Analysis of Coupled Vibration between Railway Vehicle Wheels and a Rail in Both Vertical and Horizontal Directions Using an Approximate Model with an Infinite Number of Vehicles*

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Rolling noise arises from the interaction of the vibration characteristics of rails and railway vehicle wheels through the points of contact. This paper deals with the coupled linear vibration between wheels and a rail using an approximate model with an infinite number of vehicles which are arranged with a certain periodicity on an infinitely long and straight rail installed on three distributed springs, taking the coupling in both vertical and horizontal directions into account. The vertical irregularity on the head surface of the rail is regarded as a source of excitation. As a result, it was clarified that (1) the vertical vibration of the rail, the horizontal vibrations of wheel and rail, and the vertical vibration of the wheel contribute to the rolling noise according to the high vibration level, and (2) the vibration levels are high on the rail within the narrow intervals between adjacent wheels.

Key Words: Vibration of Continuous System, Railway, Coupled Vibration, Vibration, Rail and Wheel, Rolling Noise

1. Introduction

The authors formulated and analyzed the coupled vibrations between railway vehicle wheels and rail in the vertical and the horizontal directions independently in the previous reports^{(1),(2)}, using an approximate model with an infinite number of vehicles. The approximate model is an analytical one in which the infinite vehicles are coupled in a regular arrangement and travel at a constant speed on the head of an infinitely long rail, as shown in Fig. 1.

Since the vibration characteristics of wheels and rail are almost periodic with the period of one vehicle in the actual train composition coupling a number of cars^{(3),(4)}, applying the approximate model with an infinite number of vehicles has the following great

advantages:

(1) the periodic vibration characteristics between wheels and rail under the intermediate vehicles can be effectively extracted by use of this approximate model, and

(2) the computation time to analyze the present coupled vibrations can be markedly reduced by use of the approximate model, compared with the analysis by the model of the actual train composition with a great number of cars.

In this report, the authors treat analytically the coupled vibrations between railway vehicle wheels and a rail in both vertical and horizontal directions using the approximate model with an infinite number

Fig. 1 Arrangement of the wheels (units in m)

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of vehicles, regarding the forced displacement attributable to the irregularity on the head surface of the rail in the vertical direction as an exciting source, and they present some numerical computational results applied to the Shinkansen.

In the analysis, the rail is infinitely long and is modelled by a straight Timoshenko beam installed on distributed springs. In the vertical direction, the wheels are regarded as rigid bodies neglecting the in-plane deformation, but the bending deformation of the rail is considered. In the horizontal direction, the rotating wheels are regarded as Mindlin's circular-plates, and the coupling between bending and torsion is considered in the deformation of the rail. The elastic contact spring force and the creep force attributable to the rolling contact between wheel and rail act at every point of contact in the vertical and the horizontal directions, respectively.

2. Fundamental Equations of Rail and Wheel in the Vertical and the Horizontal Directions

As shown in Fig. 2, the moving coordinate $\bar{x} = x - vt$ (t is time) is equally divided into an infinite number of sections of finite length l , and in every section, an equal number of wheels are arranged in the same arrangement. Every wheel travels in the longitudinal direction of the rail (the x -direction) at a constant speed v . That is, we treat a system in which such unit sections are repeated indefinitely along an infinitely long rail.

We define the local coordinate from the left side of each section as ξ ($0 \leq \xi \leq l$). In every section, N wheels are arranged and the corresponding local coordinates are denoted as ξ_n ($n=1, \dots, N$).

First, for preparation of the analysis of the coupled vibrations in both directions using the approximate model with an infinite number of vehicles, we summarize the fundamental equations of the responses for the wheel and the rail in the vertical and the horizontal directions by virtue of dynamic

Fig. 2 Approximate model with an infinite number of vehicles, and shearing forces and moments acting at point of contact

influence coefficients which have been obtained independently in the previous reports^{(1),(2)}. In the following, the notations with subscripts “ v ” and “ h ” represent the physical variables in the vertical and the horizontal directions, respectively. Some assumptions used in the present analysis are the same as in the previous reports⁽⁴⁾⁻⁽⁷⁾.

2.1 Fundamental equation of rail^{(1),(2)}

The rail is an infinitely long Timoshenko beam and the cross section is symmetric with respect to the vertical axis. The rail is supported with the foundation composed of the shearing distributed spring constants in the vertical and the horizontal directions k_z and k_y , respectively, and the torsional distributed spring constant k_ϕ (see Fig. 3).

