Robust Control of Flexible Rotor–Magnetic Bearing Systems Using Discrete Time Sliding Mode Control*

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This paper is concerned with the computer-based sliding mode control of flexible rotor–magnetic bearing systems (FR–MBS). The plant dynamics consisting of actuator dynamics and flexible rotor dynamics are described. The reduced-order model for controller design is given by eliminating higher-order modes of the mechanical and electrical magnetic interaction system. A discrete time sliding mode controller with reduced-order model is proposed and its robust performance is evaluated with several simulations based on a calculation model. This digital controller is implemented to replace a linear analog PID compensator. Levitation tests using the proposed digital controller are performed and compared with those of the PID compensator. With a discrete time sliding mode controller, the running test with high-speed rotation is successfully increased up to 35 000 rpm without unstable vibration.

Key Words: Magnetic Bearing, Sliding Mode Control, Discrete Time System, Robustness, Experiments

1. Introduction

Magnetic bearings are used in high-speed rotating machinery with many advantages. The advantages of magnetic bearings applied to support a rotor system are their contactless nature, the capability of high-speed rotation and active vibration control. However, active magnetic bearings are inherently unstable due to the negative stiffness elements caused by the electromagnetic field. Also, the degree of instability of the system increases due to the gyroscopic effect. The nonlinearity of the electromagnetic fields is another important factor that needs to be controlled with strong stability. The magnetic force produced by an electromagnet is proportional to the square of the coil current and inversely proportional to the square of the air gap. This nonlinearity makes for a strong interaction between displacement and current. Also, there are force coupling effects among various axes.

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The system has a complicated dynamic model due to the nonlinear nature mentioned above, and it becomes difficult to control. The third problem is to control the flexible modes of the rotor in passing through critical speeds. The flexible rotor generally has infinite vibration modes in the case of free-free support and its flexibility has some uncertainty. There is also an unpredictable disturbance synchronizing rotating speed due to mass imbalances of the rotor. However, the controller should be constructed with reduced-order models to control the full-order vibration system out of consideration for computation time. The designed controller must have robustness with spillover suppression of the poorly damped uncertain vibration dynamics at higher frequencies.

As mentioned above, it is necessary to use an asymptotically stable and robust controller for magnetic bearings to support rotor systems. Now the designs for such controllers have been developed by using state-space model approaches. These control schemes can easily accommodate the cross feedback capability through the utilization of the full-state feedback elements with observers. However, it is not easy to satisfy the performance of control systems.
using the linear control law. One of the main claims against these approaches is that the robustness is not guaranteed over the entire operating range since modeling errors and unpredictable disturbances may occur. As one means of robust control, the control theory based on the linear control system has recently been developed and used to control the flexible magnetic bearing systems\textsuperscript{(8)(9)}. But the plant modeling used in these references was simplified and idealized because several strong nonlinear factors exist in the magnetic bearing system.

Recently some research has been conducted on control of magnetic suspensions\textsuperscript{(9)} and rotor-magnetic bearing systems with sliding mode control\textsuperscript{(6)(7)}. However, most such studies did not consider the flexibility of the plant and only dealt with simulations. A discontinuous control law is obtained using a sliding mode controller which switches when the trajectory crosses a certain chosen hyperplane, which is called the sliding manifold, and reaches the origin of state space\textsuperscript{(9)}. The main advantage of this nonlinear control method is that the system can be easily designed to be robust with respect to parametric uncertainties and external disturbances. In a recent paper\textsuperscript{(9)}, the design approach using sliding mode control was extended to flexible rotor–magnetic bearing systems. The theoretical analysis was confirmed by extensive results from numerical simulations. However, sliding mode control based on a continuous system is usually implemented using a digital computer. The sampling interval in continuous systems may bring about chattering along the presdesigned sliding mode and make the system unstable. Therefore, a discrete time sliding mode controller using constant sampling should be used for computer-based control of a practical system\textsuperscript{(10)}.

Based on an early work\textsuperscript{(9)}, this paper describes the design method and the implementation of the discrete time sliding mode controller for an actual flexible rotor–magnetic bearing system. In section 2, a flexible rotor of continuous body based on the test rig is modelled as the discrete mass rotor using a finite-element method. The state-space model governing the dynamics of the flexible rotor is described. Then, reduced-order models which include both the rigid modes and the first-order flexible mode are proposed for the design of the controller. The design of the sliding mode controller for a continuous system is briefly discussed in section 3. Section 4 presents the new design method for the discrete time sliding mode controller, which takes into account the external disturbances. Some simulations for this discrete time sliding mode controller are given in section 5. Section 6 describes the implementation of this controller and comparison of its performance with that of a conventional analog compensator. Some conclusions will be presented in section 7.

