Rotational Vibration of a Helical Gear Pair with Modified Tooth Surfaces*

(Verification of the New Theory by Experiment and a New Design Method for Dynamic Performance)

Sho HONDA**

The dynamic increment of tooth load derived from the new theory is verified by experiment, which is expressed as a function of two variables, velocity and operating load, with the parameter of tooth surface modification, when the dimensions and the tooth-pair spring constant of the gear pair are given. This dynamic incremental load can be used as an index of the dynamic performance of a gear pair, such as noise, vibration and dynamic load. A gear design method to minimize the index is discussed theoretically, which clarifies that there are combinations of tooth surface modification, tooth-pair spring constant and static load which minimize the index and that, in the gear pair which realizes no dynamic incremental load (the index=0) under any load, the tooth surface is not modified and the equivalent tooth-pair spring constant does not vary. In other words, designing a gear pair to show good dynamic performance under any load means realizing an equivalent tooth-pair spring constant with variation as small as possible by choosing the optimal dimensions and realizing tooth surface modification as small as possible by choosing proper manufacturing processes.

Key Words : Gear, Vibration, Gear Geometry, Transmission Error, Dynamic Load, Gear Noise

1. Introduction

In the first report⁽¹⁾, the engagement between modified tooth surfaces of a helical gear pair was replaced with that between the equivalent tooth profiles of a spur gear pair with the same single flank error. The characteristics of the equivalent profile and its tooth actions causing the gear pair to transmit inconstant rotational motion were analyzed. The equivalent tooth profile under a load was analyzed in the regions of both single and multiple tooth-pair mesh. The tooth deflection and the tooth-pair spring constant in the region of a multiple tooth-pair mesh were clarified and the fundamental requirements for gear-tooth contact under load were also obtained.

In the second report⁽²⁾, the rotational motion of a gear pair with modified tooth surfaces under load was analyzed using their equivalent profiles and the

differential equations of motion were solved to obtain the dynamic increment of tooth load. The newly developed method was applied to a gear pair with symmetrical convex tooth surfaces by which most tooth surfaces modified for practical use can be approximated, and an analytical solution and a numerical example of the dynamic increment of tooth load were presented.

In this report, the dynamic increment of tooth load derived from the new theory is verified by experiment and a gear design method to minimize the noise and the dynamic load is presented by using it as an index of the dynamic performance.

2. Experiment

2.1 Test gear pairs

Table 1 shows the dimensions of test gear pairs and the tooth-pair spring constant $K(\theta_2)$ which is given by Umezawa et al.'s approximate equation⁽³⁾, where θ_2 is angle of rotation of gear II. They are ground after heat treatment into three different convex tooth surfaces, accuracies of which are shown in Fig. 1. The surface variation and the pitch error in the same gear pair are negligibly small. Each broken

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** Mechanical Engineering Div. III, Toyota Central R & D Labs. Inc., Nagakute-cho, Aichi-gun, Aichi 480-11, Japan

Table 1 Dimensions of test gear pairs

	Gear I	Gear II
Number of teeth	37	35
Center distance (mm)	122.00	
Normal module	2.895	
Normal pressure angle (deg.)	16.779	
Helix angle (deg.)	31.306	
Pitch diameter (mm)	125.39	118.61
Base diameter (mm)	118.24	111.85
Outside diameter (mm)	133.05	125.70
Normal tooth thickness (mm)	4.44	4.57
Face width (mm)	20.00	
Transverse contact ratio	1.740	
Overlap contact ratio	1.085	
Spring const. (N/ μm) $K(\theta_2) = 103 \cdot \exp(-185 \theta_2 ^3)$

Fig. 1 Accuracies of tooth surfaces of test gear pairs

parabola indicates the single flank error of the equivalent tooth profiles corresponding to that of the test gear pair and a_1 denotes the base circle variation coefficient.

