Fuzzy Sliding-Mode Control of Gasoline Fuel-Injection System with Oxygen Sensor*

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The fuzzy "sliding mode" control method is proposed in this paper for the design of a closed-loop fuel-injection system. A fuzzy logic control using variable structure system is combined with current oxygen sensors to maintain the stoichiometric air-to-fuel ratio. The fuzzy cell technique is also proposed to implement the designed controller on nonfuzzy processors. The proposed method offers good potential for the design of an engine controller since: (1) bias and large chattering errors due to the hysteresis of the oxygen sensor can be reduced by utilizing fuzzy logic control inside the boundary layer of the sliding mode; (2) robustness and stability are analytically guaranteed by the hybrid control structure. Simulation studies based on a typical engine-only model demonstrate the usefulness of the proposed method.

Key Words: Fuzzy Set Theory, Internal Combustion Engine, Nonlinear Control, Gasoline Engine, Fuel Injection, Air-to-Fuel Ratio, Fuzzy Control, Oxygen Sensor

1. Introduction

The stringent automobile emission standards necessitate a fuel-injection controller which can maintain the stoichiometric air-to-fuel ratio at the desired level. Generally speaking, this controller has only an add-on function which can supplement a fuel-management system for operating the engine with the least pollutant emissions. However, this kind of controller design is not a trivial task for the inherently nonlinear and cyclic nature of a four-stroke gasoline engine under a wide operating range. Current fuel-injection controllers heavily rely upon open-loop gain scheduling along with proportional plus integral action control using an exhaust gas oxygen (EGO) sensor. This approach suffers since a lengthy procedure is required to create the look-up tables of open-loop engine calibration⁽¹⁾. Other well-known nonlinear control

techniques such as sliding mode control (SMC) and fuzzy logic control (FLC) may be applied to obtain a robust fuel injection controller⁽²⁾. However, non-differentiable nonlinearities and imprecisions of most EGO sensors combined with complex engine dynamics are extremely idealized for the convenience of nonlinear control design. Hence, designers cannot utilize even the useful information except for the binary portion of the sensor output. As a result, SMC has the shortcoming of large chattering, and FLC is also not adequate to reduce the magnitude of oscillation in steady-state response. Although less research has been done on the fuel injection controller, taking into consideration the hysteresis effect of the EGO sensor, studies to date have shown that the evidence of this effect has increased⁽³⁾. In particular, the bias error in the SMC due to the neglect of the sensor hysteresis effect often degrades the control performance severely.

In order to overcome these limitations, the fuzzy "sliding mode" control method for designing a fuel injection system is proposed in this paper. The essence of the proposed method is to use the qualitative output information of the EGO sensor by adopting

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a rule base. Using the dual concept of fuzzy implication and inference, a rule base is constructed to include the properties of performance and robustness under sensor hysteresis. The fuzzy cell technique is also adopted to improve the computational efficiency of fuzzy inference. Thus, it becomes very simple to implement a fuzzy "sliding mode" controller based on nonfuzzy-processors. Subsequent sections present the flexible design procedure and the advantages of employing the proposed methodology.

2. System Model

A nonlinear model is used for designing a closed-loop fuel-injection system as depicted in Fig. 1, which is a modified version of the three-state engine model by Cho and Hedrick⁽⁴⁾. Three states with two time delays are presented as below. The mass of air in the intake manifold, M_a , is obtained by the empirical equations reduced from the conservation of mass:

$$\begin{aligned} dM_a/dt = & M_{ai}[\alpha, P_m, P_a, t] \\ & - M_{ao}[\eta(W_e, M_a), W_e, M_a, t], \end{aligned} \quad (1)$$

where the mass flow rate of air entering the intake manifold, M_{ai} , is expressed by the normalized throttle characteristics and the choked flow influence as a nonlinear function, and α , P_m , P_a and t denote the throttle angle in degrees, the manifold pressure, the atmospheric pressure and continuous time, respectively. The mass flow rate of air leaving the intake manifold, M_{ao} , can be expressed by the engine speed in radians per second, W_e , and the volumetric efficiency function, $\eta(W_e, M_a)$, which is generally determined by steady-state tests. Under the assumption of adiabatic compressible flow and uniform pressure distribution, M_a is calculated from the ideal gas law. Thus an estimate of M_{ao} can be expressed by the measurable states of W_e , P_m , and the manifold temperature, T_m . Next, the rotational dynamics of the engine with indicated torque, T_i , is generated in the continuous-time domain by the combination of a curve-fitted air-to-fuel ratio (A/F) influence function, M_{ao} , and

