Rotordynamic Analysis of Submerged Motor Pumps*

(Influence of Long Seal on the Stability of Fluid Machinery)

Yuji KANEMORI** and Takuzo IWATSUBO***

The influence of dynamic fluid reaction forces and moments of a long annular seal on the stability of fluid machinery was investigated. The stability analysis was carried out by using a submerged motor pump in a circulation boiler, in order to explain the effect of the long seal. In the stability analysis, the individual contributions of the fluid forces and moments due to rotor conical and cylindrical whirl movements were evaluated according to logarithmic decrements. As a result, the individual contributions depend on the mode shapes of the vibration. Fluid forces acting on the long seal due to conical and cylindrical whirl movements should not be ignored because they play an important role in the stability.

Key Words: Stability, Vibration of Rotating Body, Damping, Self-Excited Vibration, Whirling, Hydraulic Machine

1. Introduction

The pump is one of the important fluid machines of plants in many industrial fields. Many types of pumps have been developed and efforts are being made to increase the pump speed. When the pump runs at a high speed or the axial length of the non-contacting seal is very long, such as in balance pistons, fluid moments are induced in the long seal because the flow velocities at the seal inlet and the outlet are not equivalent. Kanemori and Iwatsubo have also experimentally found that large restoring moments and lateral forces are exerted on the long seal as a result of the conical whirl of the rotor⁽¹⁾⁻⁽⁴⁾.

The long seal is likely to have moments or is strongly affected by the reaction forces due to conical whirl and therefore requires 4×4 stiffness, damping and added mass coefficient matrices, in addition to the 2×2 short seal matrices⁽⁵⁾. The notation " 4×4 " refers to a rotordynamic model which defines the force and moment components $\{F_x, F_y, M_x, M_y\}$ as linear func-

tions of the displacements and rotations, namely the cylindrical and conical whirls $\{x, y, \alpha_x, \alpha_y\}$.

However, the effect of the moments occurring in a long seal and the fluid forces associated with the conical whirl of the rotor on the pump rotor stability has not yet been clarified as far as we know. In this report the stability of a wet-motor-type submerged pump in a circulation boiler is analyzed, in order to explain the effect of the long seal.

The chief characteristic of the wet motor is the fluid-filled annular clearance around the rotor of the submerged motor. Cool water fills the long annular clearance between the rotor and the stator and is forcibly circulated in the axial direction to carry away the heat generated by the motor and the heat transmitted from the pump. (The long annular clearance will be called the long seal in this study.)

This report describes the influence of the long seal of the wet motor on the stability of rotor vibration based on the individual studies⁽¹⁾⁻⁽⁴⁾ on the long seal by Kanemori and Iwatsubo. Logarithmic decrements associated with the fluid forces and the moments exerted on the long seal due to the cylindrical and conical whirls of the rotor are derived individually. The individual logarithmic decrement is adopted for evaluating the contribution of fluid elements on the

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** R & D Dept., Torishima Pump Mfg. Co., Ltd., 1-1-8 Miyata-cho, Takatsuki, Osaka 569, Japan

*** Faculty of Engineering, Kobe University, Rokko, Nada, Kobe 657, Japan

rotor stability, and the sum of the individual decrements represents the total logarithmic decrement of the rotor system. The individual contributions of the forces and moments are investigated in the stability analysis.

2. Nomenclature

- D, L : Diameter and length of long seal
 j : Imaginary unit $=\sqrt{-1}$
 V : Seal inlet swirl velocity $=V_i/R\omega$
 $\mathbf{X}$: Translational vector of rotor system
 $[\mathbf{K}]$: Stiffness matrix
 $[\mathbf{C}]$: Damping matrix
 $[\mathbf{M}]$: Mass matrix
 $[\mathbf{K}^h]$: Symmetric parts of stiffness matrix
 $[\mathbf{C}^h]$: Symmetric parts of damping matrix
 $[\mathbf{M}^h]$: Symmetric parts of mass matrix
 $[\Delta\mathbf{K}]$: Unsymmetric parts of stiffness matrix
 $[\Delta\mathbf{C}]$: Unsymmetric parts of damping matrix
 $[\Delta\mathbf{M}]$: Unsymmetric parts of mass matrix
 $\{\boldsymbol{\eta}_e\}$: Translational vector of element $=\{x, y\}^T$
 $\{\boldsymbol{\eta}\}$: Translational vector $=\{x, y, \alpha_x, \alpha_y\}^T$
 ω : Rotational angular speed
 λ_i : i -th eigenvalue $=\gamma_i + j\beta_i$
 δ_i : i -th logarithmic decrement
 δ_{ee} : Individual logarithmic decrement associated with forces due to cylindrical displacement
 δ_{ae} : Individual logarithmic decrement associated with moment due to cylindrical displacement
 δ_{ea} : Individual logarithmic decrement associated with forces due to conical rotation
 δ_{aa} : Individual logarithmic decrement associated with moment due to conical rotation
 Ψ : Phase difference between the seal inlet and outlet whirling movements, as defined in Fig. 3.