2.2 Calculated results of the amplitude F_{dn} of the dynamic increment of tooth load of the n -th order

The amplitude F_{dn} of the dynamic increment of tooth load of a gear pair with symmetrical convex tooth surfaces is given as follows⁽²⁾.

$$F_{dn} = 2M\sqrt{(\omega_{20}^2 U_n)^2 + \{(T_2 \theta_{2p}/J_2) V_n\}^2}, \quad (1)$$

where

$$M = J_1 J_2 / \{R_{b10}^2 (i^2 J_1 + J_2)\},$$

$$i = R_{b20}/R_{b10},$$

$$U_n = \{(a_{1x} - a_{1y})\zeta + a_{1y}\} \cos(n\pi\zeta) - \{(a_{1x} - a_{1y})/n\pi\} \sin(n\pi\zeta) \quad (1 \leq m_p \leq 2)$$

$$\begin{aligned} &= \{(a_{1z} - a_{1y})\zeta + a_{1y}\} \cos(n\pi\zeta) \\ &\quad - \{(a_{1z} - a_{1y})/n\pi\} \sin(n\pi\zeta) \quad (2 \leq m_p \leq 3) \\ &= \{(a_{1z} - a_{1w})\zeta + a_{1w}\} \cos(n\pi\zeta) \\ &\quad - \{(a_{1z} - a_{1w})/n\pi\} \sin(n\pi\zeta) \quad (3 \leq m_p \leq 4), \\ V_n &= (1/n\pi) \{(a_{1y} - a_{1x})\zeta - a_{1y}\} \cos(n\pi\zeta) \\ &\quad + \{(a_{1y} - a_{1x})(\zeta^2/3 - 1/n^2\pi^2) \\ &\quad + a_{1y}(1 - 2\zeta)/3\} \sin(n\pi\zeta) \quad (1 \leq m_p \leq 2) \\ &= (1/n\pi) \{(a_{1y} - a_{1z})\zeta - a_{1y}\} \cos(n\pi\zeta) \\ &\quad + \{(a_{1y} - a_{1z})(\zeta^2/3 - 1/n^2\pi^2) \\ &\quad + a_{1y}(1 - 2\zeta)/3\} \sin(n\pi\zeta) \quad (2 \leq m_p \leq 3) \\ &= (1/n\pi) \{(a_{1w} - a_{1z})\zeta - a_{1w}\} \cos(n\pi\zeta) \\ &\quad + \{(a_{1w} - a_{1z})(\zeta^2/3 - 1/n^2\pi^2) \\ &\quad + a_{1w}(1 - 2\zeta)/3\} \sin(n\pi\zeta) \quad (3 \leq m_p \leq 4), \\ n &= 1, 2, 3, \dots, \\ \zeta &= 2 - m_p \quad (1 \leq m_p \leq 2) \\ &= m_p - 2 \quad (2 \leq m_p \leq 3) \\ &= 4 - m_p \quad (3 \leq m_p \leq 4), \end{aligned}$$

when $m_p \geq m_{p0}$, $m_p = m_{p0}$,

J_1, J_2 : moments of inertia of gears I and II, respectively,

θ_{2p} : half of the angular pitch of gear II,

R_{b10}, R_{b20} : radii of the theoretical base circles of gears I and II, respectively,

T_2 : transmitted torque of gear II,

ω_{20} : mean angular velocity of gear II,

m_p, m_{p0} : actual and theoretical contact ratios, respectively, and

$a_{1x}, a_{1y}, a_{1z}, a_{1w}$: base circle variation coefficients under load in the regions of one-pair, two-pair, three-pair and four-pair mesh, respectively.

When the dimensions of a gear pair are given, U_n , V_n and ζ in Eq. (1) are determined by transmitted torque T_2 , tooth-pair spring constant $K(\theta_2)$ and base circle variation coefficient under no load a_1 which represents tooth surface modification. Therefore, F_{dn} is a function of five variables, M (mass of inertia of the gear pair), $K(\theta_2)$, a_1 , T_2 and ω_{20} . In this experiment, the gear pairs made of the same material have the same dimensions and three different surface modifications, which means that F_{dn} becomes a function of two variables, T_2 and ω_{20} , with a parameter a_1 .