W_e :

$$\begin{aligned} I_e \cdot dW_e/dt = & T_i[A/F, M_{ao}(t - \Delta t), \\ & W_e(t - \Delta t)] - T_f, \end{aligned} \quad (2)$$

where the torque production delay, Δt , captures the cyclic nature of a four-stroke engine, I_e is the effective rotational inertia of the engine, and T_f is the resultant friction force. The last state is associated with fueling dynamics. Lag and time delay are combined into the discrete form of the first-order filter using the approximation by the Euler method:

$$M_{fi}(k+1) = M_{fi}(k) + h \cdot [M_{fc}(k) - M_{fi}(k)]/\tau_f(k), \quad (3)$$

where M_{fi} is the actual rate of fuel entering the cylinder, and $\tau_f(k)$ is the effective fueling time constant at k th discrete time with sample interval, h . Physical parameters of the above three-state model are determined empirically for different engines. More detailed modeling of a typical mid-sized vehicle engine can be found in Ref. (5). Ultimately the three-state engine model in this work is modified to be suitable for the simulation of discrete fuel-injection controllers.

In order to reduce the level of pollutants in the exhaust gas, a catalytic converter converts CO, HC, NO_x to less harmful substances. These converters can only operate efficiently if combustion is as complete as possible. Hence, the objective of the fuel injection controller is the maintenance of A/F at the stoichiometry window such that the air-fuel mixture enables virtually complete combustion.

$$M_{ao}(k)/M_{fi}(k) = \beta \pm 0.3, \text{ for } k > 0, \quad (4)$$

where β is the stoichiometric ratio (14.64). The above instantaneous ratio of rate is modeled as A/F instead of the actual ratio integrating rate from the intake valve, open to closed. However, control objective is naturally satisfied since the employment of the instantaneous A/F can achieve the more stringent requirement. A ZrO₂-EGO sensor is widely used for the precise control of A/F in spite of inherent limitations due to the imprecision of the sensor. In particular, the sensor hysteresis as shown in Fig. 2(b) is yielded from the switching behaviors which contain the out-

Fig. 1 Schematic of closed-loop fuel-injection system

Fig. 2 Typical ZrO₂-EGO sensor characteristics

put voltage switching rate and response time delay with respect to the intake A/F transients⁽⁶⁾. This implies that the voltaic output of the EGO sensor does not change at exactly the same point for the changes of increasing A/F and decreasing A/F.

3. Sensor Hysteresis Effects in Sliding-Mode Control

SMC has recently been employed for designing a closed-loop fuel-injection system, since the method is directly applicable to a nonlinear engine system. Its design procedure is briefly summarized as follows. Let a sliding surface be defined as:

$$S(k) \equiv M_{ao}(k) - \beta \cdot M_{fi}(k). \quad (5)$$

Then, the following SMC law is obtained by considering the robustness for the worst case of model error combination.

$$M_{fc}(k) = M_{fi}(k) + \tau_f(k) [\dot{M}_{ao}(k) + K(k) \cdot \text{sign}(S(k))] / \beta, \quad (6)$$

where the robustness parameter, $K(t)$, is designed as:

$$K(k) = (\gamma + \delta + \gamma \cdot \delta) \cdot |\dot{M}_{ao}(k)| + (1 + \delta) \cdot \gamma_d \cdot M_{ao}(k), \quad (7)$$

providing that multiplicative model errors in the induction process, $\gamma(k) = M_{ao}(k) / \hat{M}_{ao}(k) - 1$, and fuel delivering, $\delta(k) = \tau_f(k) / \hat{\tau}_f(k) - 1$, can have upper bounds of γ and δ . The upper-bound value of $\gamma(k)$'s derivative is denoted by γ_d . Symbols $\hat{\cdot}$ and $\dot{\cdot}$ represent the estimated quantity and the differential state, respectively. The terms of control Eq. (6), $\tau_f(k)$, $M_{ao}(k)$ and $M_{fi}(k)$, are dynamically determined from Eqs. (1), (2) and (3). A backward difference approximation is used to translate a differential filter, for the mass flow rate of air, to a digital filter as:

$$\dot{M}_{ao}(k) = [\hat{M}_{ao}(k) - \hat{M}_{ao}(k-1)] / (1 + Nh), \quad (8)$$

where the value of N for the time constant is usually fixed to $10^{(7)}$. In practice, the following configuration of $S(k)$ is chosen for a binary sensor model:

