Optimum Motion Control of a Redundant Robot with the Objective Function of Dexterity*

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This paper describes an optimum motion control method for robotic serial manipulators with multiple redundant degrees of freedom. The manipulability measure, which is given by a function of the Jacobian matrix concerning the position and posture of the end effector of a serial manipulator, is proposed as the index to evaluate the dexterity of the manipulator, and it is defined as most dexterous when the manipulability measure takes the maximum value. New methods to drive serial manipulators most dexterously, namely the optimum CP control methods with and without consideration of the posture angle of the end effector, are proposed and experimentally examined for planar 3R-, 4R- and 5R-redundant manipulators. Consequently, a better result was obtained by the optimum CP control method with consideration of the posture angle than by the method without consideration of the posture angle: the manipulators were driven most dexterously over most of the driving range, and high response was obtained.

Key Words: Robot, Manipulator, Motion Control, Dexterity, Redundant Manipulator, Optimum Control, Manipulability

1. Introduction

Future robots such as welfare robots and leisure robots working in our daily life will be composed of mechanisms with huge degrees of freedom and will be required to engage in dexterous motion. It is hence desirable to quantitatively evaluate the dexterities of the mechanisms and then to determine the dimensions and to design the control systems based on the evaluation.

The measures for evaluating the dexterity of a robotic mechanism, which have been reported by many researchers, are divided into two groups^{(1),(2)}: measures for evaluating the workspace⁽³⁾⁻⁽⁶⁾, and

measures for evaluating the manipulation ability of a robotic mechanism^{(7),(8)}. The first group of measures involve a difficulty in that inverse kinematics makes it very difficult to calculate the workspace; on the contrary, the second group of measures can be easily calculated with the Jacobian matrix of a robotic mechanism.

To achieve optimum motion control of a robotic manipulator, most of the papers have aimed to minimize the operating period⁽⁹⁾ or the energy consumption⁽¹⁰⁾. Some papers reported the optimum control of a robotic manipulator based on dexterity measures^{(11),(12)}, but they reported computer simulations only and no experiments on real-time control.

In this paper, the manipulability measure, which is given by a function of the Jacobian matrix with respect to both the position and the posture of the end effector, is proposed as an index to evaluate the dexterity of a robotic manipulator quantitatively. Also, the optimum motion control is proposed to drive a redundant manipulator in the most dexterous state based on the index, and was experimentally examined for planar 3R-, 4R-, 5R-redundant manipulators. In addition, the relation between the redundant degree of

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freedom and the effectiveness of the proposed control methods was investigated.

2. Manipulability Measure

The velocity vector, $\dot{\mathbf{r}}_p$, of the end effector of a robotic manipulator is described with the translational velocity, $\dot{\mathbf{p}}$, at the operating point P on the end effector, and the angular velocity, $\dot{\phi}$, of the end effector as

$$\dot{\mathbf{r}}_p = \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\phi} \end{bmatrix} = \mathbf{J}_p \dot{\boldsymbol{\theta}} \quad (1)$$

where $\mathbf{J}_p$ is a Jacobian matrix, and $\dot{\boldsymbol{\theta}}$ is a joint input velocity vector. Manipulability measure, w_p , to evaluate the dexterity of the manipulator quantitatively is defined as

$$w_p = \sqrt{\det(\mathbf{J}_p \cdot \mathbf{J}_p^T)} \quad (2)$$

It is well known that the manipulability measure is proportional to the volume of a manipulability ellipsoid which is formed by connecting the end point of output velocity vectors generated by the combinations of joint input velocities⁽¹³⁾. The manipulability measure, w_p , defined by Eq.(2) hence indicates the magnitude of velocities which can be generated in all directions, including posture angular velocities. In other words, w_p shows the ability of the manipulator to handle objects. Thus, w_p is adopted as the index to evaluate the dexterity of a redundant manipulator, and it is defined as most dexterous when w_p takes the maximum value, $w_{p,\max}$, while the manipulator moves and the end effector reaches an object.

As an example, in Fig. 2, $w_{p,\max}$ are shown for 2R-, 3R-, 4R-, and 5R-serial planar manipulators with the same workspace (see Fig. 1), when their operating points P move along the X-axis. For the 2R-manipulator, which is not a redundant manipulator, $w_{p,\max}$ is always zero. For the other manipulators, which are all redundant manipulators, $w_{p,\max}$ is not zero and increases as the redundant degree of freedom increases. It is thus found that the maximum value, $w_{p,\max}$, of the manipulability measure defined by Eq. (2) reflects the redundancy of a manipulator and is suitable to evaluate the dexterity of the manipulator.

