Robust Control of Magnetic Levitation Systems by Means of H^∞ Control/ μ -Synthesis*

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This study is concerned with an application of H^∞ control and μ synthesis to magnetic levitation systems which have special flexible structures and the solution of the spillover problem. At first, we show the typical spillover phenomenon caused by the interaction between the control system and the flexible structure system using PID control. Next, applying mixed-sensitivity H^∞ control to the magnetic levitation system, we design the control system for it without spillover and describe in detail how to design an H^∞ controller. Finally, we analyze the structured singular value and design the μ controller using D - K iteration. In this paper, we assume the magnetic levitation system as an uncoupled system between the X and Y directions for simplicity, which is the third-order system with the exception of the guideway. As a result, it has been clarified that the H^∞ controller and μ -controller have the characteristics of two kinds of filters, a low-pass filter and a notch filter. It is found that the μ controller is superior to the H^∞ controller in terms of robust stability and robust performance. From experiments, we obtained good data on the magnetic levitation without spillover.

Key Words : Magnetic Levitation System, Spillover, H^∞ Control, Mixed-Sensitivity Problem, Experiments, Flexible Slider, Flexible Guideway

1. Introduction

The magnetic levitation systems or magnetic bearing systems levitate against support structures without any mechanical contacts by active control. However, since most support structures are not a thoroughly rigid body, if natural frequencies of the structures exist in the bandwidth of the controller, violent vibrations will be induced by the high feedback gains of the controller. This is the so-called spillover problem. It is a challenging and important problem for magnetic levitation systems or magnetic bearing systems to suppress the spillover phenomenon. Since there are too many uncertainties in magnetic levitation systems, the robust control theory must be applicable to these areas to solve above-mentioned problem.

The robust control theory is classified into two

regions, namely, linear robust control and nonlinear robust control. It is known that the H^∞ control theory and the μ synthesis theory are currently very powerful for linear robust control, whereas the variable structure system (VSS) like sliding mode control is very useful for nonlinear robust control. There are several papers⁽¹⁾⁻⁽⁹⁾ concerning magnetic levitation systems and magnetic bearing systems which apply the H^∞ control theory, μ synthesis theory and sliding mode control. Though a levitation system is flexible and the guideway is assumed to be rigid in most research works, we consider the opposite case where the levitation system is rigid and the guideway is flexible. Furthermore, we attempt to study the more difficult case wherein both levitation system and guideway are flexible. We design the control system and overcome the spillover problem through simulations and experiments.

2. Test Rig

The test rig is shown in Fig. 1. It is composed of a slider (levitation system), guideway (support structure) and control system. The guideway is a flexible structure likened to a thin plate having a concentrated

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mass at the middle of the span. The slider is divided into upper and lower sliders which are connected by two thin metal plates. Thus it is easy to disconnect the lower slider in the case of experiments concerned with the rigid slider. The upper slider is equipped with electromagnets and is levitated by control currents in the X and Y directions, while the Z direction is uncontrolled and the slider is supported by hanging wires. Both the guideway and the slider have very low natural frequencies caused by their shape in the X direction. Photosensors are used to detect the relative

Fig. 1 Test rig with flexible guideway and flexible slider

(a) Case when guideway is assumed to be rigid

(b) Case when guideway is assumed to be flexible

Fig. 2 Dynamical model for magnetic levitation system

displacements between the guideway and the upper slider. The air gap between the slider and the guideway is 1 mm and the slider mass including the lower mass is about 3 kg. Simultaneous digital control of the X and the Y directions is achieved by DSP.

3. H^∞ Control System Design and Experiments for a Rigid Slider

3.1 Modeling

First of all, we design the control system for a reduced-order model (a) assuming that the guideway and the slider are both rigid as shown in Fig. 2(a). The full-order model (b) is shown in Fig. 2(b).

