Dynamic Analysis of a Rotor-Journal Bearing System with Large Dynamic Loads* (Stiffness and Damping Coefficient Variations in Bearing Oil Films)

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This paper describes the variations in stiffness and damping coefficients in bearing oil films under the influence of large dynamic loads. A rotor-journal bearing system in which large dynamic loads act on the rotor is nonlinear, so it is analyzed as a coupled problem of the equations of motion for the rotor combined with Reynolds equations for all the bearings. By making use of the displacements and velocities of the rotor center obtained through analysis, the stiffness and damping coefficients have been numerically calculated as an unsteady state along the locus of the rotor center. The variations in these coefficients over one rotor revolution are shown for the journal bearings in a rotary compressor. Our results demonstrate that most of the coefficients for all the bearings vary by more than one order of magnitude, and that oil film nonlinearities seriously influence the rotor motion.

Key Words : Lubrication, Journal Bearing, Nonlinear Theory, Numerical Analysis, Dynamic Characteristics, Stiffness and Damping Coefficients, Compressor

1. Introduction

When dynamic loads act on a rotor without static equilibrium, or when these loads are relatively greater than the static loads, the rotor-journal bearing system becomes nonlinear due to the variation in the stiffness and damping coefficients of the bearing oil films. For example, a rotor in a rotary compressor for air-conditioners whirls largely by unbalanced forces due to the eccentric rotation of the rolling pistons, and by gas forces induced by the difference in pressure between compression and suction gases. Thus, the journal bearings supporting the rotor are placed under severe operating conditions.

Such nonlinear rotor systems have been analyzed dynamically by considering them as a coupled problem of equations of motion for the rotor combined with Reynolds equations for all the journal bearings. The authors have reported this analytical procedure

and its results in previous papers^{(1)~(3)}. With this technique, which we call nonlinear system analysis, all the Reynolds equations are mutually linked by the equations of motion for the rotor; thus the oil film pressures in all bearings are simultaneously obtained along with their correlation.

The stiffness and damping coefficients of bearing oil films do not, in fact, explicitly appear in the governing equations for nonlinear system analysis. However, it is very important to examine how the coefficients actually vary in the rotor system. If this could be observed, it would be useful in evaluating the degree of linearity, or nonlinearity, caused by the bearing oil films. Furthermore, if the stiffness and damping coefficients are taken to be constant values, such as when analytically investigating the vibration of machines as a whole including nonlinear rotor systems, it is thought that the examined results may be used as a criterion for determining the constant values while the variation is taken into consideration.

In this paper, with the results of nonlinear system analysis, the variations in oil film stiffness and damping coefficients are numerically analyzed for a rotor-journal bearing system under large dynamic loading.

* Received 18th June 1992. Paper No. 91-0596 A

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This analysis is then applied to the rotor system in a rotary compressor, and the coefficient variations over one rotor revolution in all bearings are discussed.

2. Analysis Concept

It is necessary to clarify dynamic characteristics of journal bearings, namely, the oil film coefficients, in order to calculate the critical speed of a rotor system. Many papers⁽⁴⁾⁻⁽¹¹⁾ regarding the analysis of dynamic characteristics have already been published. In most of them, the calculating equations are derived from an oil film perturbation using a Taylor series expansion around the static equilibrium position of the rotor center. This assumes that the dynamic amplitude of the rotor center is negligible. Namely, the calculated oil film coefficients are the values for the case where the rotor is in the static equilibrium position, and are for use in linearized rotor systems.

However, when large dynamic loads act on a rotor, the rotor amplitude cannot be ignored; to calculate the coefficients at one typical point (the static equilibrium position) is meaningless. Therefore, in this case, the coefficient calculation is repeated sequentially at times $t-1$, t , $t+1$..., etc., along the locus of the rotor center, as shown in Fig. 1. This process is one of unsteady analysis⁽⁴⁾, and the result ought to give the oil film coefficient variation over one rotor revolution (one cycle).

The short approximated bearing, as used in the above-mentioned nonlinear system analysis, is employed in this paper. Analysis is performed according to the oil film perturbation method⁽⁵⁾.