Subscripts

B, I , and S : Bearing, impeller, and seal, respectively

3. Characteristics of Long Seals

The slenderness ratio (L/D) of the long seal is large, which means that the flow pattern at the seal inlet differs from that at the outlet. As a consequence, fluid forces acting on the seal change in the axial direction. Distribution of stiffness, damping coefficient, and added mass in the axial location obtained from the finite length solution (5) is shown in Fig. 1. The horizontal axis expresses the distance from the long seal inlet to the outlet, and the vertical axis indicates dynamic coefficients belonging to the 10 axially divided elements of the long seal.

If the seal inlet swirl is opposite to the rotational direction of the rotor, the sign of cross-coupled

stiffness coefficient k is negative at the inlet but becomes positive at the outlet. On the other hand, the sign of the diagonal stiffness coefficient K is positive at the inlet but becomes negative at the outlet. The signs change in the axial direction because the circumferential velocity in the long seal at the outlet is different from that at the inlet.

For the short seal, it may be assumed that the circumferential velocity in the seal is almost uniform and the fluid forces do not change significantly in the axial direction. In this case, the fluid forces acting on the rotor may be considered uniform without causing a significant error. In the case of the long seal as shown in Fig. 1, however, the fluid forces change considerably in the axial direction. This requires consideration of the effect of the moments, and it is necessary to divide the long seal into some elements axially for modelling. The long seal is divided axially into 4 elements in this analysis.

4. Theoretical Analysis

4.1 Analytical model

A submerged motor pump in circulation boiler is employed as an analytical model to investigate the effect of the long seal on the stability of the pump. The schematic model of the submerged motor pump is shown in Fig. 2, and the major specifications are described in Table 1.

A heat barrier is provided to prevent the motor from being affected by heat transfer from the high-temperature pumping fluid. There is an annular clearance between the heat barrier and the rotor. Water does not flow axially in the annular clearance and its dynamic characteristics are obtained from the Fritz theory, whose accuracy is considered to be relatively high⁽¹⁾.

As shown in Fig. 2, an impeller is attached to the bottom of the rotor to forcibly circulate the water in

Fig. 1 Dynamic characteristics of long seal

the axial direction to carry away the heat in the motor and the heat transmitted from the pump.

Because the pump is operated in the vertical position, the radial bearing which supports the rotor does not carry the weight of the rotor. Magnetic pull, however, actually works between the rotor and stator, and radial thrust is exerted on the impeller. These forces act as a static load on the bearings. Taking the magnetic pull⁽⁶⁾ into consideration, it is assumed that the load shown in Table 1 is supported by each bearing.

Furthermore the magnetic pull also drives the

Table 1 Specifications of submerged motor pump

Parts	Diameter (mm)	Radial clearance (mm)	Length (mm)
Bearing	150	0.15	220
Wear-ring seal (Groove)	278	0.50	40
Long seal(Plain) (Rotor&stator)	250	1.2	960
Annular-clearance	123	1.2	360
Impeller	530	55 : Width at outlet	
Pressure difference ΔP (MPa) (at ω =1770 cpm)			
Wear-ring seal	0.54	Long seal	0.35
Rotor dimensions			
Length (mm)	2556	Mass (kg)	665
Bearing span (mm)	1508	Bearing load (N)	2200
Magnetic pull		$K_m = -2.12 \times 10^7$ (N/m)	
Natural frequency in air		$\beta_o = 3220$ (cpm)	
Water temp. in motor		$T_w = 303$ (° K)

Fig. 2 Cross section of submerged motor pump

rotor eccentrically, and the force is modelled with a negative stiffness coefficient K_M in the analysis. It is also assumed that the cross-coupled term of the magnetic pull does not exist because it is negligibly small.

Since the bearings are lubricated with water, the inertial force of each bearing due to water mass is large and not negligible. Added mass and effective viscosity coefficients of the bearing are obtained from Ref. (7). Approximate values of the bearing stiffness and damping coefficient are obtained from the laminar flow theory taking the effective viscosity into account (8). The dynamic characteristics of the impeller and wear-ring seal are obtained from Refs. (9) and (10), and those of the long seal are derived from the finite length solution⁽⁵⁾. The eigenvalue of the rotor is calculated ignoring the stiffness of the armature but only accounting for the influence of the mass.

4.2 Stability analysis

As illustrated in Fig. 3, the rotor is supported by bearings and seal, and the impeller is subjected to fluid forces. The forces and moments exerted on the long seal F_s are expressed by 4×4 coefficient matrices considering that fluid forces due to the cylindrical displacement and those due to the conical rotation are combined⁽⁴⁾ as follows:

$$- [F_s] = - \begin{Bmatrix} F_x \\ F_y \\ M_x \\ M_y \end{Bmatrix} = \begin{bmatrix} K & k & -k_{ea} & K_{ea} \\ -k & K & -K_{ea} & -k_{ea} \\ k_{ae} & -K_{ae} & K_a & k_a \\ K_{ae} & k_{ae} & -k_a & K_a \end{bmatrix} \begin{Bmatrix} x \\ y \\ \alpha_x \\ \alpha_y \end{Bmatrix}$$