(a) Case when the guideway is assumed to be a rigid body

The equations of motion are written as follows:

$$f_1 = f_0 + 2f_0 \left(\frac{i_1}{i_0} - \frac{h_0 - x_1}{h_0} \right), f_2 = f_0 + 2f_0 \left(\frac{i_2}{i_0} - \frac{h_0 + x_1}{h_0} \right),$$

$$M\ddot{x}_1 = f_1 - f_2 + w = 4f_0 \left(\frac{i}{i_0} + \frac{x_1}{h_0} \right) + w, i = i_1 = -i_2. \quad (1)$$

From Eq.(1), the state equation and output equation are as follows:

$$\dot{X}_a = A_a X_a + D_a w + B_a u, Y_a = x_1 = C_a X_a \quad (2)$$

where, $X_a = [x_1 \dot{x}_1 i]^T$,

$$A_a = \begin{bmatrix} 0 & 1 & 0 \\ \frac{4f_0}{Mh_0} & 0 & \frac{4f_0}{Mi_0} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix},$$

$$B_a = \begin{bmatrix} 0 & 0 & \frac{10}{L} \end{bmatrix}^T, C_a = [1 \ 0 \ 0],$$

$$D_a = \begin{bmatrix} 0 & \frac{1}{M} & 0 \end{bmatrix}^T.$$

(b) Case when the guideway is assumed to be a flexible body

From Fig. 2(b), the equations of motion are written as follows:

$$f_1 = f_0 + 2\frac{f_0}{i_0}i_1 - 2\frac{f_0}{h_0}(h_0 - x_1 + x_2),$$

$$f_2 = f_0 + 2\frac{f_0}{i_0}i_2 - 2\frac{f_0}{h_0}(h_0 + x_1 - x_2), i = i_1 = -i_2,$$

$$M\ddot{x}_1 = f_1 - f_2 + w = 4f_0 \left(\frac{x_1 - x_2}{h_0} + \frac{i}{i_0} \right) + w,$$

$$m\ddot{x}_2 = -c\dot{x}_2 - kx_2 + f_1 - f_2$$

$$= \frac{4f_0}{h_0}x_1 - \left(k + \frac{4f_0}{h_0} \right)x_2 - c\dot{x}_2 + \frac{4f_0}{i_0}i, \quad (3)$$

so we have the state equation and output equation as follows:

$$\dot{X}_b = A_b X_b + B_b u + D_b w, Y_b = x_1 - x_2 = C_b X_b, \quad (4)$$

where, $X_b = [x_1 \dot{x}_1 x_2 \dot{x}_2 i]^T$,

$$A_b = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{4f_0}{Mh_0} & 0 & -\frac{4f_0}{Mh_0} & 0 & \frac{4f_0}{Mi_0} \\ 0 & 0 & 0 & 1 & 0 \\ \frac{4f_0}{mh_0} & 0 & -\frac{k}{m} - \frac{4f_0}{mh_0} & -\frac{c}{m} & \frac{4f_0}{mi_0} \\ 0 & 0 & 0 & 0 & -\frac{R}{L} \end{bmatrix},$$

$$B_b = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{10}{L} \end{bmatrix}^T, C_b = [1 \ 0 \ -1 \ 0 \ 0],$$

$$D_b = [0 \ 1/M \ 0 \ 0 \ 0]^T.$$

x_1 is the absolute displacement of the slider, x_2 the absolute displacement of the guideway, f_1 the attractive force of electromagnets, f_2 the attractive force of electromagnets on the opposite side of the slider, M the levitation mass, m the equivalent mass in a single degree of freedom for the guideway, k the equivalent spring constant for the guideway, c the equivalent damping constant of the guideway, i_0 the bias current, h_0 the steady-state gap length, f_0 the steady-state attractive force, i the coil current, L the inductance of the coil, R the resistance of the coil, u the input voltage, and w the system disturbance.

3.2 H^∞ Control system design

Magnetic levitation systems themselves are an unstable system. Therefore it is very important to stabilize the closed-loop system including the unstable plant and the controller. Here we apply the H^∞ con-

Fig. 3 Generalized plant and closed-loop system

Fig. 4 Frequency responses of uncertainty and weighting functions

trol theory for the mixed-sensitivity problem to the magnetic levitation system.

