Dynamic Behavior of a Rotor with Time-Dependent Parameters*

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In this paper the dynamic behavior of a rotor with variable parameters and small nonlinearity is analyzed. Besides the linear damping, rigidity and gyroscopic force that are time-dependent, some nonlinear forces exist. They cause self-excited vibrations of the rotor. The vibration problem is solved analytically as well as numerically. A new procedure based on the Krylov-Bogolubov method is developed for solving a differential equation with a complex deflection function, time variable parameters and small nonlinearity. The asymptotic analytical solutions are in good agreement with the numerical results. For the rotor with variable parameters, the stability of the pure rotational motion is also analyzed. The direct Lyapunov stability and instability theorems are applied. The methods developed in this paper are applied to three special cases: 1. the mass of the rotor is time-dependent, 2. a linear damping force is applied, 3. a linear gyroscopic or a “cross coupling damping” force is applied.

Key Words: Rotor, Time Variable Parameters, Vibrations, Stability

1. Introduction

In some actual mechanical systems, rotors with variable parameters are mounted. The variable properties of the rotors are usually stiffness, damping, cross-coupling damping and mass. The parameters may be time-dependent. In general the parameter variation is slow. The variation is not periodical. The time-dependent forces may be linear or nonlinear. The nonlinearity is usually weak. These forces cause self-excited vibrations which represent a “side effect” of the pure rotational motion (required regime) of the rotor. The vibrations draw energy from the rotation and ultimately decrease the machine performance efficiency. The “factors” which transform rotative energy into shaft lateral vibration modes are most often associated with forces with variable parameters. Rotor lateral self-excited vibrations are usually represented by their precessional (orbital) motion. The question is whether these vibrations destabilize the pure rotation of the rotor or not. Analyzing the vibrations, we can conclude that the rotation is stable when the amplitude of vibration decreases during a certain time, and unstable when it increases. However, it is necessary to form criteria for instability of rotors with variable parameters. To discuss the dynamic behavior of the rotor with slow variable parameters and weak nonlinearity, it is necessary to obtain the general solution for the motion and to form stability criteria.

In addition to the numerical solution, the analytical solution is obtained. Using the analytical solution we discuss the influence of parameter variation on the amplitude and phase of vibrations and frequencies and damping decrements. The analytical solution is obtained by a generalization of the Krylov-Bogolubov (K-B) method. The numerical solution is obtained through the Runge-Kutta procedure.

The stability of rotation is analyzed by the direct Lyapunov methods. The procedure gives the criteria for stable and unstable rotation of the rotor with variable parameters and zero deflection of the mass center.

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Three special cases are considered in detail:
1. the mass of the rotor is assumed to be time-dependent,
2. a linear damping force is considered,
3. a linear gyroscopic or a "cross-coupling damping" force is applied.

The rotor represents a one-mass system with two degrees of freedom. The mathematical model is a system of two coupled second-order differential equations with weak nonlinearity and slow variable parameters:

\[ \ddot{x} + d(\tau) \ddot{x} + c(\tau) x + g(\tau) \dot{y} = \varepsilon f(x, \dot{x}, y, \dot{y}, \tau) \]
\[ \ddot{y} + d(\tau) \ddot{y} + c(\tau) y - g(\tau) \dot{x} = \varepsilon f(y, \dot{y}, \dot{x}, \tau), \]

where \( d(\tau), c(\tau), g(\tau) \) are variable damping, stiffness, gyroscopic coefficients, respectively, \( x, y \) are coordinates of the rotor center, \( \varepsilon \) is a small parameter, \( \tau = \varepsilon t \) is a slow variable time, \( t \) is time, and \( f_1 \) and \( f_2 \) are arbitrary functions of indicated variables.

Introducing the complex deflection function \( z = x + iy \) and complex nonlinear function

\[ f = f_1 + if_2, \]

where \( i = \sqrt{-1} \) is an imaginary unit, the system of Eqs. (1) becomes

\[ \ddot{z} + d(z) \ddot{z} + c(z) z - ig(\tau) \dot{z} = \varepsilon f(z, \dot{z}, \tau). \]

There are only a few papers dealing with the dynamics of rotors with variable parameters. They consider the influence of mass variation on dynamic behavior\(^{21-40}\). Their results correspond to the case of the one-frequency system and special initial conditions. On the contrary, in this paper a generalization is made based on the fact that the rotor is a two-frequency system.