3. Analysis Method

3.1 Equations for oil film coefficients

The coordinate system for a journal bearing is defined in Fig. 2. The oil film pressure gradient in the circumferential (θ) direction can be ignored for short approximated bearings. Then, the reduced Reynolds equation is

$$\frac{\partial}{\partial z} \left(\frac{h^3}{12\mu} \frac{\partial p}{\partial z} \right) = \frac{1}{2} \omega \frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial t}, \quad (1)$$

where

$$h = c - x \cos \theta - y \sin \theta \quad (2)$$

Fig. 1 Concept of analysis

$$\frac{\partial h}{\partial \theta} = x \sin \theta - y \cos \theta \quad (3)$$

$$\frac{\partial h}{\partial t} = -\dot{x} \cos \theta - \dot{y} \sin \theta, \quad (4)$$

and p is the oil film pressure, μ is the oil viscosity, ω is the angular speed of rotation, c is the radial clearance, x and y are the rotor center displacements, and $\dot{x}$ and $\dot{y}$ are the rotor center velocities in the x and y directions, respectively.

Bearing reaction forces F_{Bx} and F_{By} at time t are obtained by integrating the oil film pressure on the bearing inner face at that time:

$$\begin{Bmatrix} F_{Bx} \\ F_{By} \end{Bmatrix} = -R_B \int_0^L \int_{\theta_2}^{\theta_1} (p - p_a) \begin{Bmatrix} \cos \theta \\ \sin \theta \end{Bmatrix} d\theta dz \quad (5)$$

where R_B is the rotor radius, L is the bearing axial length, p_a is the pressure at bearing ends, and θ_1 and θ_2 are the boundary conditions in the θ direction.

It appears that the reaction forces F_{Bx} and F_{By} are functions of x , y , $\dot{x}$, and $\dot{y}$. When the rotor center deviates from location $O_t(x, y)$ on the locus by small amplitudes Δx and Δy , as shown in Fig. 1, the varying bearing reaction forces $F_{Bx+\Delta x}$ and $F_{By+\Delta y}$ can be described as follows using a first-order Taylor series expansion.

$$\begin{Bmatrix} F_{Bx+\Delta x} \\ F_{By+\Delta y} \end{Bmatrix} = \begin{Bmatrix} F_{Bx} + \frac{\partial F_{Bx}}{\partial x} \Delta x + \frac{\partial F_{Bx}}{\partial y} \Delta y \\ F_{By} + \frac{\partial F_{By}}{\partial x} \Delta x + \frac{\partial F_{By}}{\partial y} \Delta y \end{Bmatrix} + \begin{Bmatrix} \frac{\partial F_{Bx}}{\partial \dot{x}} \Delta \dot{x} + \frac{\partial F_{Bx}}{\partial \dot{y}} \Delta \dot{y} \\ \frac{\partial F_{By}}{\partial \dot{x}} \Delta \dot{x} + \frac{\partial F_{By}}{\partial \dot{y}} \Delta \dot{y} \end{Bmatrix} \quad (6)$$

From the definition of the stiffness and damping coefficients, K_{ij} and C_{ij} , the partial derivatives of the above are

$$\begin{Bmatrix} K_{xx} = \frac{\partial F_{Bx}}{\partial x}, C_{xx} = \frac{\partial F_{Bx}}{\partial \dot{x}} \\ K_{xy} = \frac{\partial F_{Bx}}{\partial y}, C_{xy} = \frac{\partial F_{Bx}}{\partial \dot{y}} \\ K_{yx} = \frac{\partial F_{By}}{\partial x}, C_{yx} = \frac{\partial F_{By}}{\partial \dot{x}} \\ K_{yy} = \frac{\partial F_{By}}{\partial y}, C_{yy} = \frac{\partial F_{By}}{\partial \dot{y}} \end{Bmatrix} \quad (7)$$