We consider the case in which the shearing forces $p^n(t)$ and $q_r^n(t)$, the torsional moment $m_r^n(t)$ and the bending moment $m_{r\theta}^n(t)$ act on the center line of the railhead at the position $\xi = \xi_n$ in every section simultaneously, as shown in Fig. 2.

$$\begin{aligned} & [p(t), q_r(t), m_r(t), m_{r\theta}(t)]^n \\ &= [\bar{p}, \bar{q}_r, \bar{m}_r, \bar{m}_{r\theta}]^n \exp(j\omega t) \end{aligned} \quad (1)$$

where ω is a circular frequency and $j^2 = -1$.

In what follows, unless otherwise specified, the fluctuating physical quantities are represented by the product of amplitude of the same symbols with overbar and $\exp(j\omega t)$. We now focus our attention only on the 0-th section, which is a representative section, and then the superscript 0 is omitted here. The displacement response in the K -th section is obtained from the product of that in the 0-th section and $\exp(jK\omega l/v)$ ^{(1),(2)}.

In the case that the wheels roll on the rail, the

Fig. 3 Contact model between wheel and rail

creep forces should occur at the points of contact between wheels and rail. According to Kalker's linear creep theorem⁽⁸⁾, not only the transverse forces $q_r^n(t)$ but also the spin moments $m_{ro}^n(t)$ act at the points of contact as creep forces relating to the coupled vibration in the horizontal direction. However, it was found that the effect of the latter is very slight⁽²⁾, and hence we neglect it here. That is, in this report, only the transverse creep force is considered in the analysis for simplicity.

2.1.1 Vertical direction The amplitude $\bar{z}$ of the vertical displacement $z(\xi, t)$ at any local coordinate ξ of the rail in the 0-th section is given as

$$\bar{z}(\xi) = j\Gamma_v \sum_{n=1}^N \bar{p}_n \bar{f}_v(\xi - \xi_n, A_v) \quad (2)$$

where $\Gamma_v = 4\beta_v/k_z$, $\beta_v^4 = k_z/4(EI)_v$,

$$\bar{f}_v(\eta, A_v) = \begin{cases} f_{4+}(\eta, A_v, \alpha_v, \beta_v) : 0 \leq \eta \leq l \\ f_{4-}(\eta, A_v, \alpha_v, \beta_v) : -l \leq \eta \leq 0 \end{cases}$$

$$f_{j\pm}(\eta, A, \alpha, \beta) = \pm \sum_{i=1}^4 \frac{A_i \exp(j\alpha_i \beta \eta)}{1 - \exp[\pm j(\alpha_i \beta - \omega/v)l]} \quad (3)$$

EI the flexural rigidity, and the dual signs involved in the function $f_{j\pm}$ appear in the same order. Furthermore, α_{vi} and A_{vi} ($i=1, \dots, 4$) are roots of $\Delta=0$ of Eq. (4) and complex constants of Eq. (5) in the previous report⁽³⁾, respectively, and both are functions of ω . By substituting $\xi = \xi_k$ ($k=1, \dots, N$) into Eq. (2), the amplitudes at the applied points of the shearing forces may be determined. These are represented in the matrix form as

$$\bar{\mathbf{Z}} = \mathbf{G} \bar{\mathbf{P}} \quad (4)$$

where $k, n=1, \dots, N$,

$$\bar{\mathbf{Z}} = {}^t(\bar{z}_1, \dots, \bar{z}_N), \bar{\mathbf{P}} = {}^t(\bar{p}^1, \dots, \bar{p}^N)$$

$$\mathbf{G} = \begin{bmatrix} G_{11} & \dots & G_{1N} \\ \vdots & \ddots & \vdots \\ G_{N1} & \dots & G_{NN} \end{bmatrix} \quad (5)$$

$$G_{kn} = j\Gamma_v \bar{f}_v(\xi_k - \xi_n, A_v)$$

and the symbol t denotes the transposition.

The element G_{kn} represents the displacement amplitude at the k -th acting point when a shearing force of unit magnitude is applied to the n -th one. It is the so-called dynamic influence coefficient among the acting points defined in each section and it is a function of ω .