Fig. 3 Model of submerged motor pump

$$\begin{aligned}
& + \begin{bmatrix} C & c & -c_{\epsilon a} & C_{\epsilon a} \\ -c & C & -C_{\epsilon a} & -C_{\epsilon a} \\ c_{a\epsilon} & -C_{a\epsilon} & C_a & c_a \\ C_{a\epsilon} & c_{a\epsilon} & -c_a & C_a \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{a}_x \\ \dot{a}_y \end{bmatrix} \\
& + \begin{bmatrix} M & 0 & 0 & M_{\epsilon a} \\ 0 & M & -M_{\epsilon a} & 0 \\ 0 & -M_{a\epsilon} & M_a & 0 \\ M_{a\epsilon} & 0 & 0 & M_a \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{a}_x \\ \ddot{a}_y \end{bmatrix} \\
& = [K_s]\{\eta\} + [C_s]\{\dot{\eta}\} + [M_s]\{\ddot{\eta}\} \\
& = \begin{bmatrix} [K_{\epsilon\epsilon}] & [K_{\epsilon a}] \\ [K_{a\epsilon}] & [K_{aa}] \end{bmatrix} \{\eta\} + \begin{bmatrix} [C_{\epsilon\epsilon}] & [C_{\epsilon a}] \\ [C_{a\epsilon}] & [C_{aa}] \end{bmatrix} \{\dot{\eta}\} \\
& + \begin{bmatrix} [M_{\epsilon\epsilon}] & [M_{\epsilon a}] \\ [M_{a\epsilon}] & [M_{aa}] \end{bmatrix} \{\ddot{\eta}\}. \quad (1)
\end{aligned}$$

$[K_{\epsilon\epsilon}]$ to $[M_{aa}]$ of Eq.(1) are 2×2 coefficient matrices. The fluid forces acting on the bearings and the impeller are expressed by the following equations.

$$\begin{aligned}
-[\mathbf{F}_B] &= [\mathbf{K}_B]\{\eta_\epsilon\} + [\mathbf{C}_B]\{\dot{\eta}_\epsilon\} + [\mathbf{M}_B]\{\ddot{\eta}_\epsilon\} \\
-[\mathbf{F}_I] &= [\mathbf{K}_I]\{\eta_\epsilon\} + [\mathbf{C}_I]\{\dot{\eta}_\epsilon\} + [\mathbf{M}_I]\{\ddot{\eta}_\epsilon\} \quad (2)
\end{aligned}$$

When an external force $\{f(t)\}$ acts on the model shown in Fig. 3, the motion of the rotor is given by the expression:

$$\begin{aligned}
& [\mathbf{K}_e]\mathbf{X} + [\mathbf{C}_e]\dot{\mathbf{X}} + [\mathbf{M}_e]\ddot{\mathbf{X}} - \{\mathbf{F}_s\} \\
& - \{\mathbf{F}_I\} - \{\mathbf{F}_B\} = f(t), \quad (3)
\end{aligned}$$

where $\mathbf{K}_e$, $\mathbf{C}_e$, and $\mathbf{M}_e$ are the stiffness matrix, damping matrix of gyroscopic term, and mass matrix of the rotor, respectively.

If the fluid forces expressed by Eq.(3) are included in those matrices and the right-hand term $\{f(t)\}$ is set to zero,

$$[\mathbf{K}]\mathbf{X} + [\mathbf{C}]\dot{\mathbf{X}} + [\mathbf{M}]\ddot{\mathbf{X}} = 0. \quad (4)$$

The eigenvalue analysis of Eq.(4) yields conjugate complex eigenvalues λ_i and $\bar{\lambda}_i$, and the eigenvectors ϕ_i and $\bar{\phi}_i$ in the same number as that of degrees of freedom. Focusing on the i -th unstable eigenmode, the logarithmic decrements of the fluid elements are examined.

Because the complex eigenvalue and eigenvector are conjugated, the following equation is derived from Eq.(4).

$$\begin{aligned}
& \lambda_i^2 [[\mathbf{M}^h] + [\Delta\mathbf{M}]]\phi_i + \lambda_i [[\mathbf{C}^h] + [\Delta\mathbf{C}]]\phi_i \\
& + [[\mathbf{K}^h] + [\Delta\mathbf{K}]]\phi_i = 0 \\
& \bar{\lambda}_i^2 [[\mathbf{M}^h] + [\Delta\mathbf{M}]]\bar{\phi}_i + \bar{\lambda}_i [[\mathbf{C}^h] + [\Delta\mathbf{C}]]\bar{\phi}_i \\
& + [[\mathbf{K}^h] + [\Delta\mathbf{K}]]\bar{\phi}_i = 0 \quad (5)
\end{aligned}$$

The mass, damping, and stiffness matrices — $[\mathbf{M}^h]$, $[\mathbf{C}^h]$, and $[\mathbf{K}^h]$ and $[\Delta\mathbf{M}]$, $[\Delta\mathbf{C}]$, and $[\Delta\mathbf{K}]$ — are symmetric and unsymmetric parts of $[\mathbf{M}]$, $[\mathbf{C}]$, and $[\mathbf{K}]$. However, $[\mathbf{M}]$, $[\mathbf{C}]$, and $[\mathbf{K}]$ are defined as in Eq.(6) so that the diagonal parts of $[\mathbf{M}]$, $[\mathbf{C}]$, and $[\mathbf{K}]$ shall be included in $[\mathbf{M}^h]$, $[\mathbf{C}^h]$, and $[\mathbf{K}^h]$.