2. Modelling of FR-MBS

The dynamics of the flexible-rotor magnetic bearing system will be described in this section. For simplicity, the analysis is only done in the X direction and all the coupling effects among the different axes and noncollocation are ignored. According to the test rig, which will be described in section 6, the rotor can be regarded as consisting of the six parts shown in Fig. 1.

2.1 Flexible rotor dynamics

The discrete model of fourteen orders is obtained using the finite-element method as follows:

\[ M_0 \ddot{q} + K_0 q = 0 \]  

where \[ q = [x_1 \ x_2 \ x_3 \ \theta_1 \ x_4 \ \theta_2 \ x_5 \ \theta_3 \ x_6 \ \theta_4 \ x_7 \ \theta_5]^T \]

and \[ x_i, \ \theta_i \ (i=1, \ldots, 7) \] represent displacement and angle of the mass on this rotor, respectively; in particular, \[ x_5 \text{ and } x_6 \] represent the positions where the electromagnets are located, \[ M_0 \] is the mass matrix, and \[ K_0 \] is the stiffness matrix. Figure 2 shows the mode shapes of this rotor up to the third flexible mode, which is calculated with the finite-element method.

2.2 Actuator dynamics

A schematic of a radial magnetic bearing is shown in Fig. 3. The attractive force due to an electromagnet can generally be given by

\[ P = \frac{A}{\mu_0} B^2 = \frac{A}{\mu_0} \left( \frac{N(b + i)}{1 + \frac{x_5 + x_6}{\mu_0}} \right)^2 \]

where \( P \) is the attractive force, \( \mu_0 \) is the permeability in air, \( A \) is the air gap area of one pole, \( B \) is the magnetic flux density, \( N \) is the number of winding turns, \( b \) is the steady-state current, \( x_5 \) is the steady-state gap length, \( i \) is the control current, \( x \) is the control gap, and \( \mu \) is the permeability in the magnetic body. Using the Taylor series expansion for small values of \( i \) and \( x \), we can obtain the following attractive force with linear terms

![Fig. 1 Model of flexible rotor–magnetic bearing system](image-url)
\[ P \equiv p_0 - k_1 x + k_2 i = p_0 + \bar{p} \quad (3) \]

where
\[
\begin{align*}
  k_1 &= \frac{2\text{AN}^2 \mu^2}{\mu_0 (1 + \frac{x_b}{\mu})} = \frac{2p_0}{\mu_0 (1 + \frac{x_b}{\mu})} \\
  k_2 &= \frac{2\text{AN}^2 \mu^2}{\mu_0 (1 + \frac{x_b}{\mu})} = \frac{2p_0}{\mu_0 (1 + \frac{x_b}{\mu})} - \frac{2\text{AN}^2 \mu^2}{\mu_0 (1 + \frac{x_b}{\mu})} 
\end{align*}
\]

and \( p_0 \) is the bias attractive force. Considering the pair of attractive forces, the magnetic force \( P' \) due to the electromagnet along the radial direction \( X \) can be modeled as the following equation:
\[ P' = P_1 - P_2 = -2k_i x + 2k_2 i \quad (4) \]

where \( P_1 \) and \( P_2 \) are the left and right magnetic forces, respectively. Equation (4) indicates the total actuator forces in each direction.

### 2.3 Plant dynamics

The flexible rotor shown in Fig. 1 is restricted by the attractive forces given in Eq. (4). This results in

\[ M_0 \dot{\theta} + K_0 q = F' + D \quad (5) \]

where
\[
F' = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}^T
\]
\[ p' = [P_1'] \]
\[ P_1' = 2k_i x - 2k_2 i : \text{forces of the AMB-L} \]
\[ P_2' = 2k_i x - 2k_2 i : \text{forces of the AMB-R} \]

and \( D \) represents the parameter uncertainty and external disturbance.