2.1.2 Horizontal direction The amplitudes of the lateral displacement $y(\xi, t)$ and the torsional one $\phi(\xi, t)$ at any local coordinate ξ of the rail in the 0-th section are given as

$$\begin{bmatrix} \bar{y}/e' \\ \bar{\phi} \end{bmatrix} = \sum_{n=1}^N \begin{bmatrix} \tilde{h}_{11} & \beta_h \tilde{h}_{12} \\ \tilde{h}_{31} & \beta_h \tilde{h}_{32} \end{bmatrix}_n \begin{bmatrix} \bar{q}_r \\ \bar{m}_r \end{bmatrix}_n \quad (6)$$

where e' is a dimension of the cross section of the rail (see Fig. 3),

$$\tilde{h}_{pq,n} = \tilde{h}_{pq}(\xi, \xi_n) = j\Gamma_h \tilde{f}_h(\xi - \xi_n, A_{pq})$$

$$\tilde{f}_h(\eta, A_{pq}) = \begin{cases} f_{8+}(\eta, A_{pq}, \alpha_h, \beta_h) : 0 \leq \eta \leq l \\ f_{8-}(\eta, A_{pq}, \alpha_h, \beta_h) : -l \leq \eta \leq 0 \end{cases}$$

$$\Gamma_h = \beta_h/k_y e', \beta_h^4 = k_y/4(EI)_h \quad (7)$$

$p=1, 3$ and $q=1, 2$. α_{hi} and A_{pqi} ($i=1, \dots, 8$) are characteristic roots in Eqs. (4) and (5) and complex constants in the previous report⁽⁴⁾, respectively.

The elements of the matrix on the right-hand side of Eq. (6) represent the lateral displacement amplitude and the torsional one at any local coordinate ξ when shearing forces and moments of unit magnitude are applied to the point ξ_n . These are the so-called dynamic influence coefficients and are functions of ω .

From Eq. (6), the amplitude of the displacement in the horizontal direction at the top of the rail $y_T(\xi, t) = y(\xi, t) - e' \phi(\xi, t)$ is given as

$$\frac{\bar{y}_T}{e'} = \sum_{n=1}^N (\tilde{H}_{1,n} \bar{q}_r^n + \tilde{H}_{2,n} \bar{m}_r^n) \quad (8)$$

where

$$\tilde{H}_{1,n} = (\tilde{h}_{11} - \tilde{h}_{31})_{,n}, \tilde{H}_{2,n} = \beta_n (\tilde{h}_{12} - \tilde{h}_{32})_{,n} \quad (9)$$

The absolute velocity y'_T on the center line of the railhead is obtained from

$$y'_T = \left(\frac{\partial}{\partial t} - v \frac{\partial}{\partial \xi} \right) y_T \quad (10)$$

Multiplying both sides of Eq. (8) by $\exp(j\omega t)$, the resulting expression is substituted into Eq. (10). Then substituting $\xi = \xi_k$ into the resulting Eq. (10) yields the absolute velocity amplitudes of the top of the rail in the horizontal direction at the acting points of the lateral shearing force and the moment in the form as

$$\bar{\mathbf{Y}}'_T = \tilde{\mathbf{H}}'_1 \bar{\mathbf{Q}}_r + \tilde{\mathbf{H}}'_2 \bar{\mathbf{M}}_r \quad (11)$$

where y'_{rk} is the absolute velocity at the k -th point of contact, and

$$\bar{\mathbf{Y}}'_T = {}^t(\bar{y}'_{T1}/e', \dots, \bar{y}'_{TN}/e')$$

$$\bar{\mathbf{Q}}_r = {}^t(\bar{q}_r^1, \dots, \bar{q}_r^N), \bar{\mathbf{M}}_r = {}^t(\bar{m}_r^1, \dots, \bar{m}_r^N) \quad (12)$$

Furthermore, $\tilde{\mathbf{H}}'_1 = [H'_{1,kn}]$ and $\tilde{\mathbf{H}}'_2 = [H'_{2,kn}]$ are $N \times N$ dimensional matrices whose element of (k, n) is given as follows:

$$\tilde{H}'_{1,kn} = (\tilde{h}'_{11} - \tilde{h}'_{31})_{,kn}, \tilde{H}'_{2,kn} = \beta_h (\tilde{h}'_{12} - \tilde{h}'_{32})_{,kn}$$

$$\tilde{h}_{pq,kn} = j\Gamma_h \tilde{f}_h(\xi_k - \xi_n, A_{pq})$$

$$\tilde{f}'_h(\eta, A_{pq}) = j\omega \tilde{f}_h(\eta, A_{pq}) - jv \beta_h \tilde{f}_h(\eta, \alpha A_{pq}) \quad (13)$$