$$\begin{aligned}
[\Delta\mathbf{M}] &= -[\Delta\mathbf{M}]^T, [\Delta\mathbf{C}] = -[\Delta\mathbf{C}]^T, \\
[\Delta\mathbf{K}] &= -[\Delta\mathbf{K}]^T \quad (6)
\end{aligned}$$

If Eq.(5) is premultiplied by $\bar{\phi}^T$ and ϕ^T from the left term, the matrices of Eq.(5) are transformed to Eq.(7).

$$\begin{aligned}
& \bar{\phi}^T [[\mathbf{M}^h] + [\Delta\mathbf{M}]]\phi = \phi^T [[\mathbf{M}^h] - [\Delta\mathbf{M}]]\bar{\phi} = m + \Delta m \\
& \bar{\phi}^T [[\mathbf{C}^h] + [\Delta\mathbf{C}]]\phi = \phi^T [[\mathbf{C}^h] - [\Delta\mathbf{C}]]\bar{\phi} = c + \Delta c \\
& \bar{\phi}^T [[\mathbf{K}^h] + [\Delta\mathbf{K}]]\phi = \phi^T [[\mathbf{K}^h] - [\Delta\mathbf{K}]]\bar{\phi} = k + \Delta k \quad (7)
\end{aligned}$$

where Δm , Δc , and Δk are imaginary numbers, and m , c , and k are real numbers.

Hereinafter, the subscript 'i' will be omitted as in Eq.(7). If Eq.(7) is substituted into Eq.(5),

$$\lambda^2 m + \lambda^2 \Delta m + \lambda c + \lambda \Delta c + k + \Delta k = 0 \quad (8)$$

$$\bar{\lambda}^2 m - \bar{\lambda}^2 \Delta m + \bar{\lambda} c - \bar{\lambda} \Delta c + k - \Delta k = 0. \quad (9)$$

Subtraction of Eq.(9) from Eq.(8) as well as addition of Eq.(9) to Eq.(8) yields the following equations;

$$\begin{aligned}
& (\lambda^2 - \bar{\lambda}^2)m + (\lambda^2 + \bar{\lambda}^2)\Delta m + (\lambda - \bar{\lambda})c \\
& + (\lambda + \bar{\lambda})\Delta c + 2\Delta k = 0 \\
& (\lambda^2 + \bar{\lambda}^2)m + (\lambda^2 - \bar{\lambda}^2)\Delta m + (\lambda + \bar{\lambda})c \\
& + (\lambda - \bar{\lambda})\Delta c + 2k = 0 \quad (10)
\end{aligned}$$

λ and $\bar{\lambda}$ in Eq.(10) are expressed by the following equations because $\lambda = \alpha + j\beta$ and $\bar{\lambda} = \alpha - j\beta$.

$$\begin{aligned}
\lambda^2 - \bar{\lambda}^2 &= 4j\beta\gamma, \quad \lambda^2 + \bar{\lambda}^2 = 2\gamma^2 - 2\beta^2, \\
\lambda - \bar{\lambda} &= 2j\beta, \quad \lambda + \bar{\lambda} = 2\gamma \quad (11)
\end{aligned}$$

If Eq.(11) is substituted into Eq.(10),

$$\gamma(2j\beta m + \Delta c) + (\gamma^2 - \beta^2)\Delta m + j\beta c + \Delta k = 0 \quad (12)$$

$$(\gamma^2 - \beta^2) = \frac{-(2j\beta\gamma\Delta m + \gamma c + j\beta\Delta c + k)}{m}. \quad (13)$$

Then Eq.(13) is substituted into Eq.(12), the real part γ is obtained and the logarithmic decrement of the total system is

$$\delta = \frac{\pi[(\beta c - j\Delta k) + j\Delta m(j\beta\Delta c + k)/m]}{\beta[(\beta m - j\Delta c/2) + j\Delta m(j\beta\Delta m + c/2)/m]}. \quad (14)$$

If Δm is set to zero, Eq.(14) coincides with the theory of Kurohashi et al.⁽¹¹⁾, as follows:

$$\delta = \frac{\pi\beta c - j\pi\Delta k}{\beta^2 m - j\beta\Delta c/2}. \quad (15)$$

The numerator, $\pi\beta c$, of Eq.(14) is a component acting in a stabilizing manner on the rotor, and $-j\pi\Delta k$ is a component acting as the destabilizing force associated with the cross-coupled term of stiffness coefficient, and $j\pi\Delta m(j\beta\Delta c + k)/m$ expresses the individual logarithmic decrement on the stability due to the unsymmetric part of the added mass.