Usually it is assumed that the damping force is small. An extension of the analysis is necessary when linear damping is significant. The influence of the gyroscopic force on the motion of the rotor with variable parameters has not yet been considered. The vibrations of such a rotor with gyroscopic force will be obtained not only numerically but also analytically. The stability properties of the rotor are discussed.

2. Analytical Procedure

In a previous paper\(^4\) the Krylov-Bogolubov method was extended for differential equations with complex deflection and small nonlinearity. The K-B method is used for determining approximate solutions to quasi-linear differential equations of the form

\[ \ddot{z} + d(z) \ddot{z} + c(z) z - ig(\tau) \dot{z} = \varepsilon f(z, \dot{z}), \]

where \( d, c, g \), are constant parameters and \( \varepsilon \) is a small constant.

In this paper, further extension of the method is made for the differential equation(3) with complex function and small nonlinearity having variable parameters. The generating equation corresponds to the case when

\[ \varepsilon = 0, \tau = \tau_0 = \text{const}, \]

and the steady-state periodic solution is

\[ z(t) = A_1 e^{-\lambda_1 t} e^{i(e^{i\omega_1 t} + \omega_2)} e^{-i(\omega_1 t + \phi_1)} + A_2 e^{-\lambda_2 t} e^{i(e^{i\omega_2 t} + \omega_1)} e^{-i(\omega_2 t + \phi_2)}, \]

with \( A_1, A_2, \lambda_1, \lambda_2 \) constant.

In this equation, \( \lambda_1, \lambda_2 \) are called the "logarithmic decrement" that is often used as a measure of the effective damping in the system and \( \omega_1, \omega_2 \) are damped natural frequencies.

The K-B approximate trial solution is the same but with \( A_1, A_2, \lambda_1, \lambda_2 \) being time-dependent:

\[ z(t) = A_1(t) e^{-\lambda_1(t) t} e^{i(e^{i\omega_1(t) t} + \omega_2(t))} e^{-i(\omega_1(t) t + \phi_1(t))} + A_2(t) e^{-\lambda_2(t) t} e^{i(e^{i\omega_2(t) t} + \omega_1(t))} e^{-i(\omega_2(t) t + \phi_2(t))}, \]

where

\[ a_1(t) = \int_0^t \lambda_1(t) \, dt, \quad a_2(t) = \int_0^t \lambda_2(t) \, dt, \]

\[ \phi_1(t) = \int_0^t \omega_1(t) \, dt + \theta_1(t), \quad \phi_2(t) = \int_0^t \omega_2(t) \, dt + \theta_2(t). \]

The natural frequencies and logarithmic decrements are known functions of the slow time, i.e., \( \lambda_1(t), \lambda_2(t), \omega_1(t), \omega_2(t) \).

Hence, the task of finding the solution \( z(t) \) is transformed into obtaining four functions, \( A_1(t), A_2(t), \theta_1(t) \) and \( \theta_2(t) \), so that expression (7) satisfies Eq. (3). Although one is free to choose these four functions, one must first impose an obvious constraint. This means Eq. (7) must be a solution of Eq. (3). Three additional constraints can be imposed to further restrict the arbitrariness. It is common for the K-B method that the time derivative of the trial solution must have the same form as the time derivative of the generating solution, therefore

\[ \dot{z} = -A_1 \lambda_1 e^{-\lambda_1(t) t} e^{i(e^{i\omega_1(t) t} + \omega_2)} e^{-i(\omega_1(t) t + \phi_1)} - A_2 \lambda_2 e^{-\lambda_2(t) t} e^{i(e^{i\omega_2(t) t} + \omega_1)} e^{-i(\omega_2(t) t + \phi_2)}. \]

Differentiating Eq. (7) with respect to \( t \) and using constraint (10), one obtains

\[ (A_1 + A_2 i \theta_1) e^{-\lambda_1(t) t} e^{i(e^{i\omega_1(t) t} + \omega_2)} + (A_2 - A_1 i \theta_2) e^{-\lambda_2(t) t} e^{i(e^{i\omega_2(t) t} + \omega_1)} = 0. \]

The third and fourth constraints are that the frequency-system parameters and decrement-system parameters must be the same for the trial solution as for the generating solution.