Fig. 2 Coordinate system for journal bearing

Here, the perturbation method has been applied to Reynolds equation (1). The film thickness after a rotor deviation can be written as

$$h' = h + \Delta h, \quad (8)$$

where

$$\Delta h = -\Delta x \cos \theta - \Delta y \sin \theta \quad (9)$$

$$\frac{\partial h'}{\partial \theta} = \frac{\partial h}{\partial \theta} + \Delta x \sin \theta - \Delta y \cos \theta \quad (10)$$

$$\frac{\partial h'}{\partial t} = \frac{\partial h}{\partial t} - \Delta \dot{x} \cos \theta - \Delta \dot{y} \sin \theta. \quad (11)$$

In this analysis, which differs from an analysis of the static equilibrium position, the rotor center basically moves along a locus with velocities $\dot{x}$ and $\dot{y}$. Hence, the partial derivative is retained as the first term on the right-hand side of Eq. (11).

The perturbation in film thickness gives rise to a similar perturbation in oil film pressure. With a first-order Taylor series expansion, this pressure can be written as

$$p' = p + p_x \Delta x + p_y \Delta y + p_{\dot{x}} \Delta \dot{x} + p_{\dot{y}} \Delta \dot{y} \quad (12)$$

where the coefficients indicate the partial derivatives differentiated by the individual subscripts. Substituting Eqs. (8) to (12) into Eq. (1), in which p and h are replaced by p' and h' , and by retaining first-order terms only, we obtain the partial derivatives

$$\left. \begin{aligned} \frac{\partial}{\partial z} \left(\frac{h^3}{12\mu} \frac{\partial p_n}{\partial z} \right) &= f_n \quad (n = x, y, \dot{x}, \dot{y}) \\ f_x &= 3 \frac{\cos \theta}{h} \left(\frac{1}{2} \omega \frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial t} \right) + \frac{1}{2} \omega \sin \theta \\ f_y &= 3 \frac{\sin \theta}{h} \left(\frac{1}{2} \omega \frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial t} \right) - \frac{1}{2} \omega \cos \theta \\ f_{\dot{x}} &= -\cos \theta \\ f_{\dot{y}} &= -\sin \theta \end{aligned} \right\} \quad (13)$$

The above equations demonstrate that the retained partial derivative in Eq. (11) appears in f_x and f_y , which are used to calculate the stiffness coefficients, but not in $f_{\dot{x}}$ and $f_{\dot{y}}$ used for the damping coefficient calculation.

Equation (13) is then integrated with respect to z

under the boundary conditions $p_n = 0$ when $z = 0$ and L .

$$p_n = 6\mu \frac{f_n}{h^3} z(z-L) \quad (14)$$

As in the calculation of bearing reaction forces utilizing Eq. (5), K_{xx} and K_{xy} are obtained by integrating Eq. (14),

$$\left. \begin{aligned} K_{xx} \\ K_{xy} \end{aligned} \right\} = -R_B \int_0^L \int_{\theta_2}^{\theta_1} p_x \begin{Bmatrix} \cos \theta \\ \sin \theta \end{Bmatrix} d\theta dz \quad (15)$$

and analogously for the remaining coefficients. By substituting Eq. (14) into Eq. (15) and integrating with respect to z , the oil film coefficients are

$$\left. \begin{aligned} K_{xx} \\ K_{xy} \\ K_{yx} \\ K_{yy} \\ C_{xx} \\ C_{xy} \\ C_{yx} \\ C_{yy} \end{aligned} \right\} = A \int_{\theta_2}^{\theta_1} \frac{f_n}{h^3} \begin{Bmatrix} \cos \theta \\ \sin \theta \end{Bmatrix} d\theta \quad (16)$$

where $A = \mu R_B L^3$

3.2 Analytical procedure

Given rotor center displacements x and y , and rotor center velocities $\dot{x}$ and $\dot{y}$, the stiffness and damping coefficients can be obtained by integrating Eq. (16). In the application example described below, x , y , $\dot{x}$, and $\dot{y}$ are calculated by applying nonlinear system analysis for an elastic flexible rotor system^{(2),(3)}. With the obtained displacements and velocities, the oil film coefficients are sequentially calculated at every time step along the locus of the rotor center, to determine the coefficient variation over one rotor revolution (one cycle).

