LAC/HAC System with a Variable Feedback Gain for Flexible Space Structures*

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This paper proposes a new control technique, based on the low authority control / high authority control (LAC/HAC) concept, to suppress the vibration of flexible space structures. The control system designed by the new control technique has a mechanism to adjust the HAC feedback gain between zero and one in such a manner as to enhance the effectiveness of control and to obtain global stability even when the magnitude of the LAC gain is not suitable. The paper first describes the theoretical mechanism of the new control technique and proposes a numerical method for determining the HAC feedback gain based on a neural network system. Then it shows the characteristic features of the new technique by computer simulation examples. The computer simulation shows that the new technique is quite effective in suppressing the spillover effects, and quite feasible for implementation.

Key Words: Flexible Structure, Vibration Control, Robust Control, Neural Network, LAC/HAC, Spillover

1. Introduction

The concept of modal control is frequently used for vibration control systems for flexible structures⁽¹⁾. However, modal control systems involve a spillover phenomenon which may bring about the instability of uncontrolled modes, since only a limited number of modes can be taken into account in designing a modal control system. On the other hand, the collocated direct velocity feedback (CDVFB) technique can guarantee the stability of the control system, although the effectiveness of control is not always sufficient⁽²⁾.

Thus the LAC/HAC technique is used in an attempt to obtain both high effectiveness of control and stability of the control system⁽³⁾. A control system designed by the LAC/HAC technique is composed of two parts: a HAC part designed by the modal control technique and a LAC part designed by the CDVFB

technique (Fig. 1). However, it is difficult to determine a suitable ratio of LAC feedback gain to HAC feedback gain, since spillover may destabilize many modes and also, natural modal damping ratios are uncertain, especially for high-vibration modes. A technique to improve these disadvantages of the LAC/HAC system is desired.

This paper proposes a new control technique, referred to as the VG LAC/HAC (variable gain LAC/HAC) technique. The VG LAC/HAC system has a mechanism to adjust the HAC feedback gain in such a manner as to minimize spillover effects in the shortest time possible and to maximize the effectiveness of control. The paper first describes the theoretical mechanism of the VG LAC/HAC technique and

Fig. 1 Block diagram of the LAC/HAC system

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proposes a numerical method for determining the HAC feedback gain based on a neural network system. Then it shows the characteristic features of the VG LAC/HAC system by computer simulation examples. Finally, the effectiveness of control of the VG LAC/HAC system is compared with the effectiveness of an ordinary LAC/HAC system and the time sharing LAC/HAC (TS LAC/HAC) system⁽⁴⁾, which has a switch to turn on or off the HAC part.

2. Concept and Design of the TS LAC/HAC System and the VG LAC/HAC System

2.1 TS LAC/HAC system

This section describes the concept and the characteristic features of the TS LAC/HAC system. Figure 2 shows the block diagram of the TS LAC/HAC system. The system has a switch to turn on or off the HAC part. The switching on or off of the HAC part is carried out in such a way that it satisfies the control objective:

“Suppress the vibration of a point in the structure as quickly as possible.”

The upper limit of the amplitude of the vibration of a point y_s at an arbitrary time t , $w_{\max}(y_s, t)$ is given by

$$w_{\max}(y_s, t) = \sum_{r=1}^{\infty} |\phi_r(y_s)| A_r(t) \quad (1)$$

where $\phi_r(y_s)$ is the eigenfunction of the r -th mode at point y_s , and $A_r(t)$ (>0) is the envelope amplitude of the r -th modal coordinate. Let the LAC/HAC system of Fig. 2 with the HAC part switched on operate for the duration t_1 after the initial time T_0 , and let the switch be kept turned off for the following duration t_2 after $(T_0 + t_1)$. Assuming that the envelope amplitude of the vibration of the point y_s becomes α times the initial upper limit during the $(t_1 + t_2)$ period, the following relationship holds:

$$\sum_{r=1}^{\infty} |\phi_r(y_s)| A_r(T_0) e^{-\zeta_r^{\text{LH}} \omega_r^{\text{LH}} t_1 - \zeta_r^{\text{L}} \omega_r^{\text{L}} t_2} = \alpha \sum_{r=1}^{\infty} |\phi_r(y_s)| A_r(T_0) \quad (0 < \alpha < 1) \quad (2)$$

where the superscripts LH (LAC/HAC) and L (LAC) indicate that the r -th modal damping ratio ζ_r and the r -th modal frequency ω_r corresponding to the LH condition and L condition can be used, respectively.