Figure 2 shows the calculated results of the base circle variation coefficients (a_{1x}, a_{1y} and a_{1z}), m_p , ζ , U_n and V_n of test gear pair 1. The increase of load T_2 gives little influence to a_{1x} and a_{1z} , and increases a_{1y} monotonically as shown in Fig. 2(a), while the increase of actual contact ratio m_p with the load T_2 gives a sort of periodic change to ζ in Fig. 2(b) and results the complicated characteristics of load of U_n and V_n as shown in Fig. 2(c). In this example, m_p barely reaches 2.0 at $T_2 = 130$ Nm on account of its large tooth surface modification.

Figure 3 shows the F_{dn} of test gear pair 1 calcu-

lated based on the results shown in Fig. 2, which is a function of T_2 and ω_{20} . F_{dn} is monotonic with respect to ω_{20} , while it fluctuates in a complicated manner

Fig. 2 Calculated variables of F_{dn} (test gear pair 1)

Fig. 3 Calculated results of F_{dn} (test gear pair 1)

with the increase of T_2 .

Figure 4 shows the influence of the tooth surface modification for various values of a_1 . Increase in a_1 increases the value of F_{dn} itself and the load T_2 which gives the minimum F_{dn} .

2.3 Method of verifying F_{dn}

The dynamic incremental tooth load which is the external force acting on the tooth surfaces forces the gear pair itself and the other components (the shafts, the housing and so on) to vibrate and causes gear noise and vibration.

Figure 5 shows schematically the relationships among F_{dn} , F_{vn} excited through the gear pair, F_{In} excited through the shafts and F_{Hn} excited through the housing. ω_m in Fig. 5 denotes the resonance velocity of the gear pair. F_{vn} , F_{In} and F_{Hn} can be expressed

Fig. 4 Calculated variations of F_{dn} due to a_1

Fig. 5 Relationships among F_{dn} , F_{vn} , F_{In} and F_{Hn}

as follows.

$$\left. \begin{aligned} F_{vn} &= G_n(\omega) F_{dn} \\ F_{In} &= I_n(\omega) F_{vn} \\ F_{Hn} &= H_n(\omega) F_{In} = H_n(\omega) I_n(\omega) G_n(\omega) F_{dn} \\ &= \lambda_n(\omega) F_{dn} \\ \lambda_n(\omega) &= H_n(\omega) I_n(\omega) G_n(\omega) \end{aligned} \right\} \quad (2)$$

where

ω : angular velocity of the gear pair and
 $G_n(\omega)$, $I_n(\omega)$, $H_n(\omega)$: transfer functions of the gear pair, the shafts and the housing, respectively.

Equation (2) indicates that F_{Hn} is proportional to F_{dn} when ω is constant, so that Eq. (1) can be verified by measuring F_{Hn} as follows.

- (1) Verification of the characteristic of load, $F_{dn}(T_2)$

When ω is fixed at an arbitrarily chosen value ω_{20} , $\lambda_n(\omega_{20})$ becomes constant. Therefore, measured $F_{Hn}(T_2)$ must be always proportional to the calculated $F_{dn}(T_2)$, when T_2 varies under a certain ω_{20} and a_1 .

- (2) Verification of the characteristic of velocity, $F_{dn}(\omega_{20})$

The result of item (1) gives a constant of proportionality $\lambda_n(\omega_{20})$ between $F_{Hn}(T_2)$ and $F_{dn}(T_2)$ under a certain ω_{20} and a_1 . By changing ω_{20} , the corresponding $\lambda_n(\omega_{20})$ can be obtained in the same manner, so that $F_{Hn}/\lambda_n(\omega_{20})$ must be always equal to the calculated $F_{dn}(\omega_{20})$ when ω_{20} varies under a certain load T_2 and a_1 .

- (3) Verification of the characteristic of tooth surface modification, $F_{dn}(a_1)$

Measured $F_{Hn}(a_1)$ must be always proportional to the calculated $F_{dn}(a_1)$, when a_1 varies under a certain load T_2 and ω_{20} .