The symmetric parts of the damping matrix of the bearing, seal, and impeller are expressed as $[\mathbf{C}_B]$, $[\mathbf{C}_s]$, and $[\mathbf{C}_I]$. The unsymmetric parts of the stiffness matrix are expressed as $[\Delta\mathbf{K}_B]$, $[\Delta\mathbf{K}_s]$, and $[\Delta\mathbf{K}_I]$, and the unsymmetric parts of the mass matrix of the seal are written as $[\Delta\mathbf{M}_s]$. If the complex vector at the bearings is defined as ϕ_B , that at the seal as ϕ_s , and that at the impeller as ϕ_I ; and c , Δk , and Δm are expressed as the sum of individual elements, the following equations are yielded.

$$\left. \begin{aligned}
c &= \bar{\psi}^T [C] \psi = \bar{\psi}_B^T [C_B] \psi_B + \bar{\psi}_S^T [C_S] \psi_S + \bar{\psi}_I^T [C_I] \psi_I \\
&= c_B + c_S + c_I \\
\Delta k &= \bar{\psi}^T [\Delta K] \psi \\
&= \bar{\psi}_B^T [\Delta K_B] \psi_B + \bar{\psi}_S^T [\Delta K_S] \psi_S + \bar{\psi}_I^T [\Delta K_I] \psi_I \\
&= \Delta k_B + \Delta k_S + \Delta k_I \\
\Delta m_s &= \bar{\psi}_S^T [\Delta M_S] \psi_S
\end{aligned} \right\} \quad (16)$$

If Eq. (14) of the logarithmic decrement is rewritten using Eq. (16), the following approximate equation is derived.

$$\begin{aligned}
\delta &= \frac{\left[\pi \left[(\beta c_B - j \Delta k_B) + (\beta c_I - j \Delta k_I) \right] + \left\{ \beta c_S - j \Delta k_S + j \frac{\Delta m_s}{m} (j \beta \Delta c + k) \right\} \right]}{\beta^2 m - \frac{j}{2} \beta \Delta c + j \frac{\Delta m}{m} \left(j \beta^2 \Delta m + \frac{\beta c}{2} \right)} \\
&= \delta_B + \delta_I + \delta_S \quad (17)
\end{aligned}$$

The individual contributions of the bearing, seal and impeller to the rotor stability are evaluated by the logarithmic decrement. If the stiffness coefficient in Eq. (1) is expressed through the following transformation, Δk_s , Δc_s , and Δm_s of the long seal are obtained by the following formula.

$$\begin{aligned}
[K_s] &= \begin{bmatrix} [K_{\epsilon\epsilon}^h] & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} [\Delta K_{\epsilon\epsilon}] & 0 \\ 0 & 0 \end{bmatrix} \\
&+ \begin{bmatrix} 0 & 0 \\ 0 & [K_{aa}^h] \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & [\Delta K_{aa}] \end{bmatrix} \\
&+ \begin{bmatrix} 0 & \frac{[K_{\epsilon a}]}{2} \\ \frac{[K_{\epsilon a}]^T}{2} & 0 \end{bmatrix} + \begin{bmatrix} 0 & \frac{[K_{\epsilon a}]}{2} \\ -\frac{[K_{\epsilon a}]^T}{2} & 0 \end{bmatrix} \\
&+ \begin{bmatrix} 0 & \frac{[K_{a\epsilon}]^T}{2} \\ \frac{[K_{a\epsilon}]}{2} & 0 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{[K_{a\epsilon}]^T}{2} \\ \frac{[K_{a\epsilon}]}{2} & 0 \end{bmatrix} \\
&= [K_{\epsilon\epsilon}^s] + [\Delta K_{\epsilon\epsilon}^s] + [K_{aa}^s] + [\Delta K_{aa}^s] \\
&+ [K_{\epsilon a}^s] + [\Delta K_{\epsilon a}^s] + [K_{a\epsilon}^s] + [\Delta K_{a\epsilon}^s] \quad (18)
\end{aligned}$$

where $[\Delta K_{\epsilon\epsilon}^s]$ and $[\Delta K_{aa}^s]$ are determined so that the diagonal terms of $[K_{\epsilon\epsilon}^s]$ and $[K_{aa}^s]$ are included in $[K_{\epsilon\epsilon}^h]$ and $[K_{aa}^h]$.

$$[\Delta K_{\epsilon\epsilon}^s] = -[\Delta K_{\epsilon\epsilon}^h]^T, [\Delta K_{aa}^s] = -[\Delta K_{aa}^h]^T \quad (19)$$

$[\Delta K_{\epsilon a}^s]$ and $[\Delta K_{a\epsilon}^s]$ satisfy Eq. (20).

$$[\Delta K_{\epsilon a}^s] = -[\Delta K_{\epsilon a}^h]^T, [\Delta K_{a\epsilon}^s] = -[\Delta K_{a\epsilon}^h]^T \quad (20)$$

The damping coefficient and the added mass are transformed by the same method as shown in Eq. (18). Then these matrices are substituted into the seal section of Eqs. (16) and (17), and the logarithmic decrement of the forces and moments due to the cylindrical displacement and conical rotation is yielded as follows:

$$\begin{aligned}
\delta_s &= (\delta c_{\epsilon\epsilon} + \delta k_{\epsilon\epsilon}) + (\delta c_{\epsilon a} + \delta k_{\epsilon a} + \delta m_{\epsilon a}) \\
&+ (\delta c_{a\epsilon} + \delta k_{a\epsilon} + \delta m_{a\epsilon}) + (\delta k_{aa} + \delta c_{aa}) \\
&= \delta_{\epsilon\epsilon} + \delta_{\epsilon a} + \delta_{a\epsilon} + \delta_{aa} \quad (21)
\end{aligned}$$

$$\delta m_s = \delta m_{\epsilon a} + \delta m_{a\epsilon}. \quad (22)$$

Equation (22) expresses the decrement due to the

unsymmetric part of the added mass matrix.