Fig. 2 Block diagram of the TS LAC/HAC system

The value $(\zeta_r^{\text{LH}}, \omega_r^{\text{LH}})$ and $(\zeta_r^{\text{L}}, \omega_r^{\text{L}})$ can be calculated by the characteristic equation of the closed loop system or can be identified by experiments before the system is put into operation⁽⁵⁾. When α is given, the problem is to find the values of t_1 and t_2 which minimize the cost function:

$$J_1 = t_1 + t_2. \quad (3)$$

In practice, the infinity in Eq. (2) has to be replaced by a finite number N which can be determined by the convergence of the HAC gain K_H by increasing N one by one before the system is put into operation. To suppress the vibration in the structure sufficiently, the process of switching on and off can be repeated n times. Then the upper limit of the envelope amplitude of the vibration becomes α^n times the initial upper limit.

However, the TS LAC/HAC system has the following problems:

(1) unfavorable uncontrolled modes can be excited, because the control input changes discontinuously when the switch turns on or off;

(2) the vibrations of uncontrolled modes may become very large at the time $T_0 + t_1$, depending on α ; and

(3) after n times of switching on and off are over, the effectiveness of the control may not be sufficient, because the HAC switch is turned off finally and therefore the control system becomes equivalent to the CDVFB system.

2.2 VG LAC/HAC system

This section describes the concept of the VG LAC/HAC system which partly solves the above-mentioned problems, and proposes a design method of a controller using a neural network system. Figure 3 shows the block diagram of the VG LAC/HAC system. Note that the TS LAC/HAC system is equivalent to a special case where the HAC feedback gain K_H in the VG LAC/HAC system takes only two values of zero and one.

Consider the same control objective in the previous section. Assuming that the upper limit of the amplitude of the vibration takes the minimum after duration τ , the problem is to find the HAC feedback

Fig. 3 Block diagram of the VG LAC/HAC system

gain K_H which minimizes the following cost function J_2 :

$$J_2 = \sum_{r=1}^{\infty} |\phi_r(y_s)| A_r(t) e^{-\zeta_r \omega_r t} \quad (4)$$

where the r -th modal damping ratio ζ_r and the r -th modal frequency ω_r are functions of the HAC feedback gain K_H . As described in the previous section, the infinity in Eq. (4) has to be replaced by a finite number N .

In a real control system a technique is needed which can find the HAC feedback gain K_H in a short time, since the calculation of the K_H must be finished within the sampling interval. A descent method can be used to numerically calculate the exact K_H which minimizes the cost function J_2 , but it takes a long time. Here a neural network system is applied to estimate the K_H .

Amplitude ratios A_r/A_1 ($r=1, \dots, N$) which change according to the state of the vibration are the input data to the neural network system, while the K_H calculated by the descent method is a "teacher" (see Fig. 4). Expressing the input data as the vector $\mathbf{x}$:

$$\mathbf{x} = \left[\frac{A_1}{A_1} \quad \frac{A_2}{A_1} \quad \dots \quad \frac{A_N}{A_1} \right]^T \quad (5)$$

the error function E whose gradient determines the weights' increments is given as

$$E = \frac{1}{2} \sum_{\mathbf{x}} \|K_H(\mathbf{x}) - \hat{K}_H(\mathbf{x})\|^2 \quad (6)$$

where $\hat{K}_H$ is the output of the neural network system. The weights w_{ij} and c_i in Fig. 4 are updated by the well-known backpropagation method⁽⁶⁾ using the values of K_H and $\hat{K}_H$, until each term in the summation of Eq. (6) falls within a specified limit δ . Note that the weights in the neural network system are determined before the control system is put into operation using

various vectors $\mathbf{x}$ which can occur in the vibration.