From Eq. (6), the amplitude of the torsional angle at the acting points is represented by

$$\bar{\boldsymbol{\Phi}} = \tilde{\mathbf{H}}_{31} \bar{\mathbf{Q}}_r + \tilde{\mathbf{H}}_{32} \bar{\mathbf{M}}_r \quad (14)$$

where

$$\bar{\boldsymbol{\Phi}} = {}^t(\bar{\phi}_1, \dots, \bar{\phi}_N)$$

$$\tilde{\mathbf{H}}_{31} = [\tilde{h}_{31,kn}], \tilde{\mathbf{H}}_{32} = [\beta_h \tilde{h}_{32,kn}]$$

$$\tilde{h}_{pq,kn} = j\Gamma_h \tilde{f}_h(\xi_k - \xi_n, A_{pq}) \quad (15)$$

2.2 Fundamental equation of wheel⁽⁵⁾

Every wheel has the same shape. The outer radius of the wheel is R and the wheel is rotating at an angular velocity $\Omega_r (=v/R)$. In the horizontal direction, the wheel is composed of uniform rings and the axial vibration of the wheel is analyzed by applying

Mindlin's circular plate theorem to each ring. On the other hand, in the vertical direction, the wheel is regarded as a rigid body having the mass of M , since the in-plane rigidity of the wheel is high and the contribution of the in-plane vibration to the rolling noise seems to be small.

2.2.1 Vertical direction Letting the vertical displacement of the n -th wheel be $w_v^n(t)$ and putting $\bar{W}_v = {}^t(\bar{w}^1, \dots, \bar{w}^N)_v$, we obtain

$$\bar{W}_v = S\bar{P} = \text{diag}(S, \dots, S)\bar{P} \quad (16)$$

where $S = -1/M\omega^2$ is the vertical dynamic influence coefficient.

2.2.2 Horizontal direction For the case in which a shearing force and a moment act at the point $(r, \bar{\theta}) = (R, 0)$ fixed in space on the outer circumference of the wheel, the amplitude of the axial displacement of the n -th wheel $w_h^n(r, \bar{\theta}, t)$ is given as

$$\bar{w}_h^n = u_1(r, \bar{\theta})\bar{q}^n + u_2(r, \bar{\theta})\bar{m}^n \quad (17)$$

where r is the radial coordinate, θ and $\bar{\theta}$ are circumferential coordinates fixed on the wheel and fixed in space respectively, and $\bar{\theta} = \Omega_r t + \theta$. Moreover, u_1 and u_2 are the dynamic influence coefficients and they are given as

$$\begin{aligned} u_1(r, \bar{\theta}) &= \sum_{ms} \bar{w}_{h,ms}(R) \bar{w}_{h,ms}(r) \Gamma_{ms}(\bar{\theta}) \\ u_2(r, \bar{\theta}) &= \sum_{ms} \bar{\psi}_{r,ms}(R) \bar{w}_{h,ms}(r) \Gamma_{ms}(\bar{\theta}) \\ \Gamma_{ms}(\bar{\theta}) &= \frac{1}{2} \left[\frac{\exp(jm\bar{\theta})}{\gamma_{ms}(\omega + m\Omega_r)} + \frac{\exp(-jm\bar{\theta})}{\gamma_{ms}(\omega - m\Omega_r)} \right] \\ \gamma_{ms}(\Omega) &= \omega_{ms}^2 - \Omega^2 + j2\zeta_{ms}\Omega\omega_{ms} \end{aligned} \quad (18)$$

where $\sum_{ms}$ is denoted as double summation with respect to m (the number of nodal diameters) and s (the number of nodal circles), $m, s = 0, \dots, \infty$, ω_{ms} is a natural circular frequency of (m, s) -mode, ζ_{ms} is a damping ratio for the (m, s) -mode, and $\bar{\psi}_{r,ms}$ and $\bar{w}_{h,ms}$ are the normalized shape functions in the radial direction of the (m, s) -mode for ψ_r^n (angular displacement around the θ -axis) and w_h^n (axial displacement), respectively.

The absolute velocity of the wheel in the horizontal direction is given as

$$w_h'^n = \left(\frac{\partial}{\partial t} + \Omega_r \frac{\partial}{\partial \bar{\theta}} \right) w_h^n \quad (19)$$

Multiplying both sides of Eq.(17) by $\exp(j\omega t)$, the resulting expression is substituted into Eq.(19).

Substituting $r=R$, $\bar{\theta}=0$ into the resulting Eq.(19) yields

$$\bar{W}_h' = U_1' \bar{Q}_r + U_2' \bar{M}_r \quad (20)$$

where

$$\begin{aligned} U_q' &= [j\omega u_q(R, 0) + \Omega_r u_q'(R, 0)] \mathbf{E} \\ u_1'(r, \bar{\theta}) &= \sum_{ms} \bar{w}_{h,ms}(R) \bar{w}_{h,ms}(r) \Gamma_{ms}'(\bar{\theta}) \\ u_2'(r, \bar{\theta}) &= \sum_{ms} \bar{\psi}_{r,ms}(R) \bar{w}_{h,ms}(r) \Gamma_{ms}'(\bar{\theta}) \\ \bar{W}_h' &= {}^t(\bar{w}'^1, \dots, \bar{w}'^N)_h, \mathbf{E} \text{ is unit matrix of order } N, \\ \Gamma_{ms}'(\bar{\theta}) &= d\Gamma_{ms}(\bar{\theta})/d\bar{\theta} \text{ and } q=1, 2 \end{aligned} \quad (21)$$