Using the assumed solution and relationships of constraints in Eq. (3), and separating the real and imaginary parts, one finds

\[ A_1 (\dot{\theta}_1 (\omega_1 + \omega_2) + A_1 (\lambda_1 - \lambda_2) + A_1 \dot{\lambda}_1 + A_2 \dot{\lambda}_2 \cos(\phi_1 + \phi_2) + A_2 \lambda_1 \sin(\phi_1 + \phi_2) = -\varepsilon \Re(e^{i\omega_1 e^{i\omega_2}}), \]

\[ A_1 (\dot{\theta}_2 (\omega_1 + \omega_2) - A_1 \dot{\lambda}_1 + A_1 \dot{\omega}_1 \sin(\phi_1 + \phi_2) - A_2 \dot{\lambda}_2 \sin(\phi_1 + \phi_2) = -\varepsilon \Im(e^{i\omega_1 e^{i\omega_2}}), \]

\[ A_2 \dot{\theta}_2 (\omega_1 + \omega_2) - A_2 \dot{\lambda}_1 + A_2 \dot{\lambda}_2 + A_2 \lambda_1 \sin(\phi_1 + \phi_2) = 0. \]
\[ f = -c_0(x)F(p), \]
\[ \rho = \sqrt{\bar{x}^2 + \bar{y}^2}, \]
where \( c_0(x) \) is the coefficient of the nonlinear elastic force.

Then, the mathematical model of the rotor leads to
\[ \ddot{x} + d(x)\dot{x} + c(x)x + g(y)\dot{y} = -c_0(x)F(p), \]
\[ \ddot{y} + d(y)\dot{y} + c(y)\dot{y} + g(x)\dot{x} = -c_0(y)F(p). \]

The parameters of the rotor satisfy the relationships
\[ c(x) \geq c_0 = \text{const}, \quad c(y) \geq c_0 = \text{const}. \]

For stability analysis, the Lyapunov function \( V \) is assumed as
\[ V = \frac{1}{2}(x^2 + y^2) + c(x)(x^2 + y^2)/2 + c_0(x)/2 \int_0^s F(s) ds. \]

The adjoint function \( W \),
\[ W = \frac{1}{2}(x^2 + y^2) + c(x)(x^2 + y^2)/2 + c_0(x)/2 \int_0^s F(s) ds, \]
is positive definite for
\[ c_0 > 0 \text{ and } c_0 > 0. \]

The function \( V \) satisfies the relationships
\[ V \geq W > 0 \text{ for } x, \dot{x}, y, \dot{y}, t > 0, \]
\[ V = W = 0 \text{ for } x, \dot{x}, y, \dot{y} = 0, t > 0, \]
and therefore is positive definite. The first derivative is
\[ dV/dt = d(x)(\ddot{x} + \ddot{y}) + c(x)(\dot{x}(x^2 + y^2)) + c_0(x)/2 \int_0^s F(s) ds. \]
The adjoint function \( W \),
\[ W = d(x)(\ddot{x} + \ddot{y}) + c(x)(\dot{x}(x^2 + y^2)) + c_0(x)/2 \int_0^s F(s) ds, \]
is positive definite for
\[ d_0 > 0, \quad c_0 > 0. \]

The function \( dV/dt \) satisfies the relationship
\[ dV/dt \leq -W, \quad x, \dot{x}, y, \dot{y}, t > 0, \]
\[ dV/dt = W = 0, \quad x, \dot{x}, y, \dot{y} = 0, t > 0 \]
and is negative definite for
\[ d(x) > d_0, \quad d(x)/d_0 < c_0, \quad d(x)/d_0 < c_0. \]

Applying the Lyapunov theorem of asymptotic stability\(^{30}\), it can be concluded that the rotation of the rotor with zero deflection of the mass center is asymptotically stable if Eqs. (17), (20), (24) and (26) are satisfied.

The function \( dV/dt \) satisfies the conditions
\[ dV/dt \geq W > 0, \quad x, \dot{x}, y, \dot{y}, t > 0, \]
\[ dV/dt = W = 0, \quad x, \dot{x}, y, \dot{y} = 0, t > 0 \]
and it is positive definite for
\[ d(x) > d_0, \quad d(x)/d_0 < c_0, \quad d(x)/d_0 < c_0. \]

Hence, the pure rotation of the rotor with zero deflection of the mass center is unstable if conditions (17), (20) and (28) are satisfied\(^{30}\).

Hence, the gyroscopic force or the cross-coupling
damping has no influence on the stability of rotation. The stability or instability depends only on the values of the coefficients of the rotor.