The bias attractive forces and the control forces of Eq. (5) are separated as follows:
\[ \dot{M}_0 \dot{\theta} + \dot{K}_0 q = \dot{F}_i + D \quad (6) \]

where
\[
i = [i_1 \ i_2]^T, \quad K = K_0 + K_i \]
\[ K_i = \text{diag}(0, 0, 0, 0, -2k_i, 0, 0, 0, -2k_i, 0, 0, 0, 0) \]
\[ F_i = \begin{bmatrix}
0 & 0 & 0 & 0 & -2k_i & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T \]

Using the modal analysis technique, we can choose the following normalized modal matrix:
\[ q = \Psi \ddot{\xi}. \quad (7) \]

Equation (6) is transformed to the form in model coordinate as follows:
\[ \ddot{\xi} + \Lambda \dot{\xi} + \Omega^2 \xi = f_i + d \quad (8) \]

where
\[
I = \Psi^T \Psi, \quad \Omega^2 = \Psi^T K \Psi, \quad \Lambda = 2 \ddot{\Omega}\Omega
\]
\[ f_i = \Psi^T F_i, \quad d = \Psi^T D_i \]

and where \( \Lambda \) is the damping matrix. The damping ratio is determined experimentally and is given in Table 1. The state equation of the electromagnetic-mechanical system is given by
\[ \dot{x} = A_r x_r + B_r u + D_r x_r \quad (9) \]

where
\[
\begin{align*}
  x_r &= [\xi \ \dot{\xi}]^T, \quad u = [i_1 \ i_2]^T \\
  A_r &= \begin{bmatrix}
0 & I \\
-\Omega^2 & -\Lambda
\end{bmatrix}, \quad B_r = \begin{bmatrix}
0 \\
1
\end{bmatrix}, \quad D_r = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\end{align*}
\]

If the rotor displacement at the magnetic bearings can be measured, the output equation is
\[ y = C_r x_r = [x_3 \ x_4]^T \quad (10) \]

where
\[ C_r = [F_i \ \Psi \ 0] \]

### 2.4 Reduced order model

Because this MIMO system is originally unstable in an open loop, the control objective is to levitate the rotor and maintain the stability. In this case, there are only two unstable rigid modes, and the flexible modes are essentially stable. It is complicated to design a controller including full-order models for this high-order flexible system. Therefore, the construction of the reduced-order model is considered from the standpoint of stabilizing the two rigid modes and controlling the vibration of flexible modes. The reduced-order model is constructed by truncation of the higher-order modes in modal coordinates. Here, the state
Table 1  Dimensions in simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td></td>
</tr>
<tr>
<td>$m_1$</td>
<td>0.03 kg</td>
</tr>
<tr>
<td>$m_2$</td>
<td>0.15 kg</td>
</tr>
<tr>
<td>$m_3$</td>
<td>1.0 kg</td>
</tr>
<tr>
<td>$m_4$</td>
<td>0.5 kg</td>
</tr>
<tr>
<td>$m_5$</td>
<td>1.0 kg</td>
</tr>
<tr>
<td>$m_6$</td>
<td>0.0 kg</td>
</tr>
<tr>
<td>$m_7$</td>
<td>0.1 kg</td>
</tr>
<tr>
<td>Length</td>
<td></td>
</tr>
<tr>
<td>$l_1$</td>
<td>0.09 m</td>
</tr>
<tr>
<td>$l_2$</td>
<td>0.072 m</td>
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<tr>
<td>$l_3$</td>
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<tr>
<td>$l_4$</td>
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</tr>
<tr>
<td>$l_5$</td>
<td>0.091 m</td>
</tr>
<tr>
<td>$l_6$</td>
<td>0.091 m</td>
</tr>
<tr>
<td>Diameter</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>0.0275 m</td>
</tr>
<tr>
<td>Gap</td>
<td></td>
</tr>
<tr>
<td>$x_0$</td>
<td>0.0003 m</td>
</tr>
<tr>
<td>Bias current</td>
<td></td>
</tr>
<tr>
<td>$i_0$</td>
<td>3.0 A</td>
</tr>
<tr>
<td>Bias attractive force</td>
<td></td>
</tr>
<tr>
<td>$p_0$</td>
<td>100.0 N</td>
</tr>
<tr>
<td>Damping ratio</td>
<td></td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.001</td>
</tr>
<tr>
<td>Permeability in magnetic body</td>
<td>$2\pi \times 10^{-3}$</td>
</tr>
<tr>
<td>Permeability in $\delta_t$</td>
<td>$4\pi \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Continuous functions of the state vector and has discontinuities on the surfaces in the state space.