Boundary conditions θ_1 and θ_2 are determined at the position where $p - p_a = 0$, and it is assumed that the oil film is ruptured in the negative pressure region. Accordingly, to obtain the coefficients, only the active pressure is integrated in Eq. (16). In this analysis, Simpson's formula is employed for numerical integration of Eq. (16).

4. Application Example for Analysis

The described analytical method is applied to the rotor system in a rotary compressor with two sets of compression units, as shown in Fig. 3. The oil film coefficients and their variations are examined for all the journal bearings.

The upper and lower rolling pistons with opposite phases are eccentrically mounted on the rotor. Hence, the compressor discharges twice per rotor revolution. The coordinates x , y , and θ are defined as being the same as those in Fig. 2. The origin and direction of the rotor rotating angle ψ are the same as those of θ . Thus, the lower and upper compression units begin to compress the gas at $\psi = 0^\circ$ and 180° respectively.

Fig. 3 Configuration of rotary compressor with two compression units

This analysis model for nonlinear system analysis is shown in Fig. 4. The elastic flexible rotor is supported by three short approximated bearings S_{B1} to S_{B3} . The calculation for the oil film coefficients is carried out for the three bearings individually. As outer loads acting on the rotor, the unbalanced forces F_{u1} to F_{u4} due to the eccentric masses and the gas forces F_{G1} and F_{G2} in the compression units are considered. The gas forces⁽¹⁾, which whirl with one half of ψ , are generated by the difference in pressure, $p(\psi)$ and p_a , between the two cells formed by the blade. They reach a maximum when the gases are discharged ($\psi = 40^\circ$ or 220°). The gas force profiles used in this analysis are shown in Fig. 5.

The analysis procedure for nonlinear rotor system is described in a previous paper⁽³⁾. The calculation, completed for a rotor rotational frequency of $f_0 = 150$ Hz, was implemented until rotor motion became steady, namely, after four revolutions. The oil film coefficients were calculated during the last revolution only. The other properties were as shown below.

Bearing properties (same values are used for S_{B1} to S_{B3})

$\mu : 3 \times 10^{-3}$ Pa·s, $c : 10$ μ m,

$L : 9$ mm, $R_B : 9$ mm

Material properties of flexible rotor

$E : 120$ GPa, $\rho : 6\,000$ kg/m³ (Motor rotor)

$E : 162$ GPa, $\rho : 7\,200$ kg/m³ (Remainder)

Fig. 4 Analytical model of rotor-journal bearing system

Fig. 5 Gas force profiles

F_{Gx1} and F_{Gy1} : in lower compression unit

F_{Gx2} and F_{Gy2} : in upper compression unit

Unbalanced masses in Fig.4

$F_{u1}, F_{u2} : 0.148$ kg $\times$ 3.5 mm,

$F_{u3}, F_{u4} : 0.0084$ kg $\times$ 20 mm

Here, E is Young's modulus and ρ is the density.

5. Results and Discussion

5.1 Rotor center motion in bearings

The characteristics of rotor center motion in the bearings, as obtained through nonlinear system analysis, are described first. Figures 6 to 8 show the displacements, the velocities, and the accelerations of the rotor center in the individual bearings, S_{B1} to S_{B3} , and their variations over one rotor revolution. The displacements are represented as an eccentricity ratio nondimensionalized by the radial clearance c . The values on the curves in Figs. 6 and 7 are given for calculation of the oil film coefficient at every time step (each rotor rotating angle). Here, as shown in Fig. 8, it is seen that small fluctuations are included in the varying curves of the rotor center accelerations.

Figure 9 shows the variations in bearing reaction forces F_{Bx} and F_{By} over one rotor revolution. In the example, the bearings are loaded almost equally when $\psi < 180^\circ$ (when F_{G2} acts mainly on the upper unit).

Fig. 6 Rotor center displacements in the bearings

Fig. 7 Rotor center velocities in the bearings

However, when $\psi \geq 180^\circ$ (when F_{C1} acts mainly on the lower unit), the reaction forces on S_{B1} , corresponding to the lower bearing in Fig. 3, become the largest.

