics and Automation*, (1990), p. 440.
- (10) Shoji, Y., Inaba, Y. and Fukuda, T., Impact Control of Grasping, *IEEE Transaction on Industrial Electronics*, Vol. 38, No. 3, (1991), p. 187.
- (11) Shoji, Y., Inaba, Y. and Fukuda, T., Impact/Contact Control of a Robotics Manipulator, *Journal of Computers and Electrical Engineering*, Vol. 18, No. 1, (1992), p. 99.
- (12) Walker, I.D., The Use of Kinematic Redundancy in Reducing Impact and Contact Effects in Manipulation, *Proc. of the IEEE International Conference on Robotics and Automation*, (1990), p. 434.
- (13) Gertz, M.W., Kim, J.O. and Khosla, P.K., Exploiting Redundancy to Reducing Impact Force, *Proc. of the IEEE/RSJ International Workshop on Intelligent Robots and Systems (IROS'91)*, (1991), p. 179.
- (14) Arimoto, S., Kawamura, S. and Miyazaki, F., Bettering Operation of Robots by Learning, *Journal of Robotic Systems*, Vol. 1, No. 2, (1984), p. 123.
- (15) Kawamura, S., Miyazaki, F. and Arimoto, S., Realization of Robot Manipulator Based on a Learning Method, *IEEE Trans. Syst., Man, and Cyber.*, Vol. 18, No. 1, (1988), p. 126.
- (16) Watanabe, K., Fukuda, T. and Tzafestas, S.G., An Iterative Learning Control Scheme Using the Weighted Least-Squares Methods, *Journal of Intelligent and Robotics Systems*, Vol. 4, No. 3, (1991), p. 267.
- (17) Sugie, T. and Ono, T., On a Learning Control Law, *Journal of Systems and Control*, (in Japanese), Vol. 31, No. 2, (1987), p. 129.
- (18) Anderson, B.D.O. and Moore, J.B., *Optimal Filtering*, (1979), p. 77, Prentice Hall.