### 4. Some Special Cases

#### 4.1 Rotor with variable mass

The model of the rotor with variable mass is

\[ \dot{z} + \omega^2(t)z = \varepsilon f(z, \dot{z}, z, \dot{z}, \tau). \quad (29) \]

Using the method developed in this paper the solution is

\[ z(t) = A(t)e^{-i\phi_1(t)} + A_d(t)e^{-i\phi_2(t)}, \quad (30) \]

where \( \phi_1(t) = \int_0^t \omega(t)dt + \theta_1(t), \phi_2(t) = \int_0^t \omega(t)dt + \theta_2(t) \) and \( A_1(t), A_d(t), \theta_1(t) \) and \( \theta_2(t) \) are the solutions of the differential equations

\[ A_1 = -A_1\omega/(2\omega) + \epsilon(Im(fe^{-i\omega}))/\omega, \]
\[ A_1\dot{\theta}_1 = -\epsilon Re(fe^{-i\omega})/(\omega), \]
\[ A_2 = -A_2\omega/(2\omega) - \epsilon(Im(fe^{i\omega}))/\omega, \]
\[ A_2\dot{\theta}_2 = -\epsilon Re(fe^{i\omega})/(\omega). \quad (31) \]

The amplitude of vibrations can either increase or decrease. Therefore, stability analysis also gives the criteria for either stable or unstable rotation.

When the elastic force in the shaft is non-linear, the model of the rotor can be given as

\[ m(\tau)\ddot{x} + x = -xc_3F(\rho), \]
\[ m(t)\ddot{y} + y = -yc_3F(\rho). \quad (32) \]

The mass variation satisfies the relationship

\[ m(t) \geq M > 0. \quad (33) \]

Let us apply the direct Lyapunov method for stability analysis. The Lyapunov function is

\[ V = m(\tau)(\ddot{x}^2 + \dot{y}^2)/2 + (\rho^2 + y^2)/2 \]
\[ + (c_3/2)\int_0^\tau F(s)ds, \quad (34) \]

while the adjoint function.

\[ W = M(\ddot{x}^2 + \dot{y}^2)/2 + (\rho^2 + y^2)/2 \]
\[ + (c_3/2)\int_0^\tau F(s)ds, \quad (35) \]

is positive definite for \( M > 0 \) and \( c_3 > 0. \)

The function \( V \) satisfies the relationships

\[ V \geq W > 0 \quad \text{for} \quad x, \dot{x}, y, \dot{y}, \tau \neq 0, \quad (36) \]
\[ V = W = 0 \quad \text{for} \quad x, \dot{x}, y, \dot{y}, \tau = 0, \quad (37) \]

and is also positive definite. The first derivative is

\[ dV/dt = (dm/dt)(\ddot{x}^2 + \dot{y}^2)/2. \quad (38) \]

This is nonpositive for

\[ dm/dt \leq 0, \quad (39) \]

and positive for

\[ dm/dt > 0. \quad (40) \]

According to the direct Lyapunov procedure it can be concluded that the rotation of the rotor with variable mass is stable under the condition of Eq. (39) and unstable under the condition of Eq. (40).

Two cases will be considered: a) case (1): the rotation is unstable and b) case (2): the rotation is stable.

a) Case (1): For the case when the band on the rotor is winding up the dimensionless mass variation function is given by Bessonov

\[ m(t) = 1 + r. \quad (41) \]

The dimensionless natural frequency of the rotor is

\[ \omega(t) = 1/\sqrt{(1 + r)}. \quad (42) \]

Using these functions of \( t \) and solving the differential equations for the initial conditions \( t = 0 \):

\[ A_1 = A_{10}, A_2 = A_{20}, \dot{\theta}_1 = \theta_{10}, \dot{\theta}_2 = \theta_{20}, \]
\[ z = z_0, m(m_0)^{1/4} \exp[-i2(\sqrt{m - m_0})]/\epsilon \]
\[ + \epsilon c_3\omega/A_{10}(A_{10}^2 + 2A_{20}^2)/2 + \theta_{10} \]
\[ = A_{10}(m(m_0)^{1/4} \exp[-i2(\sqrt{m - m_0})]/\epsilon \]
\[ + \epsilon c_3\omega/2(A_{10}^2 + A_{20}^2)/2 + \theta_{10}, \quad (43) \]

where \( m_0 \) and \( \omega_0 \) are the mass and natural frequency for \( t = 0 \).