3. Serial Planar Manipulators and Their Control Systems

3.1 Manipulators tested

The 2R-, 3R-, 4R-, 5R-serial planar manipulators with the same workspace shown in Fig. 1 were made of aluminum for the experiment. Their total length, l , was 432 mm. All their joints were driven by D.C. servomotors with reduction gear mechanisms. Figure 3 shows a photograph of the 4R-manipulator.

3.2 Hardware of control system

In Fig. 4 is shown a schematic diagram of a control system composed of a host controller and five local controllers assigned to each joint. The host controller, which is composed of a personal computer

$$2R : l_1 : l_2 = 1 : 1$$

$$3R : l_1 : l_2 : l_3 = 1 : 0.5 : 0.5$$

$$4R : l_1 : l_2 : l_3 : l_4 = 1 : 0.5 : 0.25 : 0.25$$

$$5R : l_1 : l_2 : l_3 : l_4 : l_5 = 1 : 0.5 : 0.25 : 0.125 : 0.125$$

Fig. 1 Serial planar manipulators and a workspace

Fig. 2 The maximum manipulability measures of 2R-, ~5R-serial planar manipulators

Fig. 3 A prototype of a 4R-serial planar manipulator

(i 80286, 16 MHz) and a transputer (T 800, 25 MHz), calculates all the joint angles with which a manipulator becomes most dexterous with respect to an operating point on a desired trajectory along which the end effector moves, and outputs the calculated angles as the desired values for the local controllers. Each local controller, which is composed of a single-board computer (Z 80, 4 MHz), obtains the calculated joint angle, and independently controls the position of the corresponding joint. A combination of proportional position feedback and integral-proportional velocity feedback⁽¹⁴⁾ was used in this positioning control. In Fig. 5 is shown a block diagram of a local controller.

3.3 Calculation for optimization

The maximum value of the manipulability measure, $w_{p,max}$, is calculated under the constraint that the operating point P has to lie at a specified position (X_p, Y_p) .

In Fig. 6, J_i ($i=0 \sim n$) denotes the i th joint of a serial planar manipulator, where n is the number of

Fig. 4 The block diagram of the control system

Fig. 5 The block diagram of a local controller

Fig. 6 Parameters for a serial planar manipulator

links. J_0 coincides with the origin of the fixed coordinate system, and J_n coincides with the operating point P. The input joint angle, θ_i , of the i th joint is given as

$$\theta_i = \phi_i - \phi_{i-1} \quad (3)$$

where ϕ_i denotes the posture angle of the i th link, which is measured from the X -axis. The position, (X_i, Y_i) , of the i th joint is given as

$$\begin{cases} X_i = \sum_{k=1}^i l_k \cos \phi_k \\ Y_i = \sum_{k=1}^i l_k \sin \phi_k \end{cases} \quad (4)$$

where l_k denotes the length of the k th link.

When the posture angles, ϕ_i ($i=1 \sim n-2$), are given, the positions of J_i ($i=1 \sim n-2$) are determined by Eq. (4). The position of J_{n-1} is then determined as the position from which J_{n-2} is connected with the $(n-1)$ th link of l_{n-1} length, and J_n , the position of which is a specified point on a desired trajectory, is connected with the n th link of l_n length.

The posture angles of the $(n-1)$ th and n th links are given by applying the second cosine formula to $\Delta PJ_{n-1}J_{n-2}$ as

$$\begin{cases} \phi_{n-1} = \alpha \pm \beta \\ \phi_n = \phi_{n-1} \pm (\gamma - \pi) \end{cases} \quad (5)$$

where

$$\begin{aligned} \alpha &= \tan^{-1} \frac{Y_p - Y_{n-2}}{X_p - X_{n-2}} \\ \beta &= \cos^{-1} \frac{L^2 + l_{n-1}^2 - l_n^2}{2Ll_{n-1}} \\ \gamma &= \cos^{-1} \frac{l_{n-1}^2 + l_n^2 - L^2}{2l_{n-1}l_n} \end{aligned}$$

$$L = \sqrt{(X_p - X_{n-2})^2 + (Y_p - Y_{n-2})^2}.$$

The posture angles, ϕ_i ($i=1 \sim n$), of all links can hence be determined, and then the elements of the Jacobian matrix of the serial planar manipulator are given as

$$\begin{cases} J_{p,1i} = -\sum_{k=1}^n l_k \sin \phi_k \\ J_{p,2i} = \sum_{k=1}^n l_k \cos \phi_k \\ J_{p,3i} = 1 \end{cases} \quad (6)$$

where, $i=1, 2, \dots, n$. The manipulability measure, w_p , is calculated by substituting Eq. (6) into Eq. (2).