5. Example of Calculation

5.1 Effect of rotational speed

Figure 4 shows a Campbell diagram which displays relationships between specific rotational speed and eigenvalues. The specific rotational speed is defined as the ratio of the rotational angular speed ω to the rotor natural frequency in air β_0 . The vertical axis indicates the nondimensional damped natural frequency in water (β_i/β_0) and the total logarithmic decrement δ_i . Because the dynamic characteristics of the long seal and the bearing change with the rotational speed ω , the complex eigenvalues change depending on ω . The synchronous vibrations correspond to a straight line with a slope of $\omega = \beta$.

The 3rd eigenmode λ_3 and the 4th eigenmode λ_4 become unstable, because the logarithmic decrements change from positive to negative, as shown in Fig. 4. The logarithmic decrement means the energy consumption in the rotor; namely the rotor system

Fig. 4 Effect of rotational speed on complex eigenvalues

releases energy to the outside when the logarithmic decrement is positive, while the rotor system becomes unstable by absorbing energy from outside if the logarithmic decrement is negative.

At the seal inlet swirl velocity V of 0.5, λ_3 becomes unstable when the rotational speed is more than $0.73\beta_0$ and λ_4 is unstable at ω over $0.93\beta_0$. The natural frequency of λ_3 and λ_4 is approximately one-half of the rotational speed, and it is observed that the rotor whirling speed is about $\omega/2$ in the unstable region. The calculated 3rd eigenmode λ_3 is assumed to be a cylindrical mode (see Fig. 5), and 4th eigenmode corresponds to a conical mode. However, a phase difference Ψ between the seal inlet and outlet whirling movements exists in the cylindrical mode λ_3 . Such geometrical complication of λ_3 shows that a conical whirl component is superimposed on the cylindrical mode. The logarithmic decrement of fluid elements is also drawn in the form of a bar graph in Fig. 5.

The cylindrical mode λ_3 is focused on in this study, because λ_3 is the first to become unstable; therefore λ_3 is most important in the stability problem.

5.2 Effect of seal pressure difference

Figure 6 shows the logarithmic decrement for the cylindrical mode λ_3 versus the seal pressure difference

Fig. 5 Eigenmode ($\omega=0.7\beta_0$, $V=1.0$, $\Delta P=0.57$ MPa)

ΔP . The magnitude of the individual contribution δ_{ee} associated with lateral force due to cylindrical whirl of the long seal is very large compared with that of the impeller wear-ring seal or annular clearance of the heat barrier. Therefore the long seal becomes dominant and includes other seal contributions.

The sign of the logarithmic decrement of the bearings δ_B is positive and the bearing force acts in a stabilizing manner. Conversely, that of the long seal δ_S is negative, revealing that a destabilizing force is exerted on the rotor. δ_{ee} , δ_{aa} , and δ_B increase with increasing of ΔP , which shows that ΔP has a stabilizing effect. However, δ_{ea} and δ_{ae} are negative and decrease with ΔP . The magnitude of the increment of δ_{ee} is nearly the same as that of the decrement of δ_{ea} . As a result, the effect of ΔP on the logarithmic decrement of the long seal becomes small. However, δ_B increases with increasing ΔP , namely the bearings have a positive effect on the rotor stability.

The effect of the long seal added mass is clarified by introducing nondimensional parameters. The ratio of the individual contribution δm_s derived from M_{ae} and M_{ea} to the total long seal contribution δ_S , $\delta m_s/|\delta_S|$, as well as the lateral force contributions relative to δ_S , $\delta_{ee}/|\delta_S|$ and $\delta_{ea}/|\delta_S|$, is adopted as the parameters. Plots of these parameters are provided for the 3rd and 4th eigenmodes λ_3 , λ_4 in Fig. 7.

Figure 7 reveals that the influence of each fluid force on the stability depends on the shape of the eigenmode. The individual contribution of added mass δm_s due to the unsymmetric parts has a negative sign and acts as a destabilizing force. According to Fig. 7,

Fig. 6 Effect of seal pressure difference on logarithmic decrement

the magnitude of δm_s is so small that the influence of the unsymmetric mass Δm on the stability may be ignored.

The logarithmic decrement calculated without the force due to conical rotation versus seal pressure difference ΔP is illustrated in Fig. 8. Comparison of Fig. 8 with Fig. 6 indicates that δ_s obtained by neglecting the fluid force due to conical rotation is considerably different from that with the fluid force taken into account when seal pressure difference ΔP is large.