3. Analysis of Coupled Vibrations in Both Vertical and Horizontal Directions

3.1 Exciting source of vibration⁽⁹⁾

In this report, we analyze the steady-state vibration taking the irregularity of the rail in the vertical direction into account as a representative exciting source of the rolling noise. That is, we consider the periodic vibration system in which an arrangement of the wheels at regular intervals continues infinitely on both sides repeatedly with the period of length l . The wheels travel by rolling simultaneously at a constant speed v along the railhead with the irregularity which is a cause of the forced displacement in the vertical direction $a \exp(j\omega t)$, where a is the amplitude of the roughness in the vicinity of the railhead, $\omega = 2\pi v/\lambda$, and λ is the wavelength of the roughness.

3.2 Contact model between wheel and rail

In the treatment of the vibration analysis in the vertical and the horizontal directions in section 2, we assumed that the shearing forces act on the center lines of both the wheel thickness and the railhead. However, we now assume that the point of contact between wheel and rail is shifted from the center line of the wheel by η_w and from that of the railhead by η_r in the horizontal direction, as shown in Fig. 3. The shift distances η_w and η_r are positive in the direction described in Fig. 3. Then, the moment $m_r^n(t)$ around the longitudinal axis of the rail is generated by virtue of the contact spring force $p^n(t)$ acting at the n -th point of contact between wheel and rail. This moment causes the vibrations in the horizontal direction of the wheel and the rail. The moment acting on the wheel $m_{rw}^n(t)$ should be distinguished from the one acting on the rail $m_{rr}^n(t)$. Here, as η_r is small, the displacement of the rail at the point of contact in the horizontal direction y_c is approximated by that, y_r , at the center of the head surface of the rail.

3.3 Equation of contact forces

First, we consider the equation in the vertical direction. All static loads are assumed to be common to each wheel. Then, the linear Hertzian contact spring constants due to elastic contact between wheels and a rail are all the same. These common spring constants are represented as k_c and they are calculated from the curvatures of the contact surfaces of wheel and rail and the static load⁽⁴⁾. The contact forces are expressed as

$$\bar{\mathbf{P}} = k_c(\bar{\mathbf{Z}}_c + a\mathbf{F} - \bar{\mathbf{W}}_v) \quad (22)$$

where

$$\begin{aligned} \bar{\mathbf{Z}}_c &= \bar{\mathbf{Z}} - \eta_r \bar{\Phi} \\ \mathbf{F} &= {}^t[\exp(j\omega T_1), \dots, \exp(j\omega T_N)] \end{aligned} \quad (23)$$

and $\bar{\mathbf{Z}}_c = {}^t(\bar{z}^1, \dots, \bar{z}^N)_c$ is the displacement amplitude vector of the points of contact in the vertical

direction, and $T_n = \xi_n / v$ is a phase difference accompanied by the intervals between wheels.

Next, we consider the equation in the horizontal direction. The lateral creep forces at the points of contact are represented by

$$\bar{\mathbf{Q}}_r = -\bar{f}_{22}(\bar{\mathbf{W}}_h - \bar{\mathbf{Y}}_r') \quad (24)$$

where $\bar{f}_{22} = f_{22}/v$, and f_{22} is Kalker's creep coefficient, which is common to each point of contact.

The moments acting on the wheels and the rail are related to the contact spring forces in the vertical direction by the following equation:

$$\begin{aligned} \bar{\mathbf{M}}_{rw} &= \eta_w \bar{\mathbf{P}}, \bar{\mathbf{M}}_{rw} = {}^t(\bar{m}_r^1, \dots, \bar{m}_r^N)_w \\ \bar{\mathbf{M}}_{rR} &= \eta_R \bar{\mathbf{P}}, \bar{\mathbf{M}}_{rR} = {}^t(\bar{m}_r^1, \dots, \bar{m}_r^N)_R \end{aligned} \quad (25)$$