The sliding mode occurs on a switching surface $\sigma_i(x_1) = 0$ when all of the trajectories are attracted to the subspace $\sigma_i = 0$. Then the state of the system slides and remains on the surface. The equivalent control law to maintain the state on the switching manifolds is given by

$$u = F_{eq} x$$

(15)

where

$$F_{eq} = - (SB)^{-1} SA.$$  

The sliding mode is chosen so that the closed loop system with $F_{eq}$ is stable.

After design of the sliding mode, the control to drive the state to the sliding mode is selected. The sufficient condition for the sliding mode to exist on the $i$-th hyperplane is given by the existence of positive $\delta_i$ satisfying

$$\delta_i < - \delta_i |x_i| \quad \delta_i > 0.$$  

(16)

Using a Lyapunov function, one can find the dimensional control $u$.

4. Discrete Time Sliding Mode Controller of FR-MBS

This section will discuss the design of the digital controller. As a limitation of the structure, this magnetic bearing system has only two output feedbacks which can be measured directly. Therefore, the adaptive variable structure system (VSS) observer designed in Ref. 9 is used for system state estimation. Considering the reduced-order model system given in Eq. (11), the equivalent discrete-time system is found as

$$x_r(k+1) = \Phi x_r(k) + \Gamma u(k) + d(k)$$

$$\left[ \begin{array}{c} y(k) \\ z(k) \end{array} \right] = \left[ \begin{array}{cc} C_r & C_p \end{array} \right] x_r(k)$$

(17)

where

$$\Phi = \exp(A_r + D)$$

$$\Gamma = A_r \left[ \exp(A_r + D) - I \right] B_r.$$  

$\Delta$ is the sampling time, $\tau$ is estimated state variables with VSS observer and $C_p \in \mathbb{R}^{m \times n}$. In this discrete time system, the following matching condition is guaranteed.

$$d(k) = \Pi f(k)$$

(18)

where

$$|f(k)| \leq F_{max}$$

and $F_{max}$ is the estimated maximum values of external disturbance.

The sliding manifold is defined by

$$\sigma(k) = S x_r(k).$$

(19)

After the system state is driven into the sliding mode at the time of $k \Delta$, using the condition of $\sigma(k) = \sigma(k + 1) (\forall k \geq k_0)$, the equivalent control in the sliding mode can be obtained as
where
\[ L = -(SΓ')^{-1}S(Φ - I). \]
The ideal sliding motion is then described by the system equation
\[ x_r(k+1) = [Φ - Γ(SΓ')^{-1}S(Φ - I)]x_r(k) = Gx_r(k) \]
\[ Sx_r(k) = 0. \]

4.1 Selection of switching manifolds

In the first step, S must be determined for a given set of stable G, which means that the eigenvalues of Eq. (21) must lie within the unit circle. Considering the reduced-order system shown in Eq. (17), where rank(B_r) = m = 2 and n = 4, the system equation can be changed into the following form
\[ x_r(k+1) = Φ_1x_r(k) + Φ_2x_r(k) \]
\[ x_r(k+1) = Φ_1x_r(k) + Φ_2x_r(k) \]
\[ + Γ_1u(k) + Γ_2f(k) \]
where
\[ Φ = \begin{bmatrix} Φ_1 & Φ_2 \\ Φ_3 & Φ_4 \end{bmatrix}, \quad Γ = \begin{bmatrix} 0 \\ I_4 \end{bmatrix} \]
\[ x_r(k+1) = [Φ_1, Φ_2, Φ_3, Φ_4], \quad f_r(k) = [f_1, f_2, f_3, f_4] \]

4.2 Design of controller

After the design of the switching surface, the next important aspect of variable structure system is guaranteeing the existence of the sliding mode. The variable structure system can be thought of as a closed-loop system with an adaptively varying state feedback gain. Therefore, the type of control law considered here consists of two independent functions: a linear state feedback control function \( u_c \) and nonlinear control function \( u_{nc} \):
\[ u(k) = u_c(k) + u_{nc}(k) \]
where
\[ u_{nc}(k) = Lx_r(k) \]
and \( L \) is given in Eq. (20). \( u_{nc} \) is the discontinuous control law which can be defined by
\[ u_{nc} = -[a(k) + β(k)] \text{sgn} (σ(k)) \]
where
\[ σ(k) \leq η |σ(k)| \quad 0 < η < 2, β(k) = F_{max}. \]
We consider
\[ V(k) = 0.5σ(k)^2 \]
as a candidate Lyapunov function; then for asymptotic stability in the wide range
\[ V(k+1) < V(k), \]
Eq. (30) satisfies the inequality and guarantees the existence condition. One can define \( Δσ(k+1) \) as
\[ Δσ(k+1) = σ(k+1) - σ(k) \]
Squaring both sides yields
\[ σ(k+1)^2 = σ(k)^2 + 2σ(k)Δσ(k+1) + Δσ(k+1)^2. \]
If the control satisfies
\[ 2σ(k)Δσ(k+1) + Δσ(k+1)^2 < 0, \]
then from Eq. (31), the inequality Eq. (29) is guaranteed.