3. Computer Simulations

This section shows the results of a computer simulation when the control system for a flexible structure is designed by the ordinary LAC/HAC technique, the TS LAC/HAC technique, and the VG LAC/HAC technique. As a flexible structure, a simple clamped-free uniform beam is treated here which has the clamped end at $z=0.0$ and the free end at $z=1.0$. The modal undamped natural frequencies of the structure are

$$\begin{aligned} \omega_1 &= 3.516, \quad \omega_2 = 22.02, \quad \omega_3 = 61.67, \\ \omega_4 &= 120.91, \quad \omega_5 = 199.85, \quad \dots \end{aligned} \quad (7)$$

Consider a control system used to suppress the vibration of the point $z=0.7$ ($=y_s$) as quickly as possible. It is assumed in the computer simulation that the flexible vibration consists of five modes, and the system has only one actuator at point $z=0.3$. Employing only the first mode as the controlled mode, the controller for a HAC system is designed by the LQR technique to minimize the cost function:

$$J_3 = \int_0^{\infty} (\mathbf{x}^T \mathbf{Q} \mathbf{x} + u_1 R u_1) dt \quad (8)$$

where $\mathbf{x}_1$ is the state variable vector composed of the first modal coordinate and the first modal velocity, and u_1 is the control input. In Eq. (8) $\mathbf{Q}$ and R are

$$\mathbf{Q} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = 1.0 \quad (9)$$

where λ_1 is the first modal eigenvalue ($=\omega_1^2$). From the output of a displacement sensor located at $z=0.3$, the state variable vector $\mathbf{x}_1$ for the controlled system is estimated by the first-order observer, whose poles are set at $-5 \pm j3.5$ by the pole allocation technique because the poles of the LAC/HAC system are in the neighborhood of $-0.2 \pm j3.5$ in every computer simulation of this work. On the other hand, the LAC controller of CDVFB consists of the same actuator and a velocity sensor located at $z=0.3$.

By the calculation of an optimal (t_1, t_2) curve of Eq. (3) by increasing N one by one, $N=4$ was chosen to replace infinity in Eq. (2) for the TS LAC/HAC system. In the same way, the infinity in Eq. (4) was replaced by the finite number 4 in the computational simulation for the VG LAC/HAC system. Thus the fourth-order observer with poles at $-20 \pm j3.5$, $-10 \pm j22.0$, $-10 \pm j61.5$, and $-10 \pm j120.9$ estimates the modal coordinates and the modal velocities from the first to the fourth mode in order to obtain the envelope amplitudes A_r ($r=1, \dots, 4$). Figure 5 shows the block diagram of the VG LAC/HAC system. In each computer simulation, an impulse force was applied at $z=0.25$ to excite the vibration. Note that the modal frequencies ω_r^{LH} and ω_r^{L} are different from the modal

Fig. 4 A neural network system

Fig. 5 Block diagram of the VG LAC/HAC system in the simulation

Fig. 6 A result of the simulation of a LAC/HAC system

Fig. 7 A result of the simulation of a CDVFB system

undamped natural frequencies in Eq. (5), although the differences are very small.

3.1 Ordinary LAC/HAC system

When the magnitude of the LAC feedback gain K_L is 0.2, the ordinary LAC/HAC system designed so as to minimize Eq. (8) has an unstable mode in the uncontrolled modes. The damping ratios of the LAC/HAC system are

$$\begin{aligned} \zeta_1^{LH} &= 0.057, & \zeta_2^{LH} &= -0.00027, & \zeta_3^{LH} &= 0.0031, \\ \zeta_4^{LH} &= 0.00059, & \zeta_5^{LH} &= 0.000088. \end{aligned} \quad (10)$$

Because of the instability of the second mode, the ordinary LAC/HAC system has a divergent response. Figure 6 shows a nondimensional displacement at point $z=0.7$ vs nondimensional time.