Substituting $\bar{\mathbf{M}}_{rR}$ and $\bar{\mathbf{M}}_{rw}$ for $\bar{\mathbf{M}}_r$ in Eqs.(11) and (14) and for $\bar{\mathbf{M}}_r$ in Eq.(20), respectively, and from Eqs.(22) to (25), the complex amplitudes of the contact forces $\bar{\mathbf{P}}$ and $\bar{\mathbf{Q}}_r$ generated in the coupled vibration in both vertical and horizontal directions are obtained from the following equation:

$$\begin{aligned} \begin{bmatrix} \mathbf{E} - k_c(\mathbf{G} - \mathbf{S} - \eta_R^2 \bar{\mathbf{H}}_{32}) & k_c \eta_R \bar{\mathbf{H}}_{31} \\ \bar{f}_{22}(\eta_w \mathbf{U}'_2 - \eta_R \bar{\mathbf{H}}'_2) & \mathbf{E} + \bar{f}_{22}(\mathbf{U}'_1 - \bar{\mathbf{H}}'_1) \end{bmatrix} \begin{bmatrix} \bar{\mathbf{P}} \\ \bar{\mathbf{Q}}_r \end{bmatrix} \\ = \begin{bmatrix} k_c a \mathbf{F} \\ \mathbf{0} \end{bmatrix} \end{aligned} \quad (26)$$

Equation (26) is the $2N$ dimensional complex simultaneous equations. If $\bar{\mathbf{P}}$ and $\bar{\mathbf{Q}}_r$ are numerically computed from Eq.(26), then the moments $\bar{\mathbf{M}}_{rw}$ and $\bar{\mathbf{M}}_{rR}$ are obtained by substitution of the $\bar{\mathbf{P}}$ and $\bar{\mathbf{Q}}_r$ into Eq.(25). The responses of the rail and the wheel at the point of contact in both vertical and horizontal directions are computed by substituting these contact forces and moments into Eqs.(2), (6), (8), (16) and (17), respectively.

4. Numerical Computational Results and Discussion

The 60 kg rail and the standard wheel with four steps of the Shinkansen are adopted as a calculation model⁽³⁾⁻⁽⁵⁾. The length of a vehicle and the arrangement of the wheels are based on the actual dimensions, as shown in Fig.1. We used the following parameter values as standard values in calculation:

$$\begin{aligned} k_z &= 1 \times 10^8 \text{ N/m}^2, k_y = 5 \times 10^7 \text{ N/m}^2 \\ k_\phi &= 1.5 \times 10^6 \text{ N/rad}, M = 500 \text{ kg} \\ v &= 210 \text{ km/h } (\Omega_r = 128 \text{ rad/s}) \end{aligned} \quad (27)$$

Viscous damping coefficients $N_z = N_y = N_\phi = 0.4^{(1),(2)}$. The values of the other parameters used in this report are the same as those given in the previous reports⁽³⁾⁻⁽⁵⁾. The static load and Kalker's creep coefficient are assumed to be common to all points of contact. According to Kalker's theorem⁽⁹⁾ and Hertz's contact theory, we obtain $f_{22,n} = 1.16 \times 10^7 \text{ N}$ and $k_c = 1.47 \times 10^7 \text{ N/m}$ for the case of the static load = 78.4 kN, the radius of wheel $R = 455 \text{ mm}$ and the radius of curvature of the rail head = 600 mm. For the devia-

tions η_R, η_w as shown in Fig. 3, we consider two kinds of models as follows:

(I) Standard contact model: from the geometric dimensions based on the design of the Shinkansen, $\eta_R = 0 \text{ mm}$ and $\eta_w = -5 \text{ mm}$,

(II) Worn contact model: Imagining that the wheels and the rail had been worn out, $\eta_R = 20 \text{ mm}$ and $\eta_w = 15 \text{ mm}$.

In the figures of the following numerical computational results, we illustrate the respective dimensionless displacement amplitudes of the wheels and the rail divided by the amplitude of the roughness a , and describe the symbols with capital letters instead of the small letters corresponding to the amplitudes. The responses in both vertical and horizontal directions are simultaneously shown in the figures.

Figure 4 shows the displacement responses of the wheel and the rail at the point of contact indicated in the figure for the standard parameter values and the standard contact model. Figure 5 indicates the characteristics of the contact forces in the vertical and the

Fig. 4 Displacement response at point of contact for standard contact model

Fig. 5 Contact forces for standard contact model

horizontal directions. The response levels at the respective points of contact between wheels and rail are almost equivalent to each other, though there are slight differences in the aspect of the fluctuations among them. Therefore, only the responses at the point of contact 1 illustrated in the figure are indicated as representative examples. The results are summarized as follows:

(1) In the high-frequency region above 100 Hz, we can see the violent fluctuations and beats of the vibration levels due to the mutual interference between wheels. The vertical vibration of the rail, the horizontal vibrations of wheel and rail, and the vertical vibration of the wheel contribute to the rolling noise due to the high vibration level. Furthermore, the vertical vibration characteristics of the rail have a gently sloping peak attributable to the coupling between wheels and rail in the vicinity of 1 kHz. This was also recognized in the coupled vibration between a wheel and a rail in both vertical and horizontal directions⁽⁷⁾. The vibration levels of the wheel and the rail in the horizontal direction are lower by about 20 dB than that of the rail in the vertical direction. The vibration levels of wheel and rail in the horizontal direction are almost equivalent to each other, because the rail, that is, the continuous body with an infinite length, is influenced considerably by the vibration characteristic of the wheel, which is a continuous body with a limited boundary.