From Eq. (19), one can find
\[ σ(k) = S[Φx(k) + Γ(u_c(k) + u_{nc}(k)) + Γf(k)] \]
\[ = σ(k) + SIu_{nc}(k) + SIΓf(k). \]
Substituting Eq. (28) and Eq. (34) into Eq. (31) results in
\[ Δσ(k+1) = SIu_{nc}(k) + SIΓf(k). \]
From Eq. (34) and Eq. (35), one can find
\[ 2σ(k)Δσ(k+1) + Δσ(k+1)^2 \]
\[ = 2σ(k)SI^2[SIu_{nc}(k) + SIΓf(k)] \]
\[ + (SI)^2[SIu_{nc}(k) + SIΓf(k)]^2 \]
\[ = 2σ(k)SI^2[SIu_{nc}(k) + SIΓf(k)] + (SI)^2[SIu_{nc}(k) + SIΓf(k)]^2 \]
\[ = -2σ(k) + SI^2σ(k) + (SI)^2σ(k)^2 \]
\[ = -2σ(k) + σ^2 < 0. \]
This means the sliding mode exists on the sliding surface and is attainable for all \( x_r \in R^n \).

5. Simulations

Table 1 shows the parameters used in the simulations. The sampling time of the discrete time sliding mode controller was 400 μs. For simplicity, only the results of position \( x_s \) will be given in both a simulations and experiments. Similar results could be found with a position of \( x_s \). The optimal switching matrix S is calculated as
\[ S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]
\[ S = \begin{bmatrix} -0.365 & 132.4 & 56.02 & 0.078 \\ 126.5 & -596.6 & 0.684 & 55.78 \end{bmatrix} \text{ for } (i = 2). \] (37)

It can be confirmed that in both cases the \((n - m)\) closed-loop eigenvalues of this discrete control system are located with the unit circle.

At first, simulations were performed for the control system including only rigid modes. Figure 4 shows the step response at the lift-off with the initial displacement \(x_i = 0.15 \text{ mm}\) and \(\theta_i = 0 \text{ deg} \) \((i = 1, 2, \ldots, 7)\). It shows that the rotor can be levitated with good stability if the rigid modes are included. It has been clarified that the robust stability of the system mentioned above is guaranteed by the sliding mode control. It has been observed that high chatter, which is often generated in continuous sliding mode control, does not occur with discrete sliding mode control. In Figure 5, the impulse response in the case of providing a disturbance at the right end of the rotor after the rotor has been levitated is shown. It is shown that the discrete time sliding mode control has low sensitivity to disturbance for an unstable system. Note that in the controller design model, only the rigid vibration modes of the rotor were included. The dynamic errors of the initial values at the starting time of these responses are due to estimated error of the VSS observer.

Considering the robustness of this discrete time sliding mode controller, Figure 6 shows the responses of initial displacement in the case where the bias attractive force is doubled. In the same case, Figure 7 gives the impulse responses of this discrete system. In the comparison of Figure 6 with Figure 4, we find that the discrete time sliding mode control has powerful robustness to parameter variation over a wide range because similar dynamic properties of the control system can be obtained. The attractive force is the nonlinear factor in this system mentioned above; however, the results in these responses indicated that the controller is also robust to a nonlinear system. The same results can also be obtained by comparing Figure 7 with Figure 5.

### 6. Experimental Results

The test rig of the magnetic bearing system under consideration is shown in Figure 8. An induction motor...
rotor is located in the middle of the shaft and two radial magnetic bearings are located on each side of the motor rotor. A thrust magnetic bearing is located at the left end of the shaft. The radial magnetic bearings together control two rotational and two translational degrees of freedom. The thrust magnetic bearing controls the displacement in the axial direction. Eddy-current-type proximity sensors are set up for the radial magnetic bearings at both sides of the bearings, but inner sensors are used in tests. The natural frequencies of the first two bending modes measured in an impulse experiment were determined to be 340 Hz and 780 Hz, respectively. They are consistent with the calculated values shown in Fig. 3.