The block diagram of the generalized plant and closed-loop system is shown in Fig. 3, where $P_a(s)$ is the reduced order model, $K(s)$ the H^∞ controller, w the disturbance input, y the measurement output, w_1 the weighting function of robust stability, w_2 the weighting function of sensitivity reduction, and z_1 and z_2 are the control values for the robust stability and the sensitivity, respectively. Now, we have the following equation,

$$G(s) = \begin{bmatrix} A_{w1} & 0 & 0 & 0 & B_{w1} \\ 0 & A_{w2} & B_{w2}C_a & 0 & 0 \\ 0 & 0 & A_a & D_a & B_a \\ C_{w1} & 0 & 0 & 0 & D_{w1} \\ 0 & C_{w2} & D_{w2}C_a & 0 & 0 \\ 0 & 0 & C_a & 0 & 0 \end{bmatrix}. \quad (5)$$

The H^∞ controller design in this paper was derived via the following steps. 1) Derive the model uncertainty between the full-order model and the reduced-order model. 2) Select suitable weighting functions. Here, we choose $W_1(s)$ covering the uncertainty in the frequency range to insure that the system has robust stability over the entire frequency range. On the other hand, we choose $W_2(s)$ which has a higher gain in the low-frequency band to stabilize the magnetic levitation system and to suppress the disturbances, and a lower gain in the high-frequency band. Moreover, $W_1(s)$ and $W_2(s)$ should be finally traded off to satisfy the following inequality,

$$J = \left\| \begin{bmatrix} W_1(s) & N(s) \\ W_2(s) & M(s) \end{bmatrix} \right\|_\infty < 1, \quad (6)$$

where $N(s)$ and $M(s)$ are defined by

$$N(s) = K(s)[I + P_a(s)K(s)]^{-1}P'_a(s),$$

$$M(s) = [I + P_a(s)K(s)]^{-1}P'_a(s). \quad (7)$$

$P_a(s) = C_a(sI - A_a)^{-1}B_a$ is the transfer function of plant from u to y and $P'_a(s) = C_a(sI - A_a)^{-1}D_a$ is the transfer function of plant from w to y .

$W_1(s)$, $W_2(s)$ and uncertainty $\Delta P_a(s)$ are shown in Fig. 4. $W_1(s)$ and $W_2(s)$ used are the following transfer functions.

$$W_1(s) = \frac{0.12(1/\omega_0 s^2 + s)}{s^2 + 1.042s + 521.24^2},$$

$$W_2(s) = \frac{2.3(s+1)}{(s^2 + 12s + 36)^2}. \quad (8)$$

Figure 5 shows the frequency response of the obtained H^∞ controller.

3.3 Experiment and evaluation of controller performances

Using the designed H^∞ controller, we have realized the stable control of the magnetic levitation system. Up to now, three control methods have been applied to the magnetic levitation system, i.e., 1) PID analog control, 2) H^∞ control for robust stability

Fig. 5 H^∞ controller based on Eq. (5) and Fig. 4

(a) Analog PID control

(b) H^∞ control for robust stability problem (c) H^∞ control for mixed-sensitivity problem

Fig. 6 Magnitude spectrum for three cases with levitation

problem, 3) H^∞ control for mixed-sensitivity problem. Now let us compare their control performances from results of the experiments.

1) Comparison of the abilities of spillover suppression

Figure 6 shows the power spectrum of the guideway in levitation obtained using the three controllers. It is found that H^∞ controllers with both robust stability and the mixed-sensitivity have the ability to suppress the spillover while the PID controller does not.

2) Comparison of the performances of disturbance attenuations

The impulse responses for two cases, namely (a) analog PID control, (b) H^∞ control for mixed-sensitivity problem are shown in Fig. 7. Here, the H^∞ controller for robust stability problem has so weak stiffness against an impulse disturbance that we did not show the impulse response of this controller. It is thus clarified that the mixed-sensitivity H^∞ controller is superior to both PID analog controller and H^∞ controller for robust stability problem in disturbance attenuation. However, it is difficult to realize a closed-loop system exhibiting a strong stability toward H^∞ controller for robust stability problem. This means that the H^∞ controller with robust stability problem is not good control system.