(2) What is evident on comparing the level of the contact forces in the vertical and the horizontal directions is that that in the vertical direction is higher about 30 dB than that in the horizontal direction, which is directly related to the difference between vibration levels. Since $\eta_R = 0$ mm in Figs. 4 and 5, it is clear from Eqs. (23) and (26) that there is one-sided coupling from the vertical to the horizontal direction.

As a result of comparing the vibration levels of the wheel and the rail in the vertical and the horizontal directions simultaneously, using the approximate model with an infinite number of vehicles, it became clear that the vibration level of the rail in the vertical direction is the highest. This fact coincides with the results of rolling noise measured in the Shinkansen⁽¹⁰⁾.

(3) Figures 6 and 7 show the displacement responses of the rail at the middle points (m_1, m_2, m_3) between wheels indicated in the figures. From the comparison between Figs. 4 and 6, the vibration levels at m_1 and m_2 in the vertical direction are almost equal to that at the point of contact in the frequency region of 0.5 to 2 kHz of the rolling noise. On the other hand, the vibration level at m_3 is remarkably low. The vibration levels at m_1 and m_2 in the horizontal direction are not higher than that at the point of contact,

Fig. 6 Displacement responses of rail in the vertical direction for standard contact model

Fig. 7 Displacement responses of rail in the horizontal direction for standard contact model

but the vibration of the rail has a tendency to become violent in the narrow intervals between wheels. This means that the narrower the distance between wheels is, the more remarked the mutual interference between wheels is.

(4) Figure 8 indicates the envelopes of velocity per vehicle along the railhead in the vertical and the horizontal directions at the frequencies shown in Fig. 4 by the arrows, where $\Omega = \omega/\omega_0$, $\omega_0 = \sqrt{k_z/\rho A}$ and

A are the characteristic circular frequency and the cross-sectional area for the standard parameters respectively, and ρ is density.

In the vertical direction, there are two regions of high vibration level on the rail, that is, one between truck wheels and the other under couplers between vehicles. In the horizontal direction, the vibration levels are about one-tenth of those in the vertical direction. In the wide intervals between wheels, the vibration levels are considerably low.

Next, we show the responses for the standard parameters and the worn contact model. Figures 9 to 13 are the results corresponding to Figs. 4 to 8, respectively. The results are summarized as follows:

(5) The displacement responses of the wheel and the rail at the points of contact, the contact forces and the displacement responses of the rail are increased by about 15 to 20 dB in the vibration level. This is because the excitation of vibration is strengthened according to the change in the contact model.

Fig. 8 Velocity distributions of rail in the longitudinal direction for standard contact model

Fig. 9 Displacement response at the point of contact for worn contact model

(6) The effect of the vibration characteristic in the horizontal direction upon the one in the vertical direction is almost undetectable because of large fluctuations and beats due to the mutual interference between wheels. Moreover, the response in the vertical direction is almost independent of the shift amounts η_w and η_R of the contact model.

Incidentally, the rolling noise is radiated not only from the points of contact but also from all the points on rail and wheels. Hence, the authors introduce the integral estimate per vehicle relating to the power of the noise radiation attributable to the vibration of the rail in the vertical direction defined by

$$J_v = \Omega^2 \int_0^l |\bar{z}/a|^2 d\xi \quad (28)$$

Figure 14 indicates the integral estimate J_v for the standard parameters and the standard contact model. Figures 15, 16 and 17 are the integral estimate J_v of the worn contact model for the standard parameters, for the parameters in which only the distributed

Fig. 10 Contact forces for worn contact model

Fig. 11 Displacement responses of rail in the vertical direction for worn contact model

spring constants k_z , k_y and k_ϕ of the foundation are set to five times as large as the standard values, and for the parameters in which only the damping ratios N_z , N_y and N_ϕ are set to twice as large as the standard ones, respectively. The results are summarized as

Fig. 12 Displacement responses of rail in the horizontal direction for worn contact model

Fig. 13 Velocity distributions of rail in the longitudinal direction for worn contact model

Fig. 14 Integral estimate of rail in the vertical direction in the region of a vehicle

follows:

(7) In general, the vibration of the rail in the vertical direction is almost independent of that in the horizontal direction^{(5),(7)}.