In Fig. 1(a) the orbital motion of the linear rotor with constant mass is plotted. Due to mass variation having small parameter \( \epsilon = 0.1 \) the motion of the rotor's center is located along an elliptic spiral (Fig. 1(b)). The existence of the nonlinearity does not vary the shape of the orbit significantly (Fig. 1(c)). Comparing the analytical and numerical solutions, it can be concluded that the difference is negligible.

For the special case when the system is considered as a one-frequency system the asymptotic analytical method based on the multiple scales method is developed. The similar analytical solution is obtained by this method when it is assumed that \( A_1 = 0 \). In this case, the motion of the rotor's center becomes a circular spiral. The equality of solution responds to the special initial conditions.

The amplitude of vibrations has an increasing tendency. This causes the rotation to be unstable.

b) Case (2): Let us assume a case when the band is rolling down from the disc. The mass variation and its first derivative is

\[ m(t) = 1 - r, \quad dm/dt = -\epsilon < 0, \quad (44) \]

and it satisfies the conditions of stable rotation. The rotor center motion is described as

\[ z = A_0(m/m_0)^{1/4} \exp[-i2(\sqrt{m - m_0})]/\epsilon \]
\[ + \epsilon c_3\omega/2(A_{10}^2 + 2A_{20}^2)/2 + \theta_{10} \]
\[ + A_{20}(m/m_0)^{1/4} \exp[-i2(\sqrt{m - m_0})]/\epsilon \]
\[ + \epsilon c_3\omega/2(A_{10}^2 + 2A_{20}^2)/2 + \theta_{10}, \quad (45) \]

From this equation, a spiral motion with an amplitude decreases as Fig. 2. It proves the statement of Lyapunov theory of stable rotation.

#### 4.2 Linear damping model

The model of the rotor is

\[ \ddot{z} + \dot{c}(\tau)\dot{z} + \epsilon f(z, \dot{z}, \tau). \quad (46) \]

The asymptotic analytic solution is

\[ z(t) = A_1(t)e^{-\alpha(t)}e^{i\psi_1(t)} + A_2(t)e^{-\alpha(t)}e^{i\psi_2(t)}, \quad (47) \]

where:
\[ a(t) = \frac{1}{T} \int_0^T \dot{a}(t) \, dt, \quad \dot{a}(t) = d(t)/2, \]
\[ \omega(t) = \sqrt{[c(t) - (d(t)/2)]^2}, \]
\[ \phi(t) = \int_0^t \omega(t) \, dt + \theta(t), \quad \phi(t) = \int_0^t \omega(t) \, dt + \theta(t). \]

The amplitude and phase variation is given as the solution of the equations:
\[ A_1 = \left( -A_1 \omega + c(\theta(t) - (d(t)/2)) \right)/(2\omega), \]
\[ A_2 = \left( -A_2 \omega + c(\theta(t) - (d(t)/2)) \right)/(2\omega), \]
\[ A_1 \theta_1 = -A_1 \theta + c(\theta(t) - (d(t)/2))/(2\omega), \]
\[ A_2 \theta_2 = -A_2 \theta + c(\theta(t) - (d(t)/2))/(2\omega). \]

For a case when the band is wound up the frequency of the system and the damping decrement leads to function of the wound mass on the rotor,
\[ \omega(t) = \left[ 1/m(t) - d^2/4m(t)^2 \right]^{1/2}. \]

In Fig. 3, the natural frequency as a function of

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**Fig. 1** Orbital motion of rotor's center: (a) linear rotor with constant parameters, (b) linear rotor on which the band is wound up, (c) nonlinear rotor on which the band is wound up: --- numerical solution, --- analytical solution

**Fig. 2** The path of the rotor's center from which the band is rolling down: --- numerical solution, --- analytical solution

**Fig. 3** The natural frequency as a function of mass value for various damping parameters
Fig. 4 The orbital motion of the nonlinear rotor with variable mass and linear damping: —— numerical solution, ——— analytical solution

Fig. 5 Frequencies of rotor with gyroscopic force: (a) $\omega_1 - m$ diagram, (b) $\omega_2 - m$ diagram

Fig. 6 Motion of rotor's center for (a) positive cross-coupling damping, (b) negative cross-coupling damping

Fig. 7 The path of the rotor with gyroscopic force: ——— with constant parameters, ——— with variable mass

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the mass value for various damping parameters is plotted. The frequency decreases with increase of the mass. The velocity of frequency decreasing is a function of damping: for higher values of damping the frequency decreases faster. The damping influence on the frequency variation is significant for a small amount of band wind up. In Fig. 4, the orbital motion of the mass center is plotted for \( \varepsilon = 0.1, \quad d = 0.1, \quad c_0 = 1 \) and initial conditions \( A_{10} = 0.1, \quad A_{20} = 0.2, \quad \theta_0 = 0, \quad \delta_0 = \pi/2 \). The analytical and numerical solutions are in good agreement.