On the other hand, the damping ratios of the LAC system are

$$\begin{aligned} \zeta_1^L &= 0.0021, & \zeta_2^L &= 0.0050, & \zeta_3^L &= 0.0037, \\ \zeta_4^L &= 0.00062, & \zeta_5^L &= 0.000089. \end{aligned} \quad (11)$$

Figure 7 shows the response at point $z=0.7$ when the control system is the LAC system with $K_L=0.2$. Although the LAC system stabilizes the system, it takes quite a long time to suppress the vibration.

Fig. 8 A result of the simulation of the TS LAC/HAC system

Fig. 9 A result of the simulation of the VG LAC/HAC system

3.2 TS LAC/HAC system

Figure 8 shows the result of the TS LAC/HAC system operation with $K_L=0.2$ for $\alpha=0.8$ and $n=10$: the upper figure indicates the on/off of the switch in the TS LAC/HAC system, and the lower figure shows the displacement at point $z=0.7$. It is seen that the TS LAC/HAC system has global stability, even if the ordinary LAC/HAC system is unstable because the magnitude of the LAC feedback gain K_L is not suitable.

3.3 VG LAC/HAC system

Figure 9 shows the result of the VG LAC/HAC system with $K_L=0.2$ for $\tau=2\pi/\omega_1$: the upper figure indicates the HAC feedback gain K_H , and the lower figure shows the displacement at point $z=0.7$. It is also seen that the VG LAC/HAC system has global stability, even if the ordinary LAC/HAC system is unstable. In Fig. 9 the HAC feedback gain K_H changes violently, but the change is continuous.

3.4 VG LAC/HAC system with a neural network controller

In a computational simulation of Fig. 9, the HAC feedback gain K_H which minimizes the cost function J_2 is calculated numerically by a descent method. However, this computational technique is not practical in a real control system, because it takes a long time to find the HAC feedback gain K_H . Thus a neural network controller is used to calculate K_H in a short time.

Table 1 Time required to suppress the vibration within 1/10 of an initial amplitude

K_L	0.2	0.3	0.4	0.5
LAC/HAC	—	18.2	12.2	10.4
TS LAC/HAC	25.4	16.5	12.2	11.1
VG LAC/HAC	25.4	16.5	12.2	10.4

Table 2 Time required to suppress the vibration within 1/20 of an initial amplitude

K_L	0.2	0.3	0.4	0.5
LAC/HAC	—	30.0	17.3	14.6
TS LAC/HAC	42.3	25.4	21.0	24.5
VG LAC/HAC	37.9	23.7	17.3	14.6

Fig. 10 A result of the simulation of the VG LAC/HAC system with the neural network controller

The neural network system updates the weights w_{ij} and c_i in Fig. 4 for about a hundred of x which can occur in the vibration. After this off-line learning process, the hidden layer has three neurons when $\delta = 0.04$.

Figure 10 shows the result of the $\hat{K}_H$ and the displacement at point $z=0.7$. This result resembles well that of Fig. 9. This indicates that the VG LAC/HAC system with the neural network system is quite feasible for implementation.

3.5 Comparison of the effectiveness of control

Tables 1 and 2 show the time required to suppress the vibration within 10 percent and 5 percent of an initial amplitude when the control system is designed by the ordinary LAC/HAC technique, the TS LAC/

HAC technique, and the VG LAC/HAC technique. The symbol “—” in these tables indicates that the control system is unstable. In Table 2, the TS LAC/HAC system requires a long time compared to the VG LAC/HAC system to suppress the vibration to within 5 percent of the initial amplitude, because of the third problem of the TS LAC/HAC system. Tables 1 and 2 indicate that the VG LAC/HAC system has high effectiveness of control in every simulation.

4. Conclusions

This paper proposes a new control technique, the VG LAC/HAC technique, based on the concept of LAC/HAC. The VG LAC/HAC system has a mechanism to adjust the HAC feedback gain. Computational simulations show the characteristic features of the VG LAC/HAC, its implementation as well as its high effectiveness and global stability even when the magnitude of the LAC gain is not suitable. Moreover, it is shown that the neural network system makes it possible to adjust the HAC feedback gain in a short period of time.

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