The objective function, w_p , can thus be calculated with the optimization variables, ϕ_i ($i=1 \sim n-2$), the number of which is the same as that of the redundant degree of freedom of the manipulator, while the constraint is completely satisfied.

The control program to optimize w_p with the simplex method⁽¹⁵⁾ was described with the C language of the transputer.

4. Experiments and Discussion

4.1 Optimum CP control

Optimum CP control, by which the configuration

of the manipulator is determined so that w_p always keeps its maximum value while the operating point moves on a given trajectory, $(X_p(t), Y_p(t))$, was attempted for five straight trajectories, A~E, connecting from one boundary point to another boundary point of a workspace, as shown in Fig. 7. The trajectories lie equally spaced in the X -direction.

For each trajectory, $X_p(t)$ was a constant, and $Y_p(t)$ was determined by a double dwelling function as follows

$$\left. \begin{aligned} X_p(t) &= X_b = \text{const.} \\ Y_p(t) &= 2Y_b \left(\frac{6t^5}{T^5} - \frac{10t^4}{T^4} + \frac{15t^3}{T^3} \right) - Y_b \end{aligned} \right\} \quad (7)$$

Fig. 7 Examined trajectories

where T is the operating period, and (X_b, Y_b) is the coordinates of a boundary point.

Figure 8 shows the experimental results obtained for the 4R-manipulator moving on the trajectory B with an operating period of 8 second. Figure 8(a) shows the skeleton diagrams of the manipulator every 0.6 second. In the figure, ● indicates the operating points. Figure 8(b) shows the position vs time of the operating point and the posture angle vs time of the end effector. Figure 8(c) shows joint angles, $\theta_1 \sim \theta_4$, vs time. In Fig. 8(d), the solid line is w_p calculated with the experimental joint angles, and the broken line is $w_p/w_{p,\max}$, where $w_{p,\max}$ was theoretically calculated.

As clearly seen from Figs. 8(a) and 8(b), the operating point of the manipulator accurately moved on the trajectory B; also, the manipulator moved

Fig. 8 An example of the experimental results obtained by the optimum CP control without consideration of posture angle (4R-manipulator)

with the most dexterous configuration, since $w_p/w_{p,\max}$ was always unity, as shown in Fig. 8(d). The optimum CP control was hence realized.

The time derivatives of the joint angles, $\theta_1 \sim \theta_4$, varied discontinuously at some points, as seen in Fig. 8(c). Accordingly, the time derivative of the posture angle, ϕ_n , of the end effector also varied discontinuously at some points, as seen in Fig. 8(a). In other words, there is a possibility that vibrations are induced on the manipulators as their operating periods become shorter.

The optimum CP control was similarly examined for 3R- and 5R-manipulators. They also moved with the most dexterous configuration on the trajectories. The calculation times to obtain an optimum configuration for a certain operating point were 15 ms for the 3R-manipulator, 60 ms for the 4R-manipulator, and 300 ms for the 5R-manipulator.

Figure 9 shows dexterity per redundant degree of freedom, $w_p/(n-2)$, of 3R- ~ 5R-manipulators. As seen in the figure, the dexterity per redundant degree of freedom slightly decreases as redundancy increases.

4.2 Evaluation of response

There is a possibility that accurate response for positioning is restrained by the complicated changes of joint angles including the discontinuities. The manipulator cannot trace the trajectories accurately at a high speed since the actuators of the joints cannot respond to the large variations of angular acceleration at those discontinuities.

Figure 10 shows the evaluation of responses for the 4R-manipulator. The horizontal axis is d/T , the average velocity of the operating point, where d is the distance of motion. The vertical axis is t_g/T , where t_g is the period in which the manipulator moves accurately on the trajectory. As clearly seen in the figure, the manipulator moves accurately on all the trajectories up to an average velocity, 0.1, since t_g/T is 1.0; but the manipulator partly strays from the trajec-

Fig. 9 Dexterity per redundant degree of freedom for 3R-, 4R-, and 5R-manipulators

tories, except for trajectory E, when the average velocity becomes larger than 0.1. Also, the fraction of time during which the manipulator strays from the trajectories becomes larger as they approach joint J_0 .

Figure 11 shows the responses of 3R- ~ 5R-manipulators for the trajectory B. Among them, the response of the 4R-manipulator is the best, and that of the 3R-manipulator is the worst. This is because as the degree of freedom increases, the actuator mounted at joint J_0 cannot respond accurately due to the large moment of inertia of the manipulator mounted with many actuators. Especially for the 3R-manipulator, the actuator mounted at joint J_2 cannot respond well because the moment of inertia of the end link is large.