2.4 Method of measuring F_{Hn}

Figure 6 schematically shows the test apparatus. Test gear pairs are placed in the same gear box

Fig. 6 Schematic view of test apparatus

Fig. 7 An example of measured F_{H1} (test gear pair 1)

Fig. 8-1 Test results of characteristic of load of F_{H1}

Fig. 8-2 Test results of characteristic of load of F_{H2}

and the sound pressure is measured through a fixed microphone under various loads and continuously changing velocity.

Figure 7 is an example of measured F_{H1} which shows the first component of the tooth mesh frequency under a certain load. One of the most remarkable resonance peaks which always appears under any load regardless of the test gear pair is selected (the peak around 2900 r/min, shown by * in this example) and F_{Hn} is measured by changing load at this peak.

3. Results of Experiment

Figure 8-1 shows measured F_{H1} s of test gear pairs with the calculated F_{d1} s as broken curves which are the first components of the tooth mesh frequency, while Fig. 8-2 shows measured F_{H2} s with the calculated F_{d2} s which are the second components of the tooth mesh frequency. In the region where the load is too small, for example 0 to 15 Nm in the case of (a), the measured values are omitted in this discussion because of their lack of reliability. Resistance torque due to lubricants and bearings is neglected.

3.1 Load characteristics

Figures 8-1(a) and 8-2(a) show that the measured curves, F_{H1} and F_{H2} have good correlation with the calculated ones, F_{d1} and F_{d2} , respectively. Good correlations are also shown in Figs. 8-1(b) and 8-2(b) and Figs. 8-1(c) and 8-2(c) for other gear pairs with different amounts of tooth surface modification.

3.2 Surface modification characteristics

Comparison among Figs. 8-1(a), (b) and (c) shows that the relationships between F_{d1} and a_1 shown in Fig. 4 are verified by those between the measured F_{H1} and a_1 . In the same manner, comparison among Figures 8-2(a), (b) and (c) shows that the relationships between F_{d2} and a_1 shown in Fig. 4 are verified.

3.3 Velocity characteristics

Figure 9 shows the characteristic of velocity of test gear pair 1 through F_{H1}/λ_1 . Six resonance points are chosen in the region between 200 and 400 rad/s, where each constant of proportionality λ_1 between F_{H1} and F_{d1} is obtained as shown in Fig. 9(a), and F_{H1}/λ_1 is plotted with the calculated F_{d1} (broken curve) under three different loads in Fig. 9(b). F_{H1}/λ_1 shows good agreement with F_{d1} except at $T_2=75.7$ Nm, where F_{d1} has a near zero value which could not be measured using the corresponding F_{H1} because it was too small compared to the surrounding noise.

3.4 Correlation between F_{Hn} and F_{dn}

Figure 10 shows the correlation between the measured F_{Hn} and calculated F_{dn} , where seven points are plotted every 9.5 Nm in the load from 28 Nm to 85 Nm. It shows that there are good correlations

(a) Constant of proportionality

(b) Correlation between F_{H1}/λ_1 and F_{d1}

Fig. 9 Test results of characteristic of velocity of F_{H1} (test gear pair 1)

Fig. 10 Correlation between F_{Hn} and F_{dn}

between F_{H1} and F_{d1} and between F_{H2} and F_{d2} .

4. Gear Design Method to Minimize Dynamic Increment of Tooth Load

4.1 Basic idea for gear design method to minimize F_{dn}

When a gear pair which has symmetrical convex tooth surfaces rotates under high speed and light load, where $T_2\theta_{2p}/J_2$ becomes negligibly small compared to ω_{20}^2 , Eq. (1) can be given approximately by Eq. (3), because the second term can be neglected.

$$\left. \begin{aligned} F_{dn} &= 2M\omega_{20}^2 |U_n\{a_1, K(\theta_2), F_s\}| \quad (m_p < m_{p0}) \\ &= 2M\omega_{20}^2 |U_n\{a_1, K(\theta_2), m_{p0}, F_s\}| \quad (m_p \geq m_{p0}) \\ K(\theta_2) &= K_p \cdot \exp(C_p |\theta_2|^3) \end{aligned} \right\} \quad (3)$$

where

a_1 : base circle variation coefficient under no load,

F_s : static tooth load ($F_s = T_2/R_{b20}$) and

K_p, C_p : constants determined by the gear dimensions.