The block diagram of the control system implemented digitally is shown in Fig. 9. A digital signal processor TMS320C30 with high-speed A/D converter and D/A converter is used to accomplish digital control and to transfer data between the host personal computer and test rig. The sampling time in experiments is set at 0.4 ms because it is not necessary to take a small sampling rate as in a continuous system. In fact, the computation time for the control algorithm was less than 0.1 ms. Therefore, the proposed discrete time sliding mode controller is expanded to control two directions \( (x \text{ and } y) \) simultaneously. Other controller parameters are the same as those used for simulations. All of the experimental results shown in this section are given for the case of \( i=2 \).

In this experimental setup, a linear analog compensator was also applied to each axis based on the models ignoring the coupling effects among various axes. We can select the control system by either the linear analog compensator, or the newly designed discrete time sliding mode controller. The compensator circuit is made up of an integration circuit to increase static rigidity, a phase lead circuit to increase the damping force of the rotor in the intermediate frequency range and a low-pass filter to attenuate the gain in the high frequency range.

Figure 10 shows the typical time history response of the discrete time sliding mode control system at lift-off. The time history response of the analog compensator in the same position is presented in Fig. 11. The time history response of the sliding mode controller shows that it takes about 80 ms to reach the steady-state position, whereas the analog controller takes over 200 ms with a big overshoot. Figure 12 gives the impulse responses of the sliding mode control when the disturbance is added at the right end of the rotor. This impulse response indicates excellent performance, considering that the controller was designed using only a rigid mode. The control current used to realize this response is also shown in the same figure. It was found that the experimental data are in the good agreement with the simulation results. Considering the robustness of sliding mode controller,
Figs. 13 and 14 respectively show the time history responses and the impulse response in the case of increasing the rotor mass by 50% of the nominal value by loading a mass at the right end of the rotor. It can be seen that the system is still stable under suspension and the newly designed discrete time sliding mode controller has superior performance in eliminating disturbances and maintaining low sensitivity to system parameter variation. It is not easy for a conventional analog compensator to realize similar good performance.

Next, we shall report the results of the rotating test with sliding mode controller for this test rig. Figure 15 shows the waveform of control current and shaft vibration in the $x$-direction at the 1st bending critical speed (20,000 rpm). In this figure, the control current generated by the discrete time sliding mode controller has the same period as the shaft vibration. We also find that the shaft vibration has a very small magnitude, though the shaft is rotated under the 1st bending mode. To show the shaft vibration in the $xy$-direction at the same time, Fig. 16 gives the orbit plot of the shaft center at position under the rigid mode (8,000 rpm) and 1st bending mode (20,000 rpm). It is shown from this figure that the vibration magnitude in the rigid mode is larger than that of the 1st bending mode, but the spillover phenomenon, which is caused by ignored higher-order modes, cannot be found in either case. Finally, a waterfall diagram of
shaft vibration in the rotation test is attained, as shown in Fig. 17. With the discrete time sliding mode controller, the rotating test was successfully rotated at 35,000 rpm without unstable vibration in this case. As a result the vibration frequency of the shaft was found to be proportional to the rotating speed over the entire frequency range, and the vibration response was fully damped even at the rigid and 1st bending critical speeds.

7. Conclusions

This study has developed an active vibration control system for actual flexible rotor-magnetic bearing systems by considering the instability, nonlinearity and flexibility of the plant. Using the finite-element method, the state-space model for the full-order system of the flexible rotor was derived. The plant dynamics, including the actuator dynamics and the flexible rotor, were described. A discrete time sliding mode controller using a reduced-order model of the plant, which can effectively control the full-order system with robustness to the spillover phenomena of the higher-order modes, was designed for active control of magnetic bearing systems. A new and simple algorithm for digital controller design was proposed. Simulations were carried out with digital controllers using two reduced-order models which showed that the unstable modes can easily be controlled and the system can be controlled with strong stability. This controller was implemented as an alternative to a conventional linear analog PID compensator. The comparisons between results of the discrete time sliding mode controller and the analog PID compensator, which were carried out in levitation tests, indicated that the digital controller has superior dynamic response properties and robustness to the system parameter uncertainty, nonlinear factors and external disturbances. With the discrete time sliding mode controller, the rotating test was successfully performed up to 35,000 rpm without unstable vibration such as chattering, which indicated that the vibration responses are fully damped even at the rigid and 1st bending critical speeds.

Acknowledgments

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