(a) Analog PID control

(b) H^∞ control for mixed-sensitivity problem

Fig. 7 Impulse response for two cases

4. H^∞ Control System Design and Experiments for a Flexible Slider

In the above-mentioned control system design, we treated the levitation system (slider) as a simple rigid body; however, in some cases, the slider is not a rigid body. Like the guideway, it has a flexible structure, that is to say, both slider and guideway have natural frequencies. The purpose of this section is devise a way to reduce the interaction of these natural frequencies induced by magnetic forces during levitation between the flexible slider and the flexible guideway, and to realize a stable levitation without the spillover phenomenon.

4.1 Modeling and control system design

We derive the equations of motion and the state equations for Figs. 8(a) and (b) as follows.

(a) Case when the guideway is assumed to be a rigid body

$$\begin{aligned} M_1 \ddot{x}_1 &= -k_2(x_1 - x_2) - c_2(\dot{x}_1 - \dot{x}_2) \\ &\quad + 4f_0\left(\frac{i}{i_0} + \frac{x_1 - x_0}{h_0}\right) + w, \\ M_2 \ddot{x}_2 &= -k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1), \end{aligned} \quad (9)$$

$$\begin{aligned} \dot{X}_a &= A_a X_a + B_a u + D_a w, \\ Y_a &= C_a X_a, \end{aligned} \quad (10)$$

where, $X_a = [x_1 \ \dot{x}_1 \ x_2 \ \dot{x}_2 \ i]^T$,

$$A_a = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \left(\frac{4f_0}{M_1 h_0} - \frac{k_2}{M_1}\right) & -\frac{c_2}{M_1} & \frac{k_2}{M_1} & \frac{c_2}{M_1} & \frac{4f_0}{M_1 i_0} \\ 0 & 0 & 0 & 1 & 0 \\ \frac{k_2}{M_2} & \frac{c_2}{M_2} & -\frac{k_2}{M_2} & -\frac{c_2}{M_2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{R}{L} \end{bmatrix},$$

$$B_a = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{10}{L} \end{bmatrix}^T, \quad C_a = [1 \ 0 \ 0 \ 0 \ 0],$$

(a) Case when guideway is assumed to be rigid (b) Case when guideway is assumed to be flexible

Fig. 8 Equivalent model system for the X direction of Fig. 1

$$D_a = \begin{bmatrix} 0 & \frac{1}{M} & 0 & 0 & 0 \end{bmatrix}^T.$$

(b) Case when the guideway is assumed to be a flexible body

$$\begin{aligned} m \ddot{x}_0 &= -k_1 x_0 - c_1 \dot{x}_0 + 4f_0\left(\frac{i}{i_0} + \frac{x_1 - x_0}{h_0}\right), \\ M_1 \ddot{x}_1 &= -k_2(x_1 - x_2) - c_2(\dot{x}_1 - \dot{x}_2) \\ &\quad + 4f_0\left(\frac{i}{i_0} + \frac{x_1 - x_0}{h_0}\right) + w, \\ M_2 \ddot{x}_2 &= -k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1), \end{aligned} \quad (11)$$

$$\begin{aligned} \dot{X}_b &= A_b X_b + B_b u + D_b w, \\ Y_b &= C_b X_b, \end{aligned} \quad (12)$$