$$S(k) = 450 \text{ mV} - V(k - \Delta k), \quad (9)$$

where voltaic sensor output, V , is associated with the measurement time delay Δk , and the value of 450 mV in Eq. (9) represents the voltaic sensor output at stoichiometric ratio, as shown in Fig. 2(a). For the comparison of A/F at the typical engine speed of 2000 rpm, the robustness parameter is designed by selecting γ and δ as 0.1. The maximum allowable γ_d that meets the design specification is also determined as 0.2π . The performance of SMC is demonstrated for a binary sensor model in Fig. 2(a). The chattering magnitude of simulated A/F depicted in Fig. 3(a) is slightly larger than the prescribed boundary of ± 0.3 in Eq. (4). Note that the SMC is compatible with current EGO sensor technology because the controller

requires only sign information of the sensor output. However, most of the current EGO sensors have evidence of the hysteresis effect. Therefore, we once apply an actual sensor model as illustrated in Fig. 2(b). The simulated A/F is shown in Fig. 3(b) for the same controller except the sensor model. It is clear that the larger chattering and bias error of A/F are unacceptable for the fuel-injection controller, although a chattering A/F of small magnitude is actually beneficial for the efficiency of the catalytic converter. This result illustrates that the A/F controlled by using an ideal on-off sign function, $\text{sign}(S(k))$, is generally unsatisfactory for the hysteresis of an actual EGO sensor model. Specifically, the bias error is mainly attributed to the difference of A/F that is the switching part of output voltage at stoichiometry between the lean to rich path and the rich to lean path. It can be seen that the feedback signal, $S(k)$, given above cannot compensate the hysteresis effect of the sensor because the $\text{sign}(S(k))$ does not contain magnitude information. Therefore, the feedback signal of $S(k)$ in Eq. (6) loses magnitude information despite the fact that the robustness parameter, $K(k)$, automatically adjusts to different operating conditions.

Fig. 3 Simulated air-to-fuel ratio : Sliding-mode control

4. Fuzzy Sliding-Mode Fuel-Injection Controller

4.1 Fuzzy logic control for boundary layer

An alternative approach proposed in this paper is the fuzzy “sliding mode” control method. Its objective is to correct bias error components due to sensor hysteresis, which is achieved by a nonlinear smoothing approximation of control discontinuities in a thin boundary layer adjacent to the switching surface.

$$B(k) \equiv \{\lambda, \Phi_1 < S[V(\lambda; k)] < \Phi_2\}, \quad (10)$$

where λ is A/F normalized by β , and the reference input λ_R becomes 1. Sliding surface $S[V(\lambda; k)]$ is represented by the voltaic output of λ at the k th discrete time. The thickness of the boundary layer for the sliding surface is defined as $|\Phi_1 - \Phi_2|^{(8)}$. Without loss of generality, the idea of the boundary layer can be extended to the linear range of the EGO sensor. Although the linear band is very narrow and sensitive to temperature, the proposed method can express rather accurate qualitative information. The proce-

Table 1 Scaling factor of fuzzy set

Adjective	NB	NS	ZE	PS	PB
V	100	250	450	750	940
VC	-15000	-5000	0	5000	15000
Λ	-1	-0.4	0	0.4	1

Fig. 4 Fuzzy logic controller using fuzzy cell

dures of design is described as follows. A boundary layer separates the state space into the FLC and SMC subspaces. When the closed-loop dynamics lies in the outside of $B(k)$, the control law is identical to that defined in Eq. (6). However, all trajectories starting inside $B(k=0)$ remain inside $B(k)$ for all $k > 0$, and the control law inside of $B(k)$ is interpolated by FLC. Thus the term, $\text{sign}(S(k))$, in the expression of the controlled fuel rate in Eq. (6) can be replaced by the following type of FLC ($S(k)$), as shown in Fig. 4:

$$\text{FLC}(S(k)) = FZ[V(v(k)), VC(vc(k))], \quad (11)$$

where FZ means the fuzzy relation defined by the rule base and appropriate scaling factors in Table 1, $v(k)$ is the crisp output of the sensor voltage at k th discrete time, and $vc(k)$ denotes the rate of change $v(k)$.