(8) In the high-frequency region of Figs. 16 and 17, the average of the vibration level is decreased and the fluctuation of the level is also small. This shows that the attenuation of vibration propagation along the rail and the decrease of the mutual interference between wheels occur due to increase of the distributed spring constants of the foundation and the dampings of the rail. On the whole, therefore, the vibration level of the rail in the region of a vehicle is lowered.

Incidentally, we consider the random vibration for the case that there exists roughness of the railhead

Fig. 15 Integral estimate of rail in the vertical direction within the region of a vehicle

Fig. 16 Integral estimate of rail in the vertical direction within the region of a vehicle

Fig. 17 Integral estimate of rail in the vertical direction within the region of a vehicle

in the vertical direction $u(x)$. The power spectrum density of the exciting source $S_u(\omega)$ and the power spectrum density for the velocity response of the rail at the k -th point of contact $S_{vk}(\omega)$ are expressed as⁽³⁾

$$S_u(\omega) = cv^{r-1}/\omega^r \quad (29)$$

$$S_{vk}(\omega) = \omega^2 |\bar{Z}_k|^2 S_u(\omega) \quad (30)$$

where c and r are both constants.

The measured results in the Shinkansen indicate that the intensity of the rolling noise is proportional to the travelling speed⁽¹¹⁾. Assuming that the power spectrum density for the velocity response of the rail is related to the rolling noise, the relation of $r \doteq 2$ is estimated from the measured results. Hence, putting $r = 2$ for the time being, the power spectrum density for the velocity response of the rail at the point of contact has the same aspect as the displacement response at the point of contact illustrated in this report. Then, the power spectrum density for the integral estimate in random vibration is obtained by passing through the filter of $1/\omega^2$. Therefore, in the power spectrum density for the velocity response of the rail at the point of contact and the power spectrum density for the integral estimate, we can see a gently sloping peak near 1 kHz, which is a peculiar feature of the rolling noise. In such a way, it is confirmed that the roughness of the railhead in the vertical direction is essential to the source of excitation of the rolling noise. The power spectra⁽¹⁰⁾ for the velocity response of the rail in the vertical direction appearing dominantly in the actually measured results of the vibration and noise can be explained by using this mechanism.

Lastly, the authors simply mention the coupled vibration mode of the wheel and the rail in the vertical direction in the high-frequency region. In the high-frequency region, the wheel does not vibrate in the vertical direction because of its large mass and therefore at the points of contact the rail moves in the opposite direction to the irregularity on the railhead. As a result, the amplitude of the contact force in the vertical direction is approximately held constant.

5. Conclusions

In the previous reports^{(1),(2)}, assuming the periodic arrangement of the vehicles, for example, the period of the vehicle length, the authors presented convenient analytical models for the respective coupled vibration analyses between wheels and rail in the vertical and the horizontal directions. In this report, on the basis of the previous reports, they analyzed the coupled vibration in both vertical and horizontal directions, regarding the roughness on the head surface of the rail in the vertical direction as a source of excitation. They estimated the vibration levels of wheels and rail in order to examine, by numerical computation, how the

wheels and the rail contribute to the vibrations of the Shinkansen. The results are summarized as follows:

(1) The violent fluctuations and beats due to arrangement of wheels are clearly recognizable in the displacement responses. According to the high vibration level, the vertical vibration of the rail, the horizontal vibrations of wheel and rail, and the vertical vibration of the wheel contribute to the rolling noise. This result with respect to the vibration level is the same as that described for the coupled vibration between a wheel and a rail in both vertical and horizontal directions⁽⁷⁾. Hence, the rolling noise may be caused mainly by the vibration of the rail in the vertical direction. This is coincident with the results of the vibration level measured in the Shinkansen⁽¹⁰⁾.

(2) Even if some of the parameters with respect to the rail are tentatively changed, the responses in the vicinity of the points of contact between wheels and rail are almost equivalent in level. However, it is effective for the attenuation of vibration propagation through the points of contact along the rail and for the reduced fluctuation of vibration level to increase the distributed spring constants of the foundation and the dampings of the rail. This gives us useful guidance as to countermeasures against vibration and noise.

(3) In the narrow intervals on the rail between wheels, in which the wheels interact strongly with each other, the rail vibrates violently. This tendency is the same in both vertical and horizontal directions.

The radiation of noise from the wheels and the rail will be dealt with in detail in the following report, using the approximate model with an infinite number of vehicles.

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