Fig. 7 Component of seal logarithmic decrement

Fig. 8 Effect of seal pressure difference on logarithmic decrement (Without fluid force due to conical whirl)

If the lateral forces due to conical rotation are ignored, δ_{ee} of the long seal becomes dominant. δ_{ee} is negative but it increases with ΔP , and therefore the effect of ΔP on δ_s is positive to stabilize the rotor. However, δ_B shown in Fig. 8 is smaller than that in Fig. 6; hence there is not much difference between the total logarithmic decrement in Fig. 8 and that in Fig. 6.

5.3 Effect of seal inlet swirl velocity

Figure 9 shows the relationships between the seal inlet swirl velocity V and the logarithmic decrements. The seal inlet swirl velocity has a significant effect on the stability of the vibration, as shown in Fig. 9. If the inlet swirl is in the same direction as the rotor rotational direction, namely positive inlet swirl ($V > 0$), it has a destabilizing effect on the stability. Conversely, if the swirl is in the opposite direction to the rotor rotational direction, namely negative inlet swirl ($V < 0$), it has a stabilizing effect on the stability.

The logarithmic decrement δ_{ee} associated with the lateral force induced by cylindrical displacement

Fig. 9 Effect of seal inlet swirl velocity on logarithmic decrement

Fig. 10 Phase and natural frequency versus seal inlet swirl

and δ_{ea} induced by conical rotation have nearly the same magnitude, and both δ_{ea} and δ_{ee} depend on the seal inlet swirl velocity V . δ_{ea} has a positive value if V is less than approximately 0.25, and has a negative value if V is over 0.25. When V exceeds 0.7, the rotor becomes unstable, since the logarithmic decrement of the long seal δ_s is negative. Hence the unstable whirl caused by δ_s is called "seal whirl."

Figure 10 shows the relationships among the seal inlet swirl velocity V , the phase difference Ψ between seal inlet and outlet, as defined in Fig. 3, and the nondimensional natural frequency (β_3/ω) .

If the inlet swirl is negative, Ψ becomes positive and the phase difference of whirling motion at the seal outlet advances relative to the inlet in the rotor rotational direction. If Ψ is positive, δ_{ea} also becomes positive, and the force due to conical whirl acts as a stabilizing force. Conversely, when V is positive, Ψ changes to negative, the phase at the seal outlet gets phase lag relative to the inlet; then δ_{ea} has a negative value. The interaction between δ_{ea} and Ψ can be observed and, if Ψ is positive, a stabilizing force is exerted on the long seal. This result is consistent with that of the individual study on the long seal⁽⁴⁾.

The natural frequency β_3 increases with the inlet swirl V , and the logarithmic decrement of the bearings δ_B becomes negative when β_3/ω is under 0.5, while δ_B changes to positive at β_3/ω over 0.5.

5.4 Effect of seal clearance

Figure 11 illustrates the seal clearance H_0 versus the individual contributions for the mode λ_3 . The logarithmic decrement δ_{ee} associated with the force due to cylindrical displacement is positive and δ_{ee}

Fig. 11 Effect of seal clearance on logarithmic decrement

enlarges with decreasing seal clearance H_0 to stabilize the rotor. By contrast, with decreasing seal clearance H_0 , the logarithmic decrement of the bearings δ_B decreases and becomes negative.

The logarithmic decrement δ_{ea} associated with the lateral force due to conical rotation is negative and the force is exerted in a destabilizing manner. The sign of δ_{ea} is negative but the absolute value of δ_{ea} reduces with increasing H_0 so that the destabilizing force also reduces. On the other hand, the sign of δ_{ee} changes from positive to negative. Therefore, there is a seal clearance H_0 at which the long seal is exerted in the most destabilizing manner. When H_0 is about 1.5 mm, the logarithmic decrement of the long seal δ_s becomes the minimum, as shown in Fig. 11. However, when H_0 is greater than 1.5 mm, δ_s changes slightly but seems to be almost constant. With increasing H_0 , the logarithmic decrement of the bearings δ_B increases and the total rotor system is stabilized.

If the seal shape is altered, the eigenvector ψ and the natural frequency β are also changed. Therefore, other factors, such as bearings $\pi\beta_{CB}$ in Eq.(17), change. Altering the seal shape influences the contribution of bearing and impeller, and the interactions between these fluid elements are found in Fig. 11.

5.5 Effect of bearing load

The dynamic stiffness and damping coefficients of the bearings are defined as functions of the bearing load⁽⁸⁾. The effect of the bearing load on the stability must therefore be investigated.

The nondimensional bearing load

$$W = \frac{W_B}{W_R} \quad (23)$$

Fig. 12 Effect of bearing load on logarithmic decrement

is a ratio of bearing static load W_B and the total weight of the rotor W_R . The relationship of the parameter W versus the individual contributions is provided in Fig. 12 for the eigenmode λ_3 .