Where $X_b = [x_0 \ \dot{x}_0 \ x_1 \ \dot{x}_1 \ x_2 \ \dot{x}_2 \ i]^T$,

$$A_b = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \left(-\frac{4f_0}{mh_0} - \frac{K_1}{m}\right) & -\frac{c_1}{m} & \frac{4f_0}{mh_0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{4f_0}{M_1 h_0} & 0 & \left(\frac{4f_0}{M_1 h_0} - \frac{k_2}{M_1}\right) & -\frac{c_2}{M_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{k_2}{M_2} & \frac{c_2}{M_2} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_b = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{10}{L} \end{bmatrix}^T, \quad C_b = [-1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0],$$

$$D_b = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{M} & 0 & 0 & 0 \end{bmatrix}^T.$$

where x_0 is the absolute displacement of the guideway, x_1 the absolute displacement of the upper slider, x_2 the absolute displacement of the lower slider, and k_1 and k_2 are the equivalent spring constants for the guideway and the slider, c_1 and c_2 the equivalent damping constants of the guideway and slider, and M_1 and M_2 the masses of the upper slider and the lower slider, respectively. m is the equivalent mass in a single degree of freedom for the guideway. The natural frequency of the slider is 11 Hz, and the natural frequency of the guideway is 83 Hz.

With the same control system design method as that mentioned above, we designed a mixed-sensitivity H^∞ controller for the flexible slider system. Using the weighting functions $W_1(s)$ and $W_2(s)$ shown in Fig. 9, we obtained the H^∞ controller as shown in

Fig. 9 Frequency responses of uncertainty and weighting functions

Fig. 10 H^∞ controller based on Eq.(12) and Fig. 9

Fig. 12 Gain of measured frequency response function of the H^∞ controller

Fig. 13 Orbit of slider with levitation using FFT analyzer

(a) Response of guideway

(b) Response of upper slider

(c) Response of lower slider

Fig. 11 Transient responses when guideway is initially deviated 0.1 mm toward the X direction

Fig. 10.

4.2 Simulation and experiment

Here, we show the results of simulations and experiments. Figure 11 shows the transient responses when the guideway is initially deviated 0.1 mm toward the X direction. Figure 12 shows the gain of frequency response function of the implemented and measured H^∞ controller. Figure 13 shows the residual orbit of the upper slider in levitation. We can verify through simulations and experiments that the spillover phenomenon is successfully suppressed by the mixed-sensitivity H^∞ controller, whether the slider is rigid or not.

Fig. 14 Block diagram of μ synthesis control system Fig. 15 Loci of eigenvalues when the mass M is varied from 1.5 kg to 4.5 kg

5. Control System Design Based on μ Synthesis Theory and Its Performances

In this section, we attempt to design the control system⁽¹⁰⁾. Figure 14 shows the block diagram for the μ control system design. The H^∞ control system is apt to be a conservative system because the uncertainties are measured by maximum singular values. We can break down the conservativeness using the μ synthesis theory, since it uses a structured singular value instead of a maximum singular value for uncertainty.

Figure 15 shows the loci of eigenvalues when the mass M is varied from 1.5 kg to 4.5 kg. k_1 means the case of the H^∞ controller above mentioned and k_4 means that using the controller by the fourth D - K iteration. From this, it is found that though the system becomes unstable in the H^∞ controller, it is still stable in the μ controller. The robust stability and robust performance are improved in the case of μ control.

6. Conclusions

We have verified a method for spillover suppression for a magnetic levitation system by applying H^∞ control. The conclusions are summarized as follows.

(1) We have realized the magnetic levitation system using the H^∞ controller for robust stability problem, and also suppressed the spillover. In particular, the magnetic levitation system was realized by the H^∞ controller based on mathematical model without the need for trial and error like tuning. This means that H^∞ control has strong robustness for parameter uncertainty.

(2) Although it is difficult to realize a closed-loop system with high stability for the robust stability H^∞ controller in the case when the guideway is flexible, the mixed-sensitivity H^∞ controller realizes a stable closed-loop system which is superior to that using a PID controller.

(3) In the case when both slider and guideway are flexible the H^∞ controller for the mixed-sensitivity problem exhibits robustness and superior control performance. By applying μ synthesis to this magnetic levitation system, its robust performance improves compared with the H^∞ controller for the mixed-sensitivity problem. We will verify their performances by means of experiments in a future study.

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