It can be concluded that the asymptotic analytical method developed in this paper is possible for application also to the case of linear damping, comparing with the method of multiple scales and the Bogolubov-Mitropolis method, for example.

### 4.3 Action of a gyroscopic force

In the rotating systems, one of the most effective parameters is the gyroscopic force. The model of the rotor is

\[ \dot{z} + c(\tau)z - ig(\tau)\dot{z} = e(f(z, \dot{z}, \tau)), \quad (52) \]

The system has two different frequencies

\[ \omega_1(\tau) = -\left(\frac{g(\tau)}{2}\right) + \sqrt{c(\tau) + \left(\frac{g(\tau)}{2}\right)^2}, \]

\[ \omega_2(\tau) = \left(\frac{g(\tau)}{2}\right) + \sqrt{c(\tau) + \left(\frac{g(\tau)}{2}\right)^2}. \quad (53) \]

In Figs. 5(a) and (b), the frequencies are plotted as a function of mass. The gyroscopic parameter has various values: 0, 0.1, 0.5 and 1. The frequencies have a decreasing tendency with increasing mass. Frequency \( \omega_1 \) has a smaller value and \( \omega_2 \) higher values for higher gyroscopic coefficient. The trial solution of Eq. (52)

\[ \dot{z}(t) = A_1(t)e^{i\phi_1(t)} + A_2(t)e^{i\phi_2(t)}, \quad (54) \]

where

\[ \phi_1(t) = \int_{t_0}^{t} \omega_1(\tau) d\tau + \theta_1(t), \quad \phi_2(t) = \int_{t_0}^{t} \omega_2(\tau) d\tau + \theta_2(t), \quad (9) \]

and the amplitudes \( A_1 \) and \( A_2 \) and the phases \( \theta_1 \) and \( \theta_2 \) are obtained from the equations

\[ A_1 = -A_1(\omega_1 + \omega_2)^{-1} \om_1 + \varepsilon(\omega_1 + \omega_2)^{-1} <Re(fe^{-i\phi_1(t)})>, \]

\[ A_1 \dot{\theta}_1 = -\varepsilon(\omega_1 + \omega_2)^{-1} <Re(fe^{-i\phi_1(t)})>, \]

\[ A_2 = -A_2(\omega_1 + \omega_2)^{-1} \om_2 - \varepsilon(\omega_1 + \omega_2)^{-1} <Re(fe^{i\phi_2(t)})>, \]

\[ A_2 \dot{\theta}_2 = -\varepsilon(\omega_1 + \omega_2)^{-1} <Re(fe^{i\phi_2(t)})>. \quad (55) \]

Analyzing the influence of the gyroscopic force, we first consider the linear rotor with constant parameters. The gyroscopic force makes the orbit of the rotor rotate as in Figs. 6(a) and (b). Usually the rotor is considered as a single-frequency system. It obtains a correct description of the amplitude but it is not possible to define the phase angle variation. The method suggested in this paper also defines this phenomenon.

The mass variation, in spite of its smallness, has a significant influence on the rotor motion (Fig. 7). (The results in Fig. 7 are numerically obtained.) In

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**Fig. 8** The orbital motion of the nonlinear rotor with gyroscopic force and variable mass: ------ analytical solution, --- numerical solution

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**5. Conclusion**

The following can be concluded.

1. The vibrations of the rotor with variable parameters and small nonlinearity can be solved both numerically and analytically. The extended Krylov-Bogolubov method yields asymptotic solutions that are in good agreement with the numerical solutions. The analytical procedure shows the natural frequencies and logarithmic decrements that are functions of the variable parameters, i.e., slow varying time.

2. This analytical procedure also gives solutions for the case when linear damping and gyroscopic force or cross-coupling damping acts in the rotor system with variable parameters.

3. The analytical procedure developed in this paper is based on the perturbation of a two-frequency steady-state solution, while the well-known Bogolubov-Mitropolis method and the method of multiple scales are based on a single-frequency solution of the differential equation with complex deflection.

4. The criteria for stable and unstable pure rotative motions of the rotor with variable parameters are affected by parameters of the system. The gyroscopic force has no influence on the stability properties of the system.

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