A fuzzy logic controller for the linear band of the ZrO_2 -EGO sensor provides a means of converting a linguistic control strategy into the sensor technology for measuring A/F information to handle the hysteresis and imprecision of the sensor easily. The basis of fuzzy logic is the representation of the rule base as membership functions shown in Fig. 4, which are related by the dual concepts of fuzzy implication and the compositional rule of inference. A general form of fuzzy rule is expressed by the following Mamdani's form⁽⁹⁾:

$$\text{ALSO } \{ \text{IF } (V \text{ is } A_j \text{ AND } VC \text{ is } B_j), \text{ THEN } (\Lambda \text{ is } C_j) \}, \quad (12)$$

where the output voltage, V , and the rate of change in output voltage, VC , are the input states to FLC, Λ is the output state, and subscript j denotes the rule number. The adjectives A_j , B_j , and C_j , are any of the linguistic attributes NB, NS, ZE, PS or PB. These adjectives used to formulate rules are more rigorously defined in membership functions. The shape of triangular-type membership functions as shown in Fig. 4 may be selected to be crossed at the grade 0.5 according to Mizumoto⁽¹⁰⁾. The rules reflect “situation-action” pairs to cope with the sensor characteristics.

A fuzzy “sliding mode” controller is designed as a block diagram shown in Fig. 5. Key to the design is a

Fig. 5 Block diagram of fuzzy sliding-mode controller

decision of the fuzzy rule base which corresponds to the qualitative characteristics of the sensor. Although exploitation of knowledge in the control design is the most advantageous aspect of fuzzy approach, the lack of a systematic design technique is a limiting factor. A good start is to make sure that all designers agree with proven input-output states, and that they also produce acceptable results for new data. Therefore, an approach for the rule base is proposed by using the systematic procedures of SMC design. A basic rule base may be created by the linearization schemes in the boundary layer, $B(k)$ whose output accords with the linguistic input of FLC. Then, using the basic rule base, precise evaluation for the compensation of the sensor's incomplete information can be performed in a trial-and-error manner. However, it can be easily modified by changing the rules. Because the evaluated rule base in the bottom table of Fig. 4 may model the decision maker rather than the sensor, sensor feedback is less critical to its hysteresis and nonlinearity. Fuzzy logic also tolerates imprecision since its decision includes contributions from parallel firing rules. Although FLC offers many advantages over conventional SMC, the two methods can be used in a complementary manner to guarantee both stability and robustness by hybrid control. In particular, FLC is beyond the proof of mathematical stability, and SMC cannot correct the sensor hysteresis. Therefore, fuzzy "sliding mode" control appears to be a practical design methodology since the stability in the sense of Lyapunov is proven by utilizing SMC outside of the boundary layer, $B(t)$:

$$dS(t)/dt = -\eta \cdot \text{sign}(S(t)), \eta > 0, \quad (13)$$

where the surface defined by " $S(t)=0$ " can be stable, and η denotes the robustness parameter for attraction. While the closed-loop dynamics lie inside the boundary layer, FLC takes over robustness for sensor imprecision.

4.2 Fuzzy cell

The fuzzy cell technique derived from the analysis of the phase plane has shown an effective computational approach to the implementation of a fuzzy system based on nonfuzzy processors. When a FLC scans an application rule base, the dominant fuzzy rules are selected by the set of three $[A_j, B_j, C_j]$, which are the nearest points to the arbitrary given states. To obtain a fuzzy cell from the rule base, the control phase space must be quantized and partitioned into cells. The range of the cell is defined by the form of a Cartesian product as:

$$\text{CELL} = [A_j, A_{j+1}] \times [B_m, B_{m+1}] \times [C_n], \quad (14)$$

where the assignment of an input state pair $[v(k), vc(k)]$ to a fuzzy set is given by

$$A_j(k) < v(k) < A_{j+1}(k) \text{ and}$$

$$B_j(k) < vc(k) < B_{j+1}(k). \quad (15)$$

Membership functions of the fuzzy cell should be selected to be disjointed at the outside of the cell boundary. Thus, fuzzy inference is implemented on a fuzzy cell instead of a global rule base. Each rule of the fuzzy cell constitutes a conclusion weighted by the degree to which the situation statement of the rule is fulfilled. The final decision is obtained by averaging the weighted membership functions of all the contributed conclusions in the cell. To show the use of the fuzzy cell, the procedure shown in Fig. 4 is expanded. Given crisp input values of $v(k)$ and $vc(k)$, the crisp output value of $\lambda_o(k)$ is inferred, after creating a 4-rule system to govern the FLC as follows. The control flow starts introducing the above dominant FLC rules in the fuzzy cell.