If the bearing load changes, the logarithmic decrement and eigenvector also change. δ_B increases by increasing of W , whether δ_B decreases to produce destabilizing force if W decreases. When the bearing load changes, the vibration shape associated with the eigenvector changes and the logarithmic decrement of the long seal δ_s changes depending on W . The increase of bearing load has a significant positive effect on the rotor stability, because not only δ_B but also $\delta_{\epsilon\alpha}$ increases.

6. Conclusions

The effect of the long seal on the stability of the submerged motor pump has been investigated by introducing the individual logarithmic decrement, and the following conclusions are yielded.

(1) If the rotor whirls cylindrically and conically, the influence of the fluid moments and forces induced by the long seal is dependent on the mode shape of the vibration. In the model of the submerged motor pump used for this study, the individual contribution of $\delta_{\epsilon\epsilon}$ is the largest, followed by $\delta_{\epsilon\alpha}$ and $\delta_{\alpha\epsilon}$, and that of $\delta_{\alpha\alpha}$ is the smallest. $\delta_{\epsilon\epsilon}$ and $\delta_{\epsilon\alpha}$ associated with the lateral forces exert a significant effect on the stability of the vibration, but $\delta_{\alpha\epsilon}$ and $\delta_{\alpha\alpha}$ associated with the moment are smaller in the cylindrical mode.

(2) Logarithmic decrement δm_s associated with the unsymmetric parts $M_{\epsilon\alpha}$ and $M_{\alpha\epsilon}$ of the added mass is small and its effect on the stability is negligible.

(3) If the seal inlet swirl velocity V is in the same direction as that of rotor rotation, logarithmic decrements $\delta_{\epsilon\alpha}$ and $\delta_{\epsilon\epsilon}$ associated with the forces due to conical and cylindrical whirls have negative values. As a result, the forces subjected to $\delta_{\epsilon\epsilon}$ and $\delta_{\epsilon\alpha}$ act in a destabilizing manner. If V is opposite to the rotor rotational direction, $\delta_{\epsilon\alpha}$ and $\delta_{\epsilon\epsilon}$ have positive values, therefore stabilizing forces are exerted on the long seal.

(4) If the phase difference of whirling motion at the seal outlet advances forward relative to the inlet in the rotor rotational direction, $\delta_{\epsilon\alpha}$ acts as stabilizing force, as a result of having a positive value. Conversely if the phase at the outlet gets phase lag relative to the inlet, $\delta_{\epsilon\alpha}$ has a negative value. This result is compatible with the individual research on the dynamic fluid forces induced by the cylindrical displacement and the conical rotation in the long seal.

(5) The effect of a long seal, such as a fluid-filled

annular clearance between the rotor and the stator of a submerged motor pump, on the stability is significant, so such long seals should be taken into account in stability analyses.

References

- (1) Kanemori, Y. and Iwatsubo, T., Experimental Study of Dynamic Fluid Forces and Moments for a Long Annular Seal, *Trans. ASME, J. Tribol.*, Vol. 114, Oct. (1992), p. 773.
- (2) Kanemori, Y. and Iwatsubo, T., Experimental Study of Dynamical Characteristics of a Long Annular Seal (Force and Moment Due to Conical Whirl Rotation), *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 55, No. 520, C(1989), p. 2974.
- (3) Kanemori, Y. and Iwatsubo, T., Experimental Study of Dynamical Characteristics of a Long Annular Seal (Influence of Inlet Swirl on the Dynamic Fluid Moment and Force), *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 56, No. 525, C (1990), p. 1273.
- (4) Kanemori, Y. and Iwatsubo, T., Experimental Study of the Dynamic Characteristics of a Long Annular Seal (Fluid Force Due to Conical and Cylindrical Whirl), *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 56, No. 532, C(1990), p. 3216.
- (5) Childs, D.W., Rotordynamic Moment Coefficients for Finite Length Turbulent Seals, *Proceedings of the IFTOMM Conference, Rome (1982)*, p. 371.
- (6) Nagaya, K. and Ikeda, S., Vibration and Vibration Isolation of a Motor Due to Whirlings of the Shaft with Consideration of Electromagnetic Forces, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 51, No. 472, C(1985), p. 3354.
- (7) Kanki, H., Ozawa, Y., Oda, T. and Kawakami, T., Study on the Dynamic Characteristics of a Water-Lubricated Pump Bearing (1st Report, Cylindrical Bearing under High Ambient Pressure), *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 51, No. 470, C(1985), p. 2650.
- (8) JSME., Data Collection of Static and Dynamic Characteristics of Journal Bearings, Publishing Company of Japan Industry (in Japanese), (1984).
- (9) Jery, B., Brennen, C.E., Caughey, T.K. and Acosta, A.J., Force on Centrifugal Pump Impellers, *Proceedings of the Second International Pump Symposium*, (1985), p. 21.
- (10) Yang, B.S., Iwatsubo, T. and Kawai, R., Effects of Seals on the Stability of Pump Rotor System, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 51, No. 470, C(1985), p. 2479.
- (11) Kurohashi, M., Iwatsubo, T., Kawabata, N., and Fujikawa, T., Evaluation of Rotor Stability by Means of Logarithmic Decrements Derived from Energy for the Individual Elements, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 48, No. 439, C (1982), p. 825.