5.2 Oil film coefficient variation

Figures 10 and 11 show the stiffness and damping coefficient variations over one rotor revolution, respectively, for the individual bearings S_{B1} to S_{B3} . In S_{B1} , which corresponds to the lower bearing, the coefficients are large when the rotor rotating angle $\psi > 180^\circ$, and the peak values of the individual coefficients appear in the region of 220° to 315° . For

the upper bearing, the coefficients in S_{B2} become large around $\psi = 40^\circ$ and 220° , when the gas forces reach a peak, but they fall near $\psi = 135^\circ$. In S_{B3} , both the stiffness and damping coefficients decrease when $\psi > 225^\circ$.

In the application example, which is a rotary compressor, the coefficient profiles have differing features among the three bearings; however, it can be said that these profiles are similar to those of the individual bearing reaction forces in Fig. 9. Here, viewing some of the coefficients, for instance, K_{xy} and

Fig. 8 Rotor center accelerations in the bearings

Fig. 9 Bearing load profiles

(a) S_{B1}

(b) S_{B2}

(c) S_{B3}

Fig. 10 Stiffness coefficient variations over one rotor revolution

(a) S_{B1}

(b) S_{B2}

(c) S_{B3}

Fig. 11 Damping coefficient variations over one rotor revolution

Table 1 Stiffness and damping coefficients in oil films
(Mean value, maximum and minimum values)

		S_{B1}		S_{B2}		S_{B3}	
		Mean V.	Max/Min	Mean V.	Max/Min	Mean V.	Max/Min
K_{xx}	$\times 10^8$ N/m	2.099	-0.492 5.205	-0.696	-0.042 -1.427	-0.823	0.149 -2.405
K_{xy}		-0.130	0.860 -2.377	-0.137	0.239 -0.791	-0.595	0.306 -2.997
K_{yx}		2.187	4.727 0.754	0.843	1.678 0.092	0.809	2.333 0.006
K_{yy}		1.242	0.404 -3.750	-0.563	0.070 -1.662	-0.560	0.378 -1.653
C_{xx}	$\times 10^5$ N·s/m	-2.664	-0.360 -8.899	-0.943	-0.107 -2.653	-1.725	-0.141 -6.279
C_{xy}		1.707	4.294 0.339	0.537	1.311 -0.022	0.326	1.808 -1.301
C_{yx}		-2.253	-0.948 -4.613	-1.138	-0.238 -2.046	-1.253	-0.143 -2.746
C_{yy}							

C_{xx} in S_{B1} , they exhibit a curve fluctuating by small amplitudes all over or locally. The reason for this is considered to be the influence of fluctuations similar to those in rotor center acceleration variations, as seen in Fig. 8.

The range of coefficient variations is very wide, as seen in Figs. 10 and 11. For instance, K_{xx} in S_{B1} has a minimum value of -5.21×10^8 N/m ($\psi=294^\circ$) and a maximum value of -4.92×10^7 N/m ($\psi=176^\circ$). As for the damping coefficients, C_{xx} in S_{B1} varies from -8.90×10^5 N·s/m ($\psi=294^\circ$) to -3.60×10^4 N·s/m ($\psi=168^\circ$). Namely, they vary by more than one order of magnitude. Most other coefficients are also similar. In some cases, a coefficient's value changes from positive to negative, or negative to positive, over a rotor revolution, since no static equilibrium position exists in this case of analysis.

To summarize, it can be said that nonlinearities due to bearing oil films seriously influence rotor motion. Hence, linearizing the rotor system is not acceptable, and nonlinear system analysis are needed to analyze the rotor-journal bearing system in a rotary compressor. The degree of linearity, or nonlinearity, caused by the bearing oil films can be evaluated by examining these results.

The maximum, minimum, and mean values of the coefficients for each individual bearing are given in Table 1. They clearly show the relative differences in the value of the same type of coefficient among the three bearings S_{B1} to S_{B3} . This result is obtained using this analysis together with nonlinear system analysis, by which the characteristics of all the bearings can be simultaneously obtained along with their correlations. It is also useful for linear rotor systems, in particular, a complex multirotor system, in order to find the oil film coefficients in all the bearings with their relative differences, as shown in Table 1.