Rule 1: IF V is NB AND VC is PS , THEN Λ is NS

Rule 2: IF V is NS AND VC is PS , THEN Λ is ZE

Rule 3: IF V is NS AND VC is PB , THEN Λ is PS

Rule 4: IF V is NB AND VC is PB , THEN Λ is ZE

Then the i th truth degree of membership functions in the input fuzzy sets of cell, α_i , is applied to determine the influence of fuzzy output sets on the final conclusion:

$$\alpha_1 = (1-d_1)(1-d_2), \alpha_2 = (1-d_2)d_1, \\ \alpha_3 = d_1 \cdot d_2, \alpha_4 = (1-d_1)d_2, \quad (16)$$

where d_1 and d_2 are the distances from rule 1 to the given states, v and vc . The following modified form of fuzzy inference with fuzzy cell is constructed by the algebraic product for fulfillment, the arithmetic mean for the rule's parallel firing and the center of gravity method for defuzzification as follows:

$$\lambda_o(k) = \frac{\sum_{i=1}^4 A_i(v(k)) * B_i(vc(k)) * C_i^d}{\sum_{i=1}^4 A_i(v(k)) * B_i(vc(k))}, \quad (17)$$

where C_i^d is the defuzzified value of membership function C_i that can best represent the linguistic description, and i denotes the i th rule of the fuzzy cell. In the conditional statement of Eq. (12), the fulfillment, α_i , can be represented by the algebraic production. Typically, we take the centroid of membership function as its value⁽¹¹⁾. As a result, the hybridization in Eq. (6) between the FLC crisp output, FLC ($S(k)$), which is substituted from the defuzzified value, $\lambda_o(k)$, in Eq. (17) and the robustness parameter in Eq. (7) can exploit the FLC tolerance for sensor imprecision. Because sensor precision is costly, it makes sense to minimize the precision needed to perform the task of reducing pollutant emissions. Furthermore, the fuzzy cell technique with the inference procedure in Eq. (17) is very suitable for practical applications because of its computational simplicity. The important advantage of FLC, which lies in the interpolation nature of the

(a) Fuzzy SMC with actual EGO sensor

(b) Fuzzy SMC for the worst model error

Fig. 6 Simulated air-to-fuel ratio: Fuzzy sliding-mode control

rules, complements the boundary layer of conventional SMC to consider hysteresis and imprecision of the EGO sensor.

For the purpose of comparison, the designed fuel-injection controller with an actual sensor model in Fig. 2 (b) is simulated under the perfect model. The resulting A/F in Fig. 6 (a) is better than those of SMC as shown in Figs. 3 (a) and 3 (b). That is, the designed controller can reduce the bias and chatter magnitudes compared to the result of conventional SMC. Finally, we enter the worst combination of the above reasonable errors in section 3 into the designed controller. Then the chatter magnitudes as depicted in Fig. 6 (b) are slightly increased, but the result is comparable to the prescribed boundary. This clearly shows that the fuzzy "sliding mode" control method is successful in maintaining the A/F within the prescribed boundary layer in spite of the worst error combination. This result shows that the fuzzy logic scheme of the proposed method provides robustness from the viewpoint of performance mainly. Of course, robustness in the sense of relative stability can also be considered in terms of the structure of SMC. However, the fuzzy "sliding mode" fuel-injection controller presented in this paper can serve as only a first step

toward demonstrating the feasibility of the proposed method. In order to meet the design objective, an actual controller may require not only additional fine tuning of the robustness parameter for reducing the model errors, but also the evaluation of the effective rule base for compensating the sensor imprecision.

In addition, the controller is programmed in floating-point arithmetic on an Intel 80386 32-bit microprocessor employing a 33 MHz clock; the required computing time is only 0.6 millisecond in spite of the fuzzy inference. This result demonstrates that the fuzzy "sliding mode" fuel-injection controller can be implemented by using the fuzzy cell technique instead of fuzzy coprocessors.

5. Conclusions

This paper has proposed a fuzzy "sliding mode" control method for designing a fuel-injection controller because of its capability to consider the incomplete information of the current oxygen sensor and its applicability to nonlinear engine models. Moreover, the designed controller can be implemented on nonfuzzy processors by adopting the new technique of the fuzzy cell. With the basic idea of incorporating the fuzzy logic into the boundary layer of the sliding mode, the proposed design methodology can guarantee the robustness and the stability of the controlled system analytically. The simulation result of this algorithm show good performance regardless of sensor hysteresis and model errors. The advantages of the proposed method are summarized as follows: it can provide accurate control using low-cost oxygen sensors and nonfuzzy processors; the evaluation of the rule base is flexible to compensate the hysteresis and imprecision of the sensor.

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