4.3 Optimum CP control with consideration of posture angle

Optimum CP control with consideration of the posture angle of the end effector was attempted in order to avoid the discontinuity problem. At the start and end points of the positioning period of a manipulator, its configuration is determined to be the most dexterous, as explained in the previous section; and, between the two points, the configuration is determined so that the posture angle of the end effector becomes the value given by the following double dwelling function:

Fig. 10 The response of 4R-manipulator

Fig. 11 The responses of 3R-, 4R-, and 5R-manipulators

Fig. 12 An example of the experimental results obtained by the optimum CP control with consideration of posture angle (4R-manipulator)

$$\phi_n(t) = (\phi_{n,f} - \phi_{n,s}) \left(\frac{6t^5}{T^5} - \frac{10t^4}{T^4} + \frac{15t^3}{T^3} \right) + \phi_{n,s} \quad (8)$$

where $\phi_{n,s}$ and $\phi_{n,f}$ denote the posture angles at the start and end points of the positioning period, respectively.

In this control method, the posture angles, ϕ_i ($i = 1 \sim n-3$), are given, unlike in the previous section, since the posture angle, ϕ_n , of the end effector is given by Eq. (8). The position, (X_{n-1}, Y_{n-1}) , of J_{n-1} is determined as

$$\begin{aligned} X_{n-1} &= X_p - l_n \cos \phi_n \\ Y_{n-1} &= Y_p - l_n \sin \phi_n \end{aligned} \quad (9)$$

since ϕ_n is given. The position of J_{n-2} is then determined as the position from which J_{n-3} is connected with the $(n-2)$ th link of l_{n-2} length and J_{n-1} is connected with the $(n-1)$ th link of l_{n-1} length. The posture angles, ϕ_{n-2} and ϕ_{n-1} , can be obtained by applying the second cosine formula to $\Delta J_{n-1}J_{n-2}J_{n-3}$. Finally the Jacobian matrix can be calculated by Eq. (6), and then the manipulability measure (the objective function), w_p , can be calculated.

Figure 12 shows an example of the experimental results obtained when the 4R-manipulator was driven on trajectory B for 5 second. Figure 12(a) shows the skeleton diagrams of the manipulator; Figure 12(b) shows the displacements of the operating point and the posture angle of the end effector; Figure 12(c)

Fig. 13 Dexterity obtained by the optimum CP control with consideration of posture angle

shows the angular joint input displacements.

As seen in Fig. 12(b), the end effector moved accurately according to the functions given by Eqs. (7) and (8). Also, all of the joint angles varied smoothly, as seen in Fig. 12(c). Smooth motion was similarly obtained by 3R- and 5R-manipulators. Thus, the optimum CP control with consideration of posture angle was effective.

In Fig. 13 is shown $w_p/w_{p,max}$ of the 4R-manipulator for the trajectories B~E, where w_p is the experimental manipulability measure, and $w_{p,max}$ is the theoretical value. The manipulability measure took the maximum value except for the range around $t/T = 0.5$ for all of the trajectories. This means that the

Fig. 14 The improved responses of the manipulators

manipulators took the least dexterous configurations when their operating points crossed the X -axis. Especially for the trajectory A, the manipulators took a singular configuration when they crossed the X -axis.

In Fig. 14 are shown the results of 3R-, 4R-, 5R-manipulators controlled by this method for trajectory B. Symbols $\circ$, $\triangle$, $\square$ and $\bullet$, $\blacktriangle$, $\blacksquare$ show the results obtained with the optimum control, with and without consideration of posture angle, respectively. As clearly seen, the responses for this method were fairly good. This is because there was no discontinuity in the time derivatives of joint angles.

Similar results were obtained for the other trajectories, C, D and E. The optimum CP control with consideration of posture angle was thus found to be effective and useful.

5. Conclusions

Two new optimum motion control methods were proposed to drive a serial planar manipulator most dexterously by using the objective function of the manipulability measure, and were experimentally examined. The main results obtained are as follows.

(1) For the optimum CP control method without consideration of posture angle, the manipulators moved most dexterously over the whole positioning period; but there were observed discontinuities of joint input angular velocity which would induce vibrations on the manipulator because of a large variation of acceleration.

(2) For the optimum CP control method with consideration of posture angle, the manipulators moved most dexterously over almost the whole positioning period. In addition, the operating points of the end effector traced the specified trajectories accurately; in other words, high response was obtained since the joint angles varied smoothly.

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