In the vibration analysis model of whole machines which incorporate nonlinear system analysis, there are cases where the stiffness and damping coefficients are roughly estimated as constants. When the

coefficients widely vary with singular profiles, as shown in Figs. 10 and 11, it is difficult to select constant values from only the analytical results. Consequently, some experimental measurements may be needed to confirm the validity of the estimated values. However, the mean values in Table 1 may be applied in the vibration analysis. It is also considered reasonable to modify the mean values by referring to the profile features or the maximum and minimum values. Thus, these analytical results can be fully used as determining criteria for constant values.

6. Conclusions

(1) The stiffness and damping coefficients of bearing oil films are obtained by means of numerical analysis for a rotor-journal bearing system with large dynamic loads. In this analysis, the variations over one rotor revolution (during one cycle) have been obtained by calculating the coefficients as an unsteady state along the locus of the rotor center.

(2) With this analysis, together with nonlinear system analysis, relative differences among the same type of coefficients in different bearings were elucidated. Furthermore, when the coefficients are given estimated constant values in the vibration analysis of whole machines including such a rotor system, these results can be used as determining criteria for the constant values, and the variations can be taken into consideration.

(3) As shown in the results, for a rotary compressor with two compression units, the profiles of the oil film coefficients differ greatly among the three bearings. However, they are similar to the profiles of the individual bearing reaction forces. Then, the coefficients vary over a wide range, and most of them vary by more than one order of magnitude. Accordingly, it is clear that oil film nonlinearities seriously influence the rotor motion.

References

- (1) Hattori, H. and Kawashima, N., Dynamic Analysis of a Rotor-Journal Bearing System with Large Dynamic Loads (Influence of Relative Misalignment between the Bearings on the Characteristics of a Rigid Rotor System), JSME Int. J., Ser. III, Vol. 34, No. 4 (1991), p. 503.
- (2) Hattori, H. and Kawashima, N., Dynamic Analysis of a Rotor-Journal Bearing System for Twin Rotary Compressors, Proc. Int. Compressor Engineering Conference at Purdue, Vol. 2 (1990), p. 750.
- (3) Hattori, H., Dynamic Analysis of a Rotor-Journal Bearing System with Large Dynamic Loads (Application to an Elastic Flexible Rotor System), Trans. Jpn. Soc. Mech. Eng., (in Japanese),

- Vol. 56, No. 531, C (1990), p. 2984.
- (4) Inagaki, T., Study on Non-steady Stability of a Rotor-Journal Bearing System (Stability of the 360 Degree Short Bearing), Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 53, No. 492, C (1987), p. 1704.
 - (5) Lund, J. W. and Thomsen, K. K., Topics in Fluid Film Bearing and Rotor System Design and Optimization, (1978), p. 1, ASME publ.
 - (6) Lund, J. W., Review of the Concept of Dynamic Coefficients for Fluid Film Journal Bearings, Trans. ASME, J. Tribol., Vol. 109, No. 1 (1987), p. 37.
 - (7) Someya, T., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 42, No. 360 (1976), p. 2599.
 - (8) Hashimoto, H., Wada, S. and Marukawa, T., Performance Characteristics of Large Scale Tilt-ing-Pad Journal Bearings, Bull. JSME, Vol. 28, No. 242 (1985), p. 1761.
 - (9) Goto, F., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 51, No. 461, C (1985), p. 88.
 - (10) Konishi, T., Baba, M. and Takehisa, Y., A Study of Journal Bearings with Circumferential Oil Grooves at Both End-Sides (3rd Report) -Theory of Dynamic Characteristics of Journal Bearings with Two Sealing Disks-, J. Jpn. Soc. Lubri. Eng., (in Japanese), Vol. 31, No. 6 (1986), p. 406.
 - (11) Kato, T. and Hori, Y., Application of the Matrix Form of the Reynolds Equation to Dynamic Journal Bearings, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 54, No. 500, C (1988), p. 935.
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