In Eq. (3), M and ω_{20} are not 0, therefore U_n must be 0 to realize $F_{dn}=0$, which means that the gear design method to minimize the dynamic increment of load involves determining a combination of $a_1, K(\theta_2)$ and F_s to realize $U_n=0$ by choosing an adequate combination of gear dimensions including m_{p0} , such as number of teeth, base circle radius, tooth thickness, whole depth, facewidth and helix angle, when the center distance and gear ratio are given⁽⁴⁾.

The dimensions of the gear pair for calculation in this section are shown in Table 2, which are the same as those used in the second report⁽²⁾, and the calculated results of the base circle variation coefficients (a_{1x}, a_{1y}, a_{1z} and a_{1w}), m_p, ζ and U_n are shown in Fig. 11.

4.2 Combinations of $a_1, K(\theta_2)$ and F_s to realize $U_n=0$

4.2.1 In the case of $1 \leq m_p \leq 2$ The static equivalent tooth profile consists of two regions of one-pair mesh and two-pair mesh, therefore $U_n=0$ implies Eq. (4).

$$\{(a_{1x} - a_{1y})\zeta + a_{1y}\} \cos(n\pi\zeta) - \{(a_{1x} - a_{1y})/n\pi\} \sin(n\pi\zeta) = 0, \quad (4)$$

where

$$\begin{aligned} a_{1x} &= a_1 + x_s''(\theta_2) = a_1 + \{F_s/K_x(\theta_2)\}'', \\ a_{1y} &= a_1 + y_s''(\theta_2) = a_1 + \{F_s/K_y(\theta_2)\}'', \\ K_x(\theta_2) &= K(\theta_2), \\ K_y(\theta_2) &= \frac{K(\theta_2 - 2\theta_{2p}) + K(\theta_2)}{1 + K(\theta_2 - 2\theta_{2p})\{\Delta p(\theta_2) - \Delta p(\theta_2 - 2\theta_{2p})\}/F_s}, \\ \Delta p(\theta_2) &= -a_1\theta_2^2/2, \end{aligned}$$

$x_s(\theta_2), y_s(\theta_2)$: equivalent static deflections in the regions of one-pair mesh and two-pair mesh, respectively,

$K_x(\theta_2), K_y(\theta_2)$: equivalent tooth-pair spring constants in the regions of one-pair mesh and

two-pair mesh, respectively, and

$\Delta p(\theta_2)$: amount of tooth surface modification.

The solutions of Eq. (4), namely the combinations of $a_1, K(\theta_2)$ and F_s (or T_2), are shown by example as follows.

(1) In the case of $F_s=0$, $a_1=0$ is the solution of Eq. (4) for any n , which means that when the load is

Table 2 Dimensions of gear pair for optimum design

	Gear I	Gear II
Number of teeth	52	55
Center distance (mm)	122	
Normal module	1.953	
Normal pressure angle (deg.)	16.35	
Helix angle (deg.)	31.06	
Pitch diameter (mm)	118.58	125.42
Base diameter (mm)	112.19	118.66
Outside diameter (mm)	124.0	131.2
Normal tooth thickness (mm)	2.97	3.08
Face width (mm)	20	
Transverse contact ratio	1.985	
Overlap contact ratio	1.597	
Base circle variation coefficient ($\mu\text{m}/\text{rad}$)	1334	
Spring const. ($\text{N}/\mu\text{m}$) $K(\theta_2) = 64 \cdot \exp(-850 \theta_2 ^3)$

(a) Base-Circle Variation Coefficient

(b) Contact Ratio & ζ

(c) U_1 & U_2

Fig. 11 Calculated variables of F_{dn} based on dimensions listed in Table 2

sufficiently light or the tooth pair is nearly rigid, the tooth surface modification must be 0 to realize $U_n=0$.

(2) In the case of $m_p=1$, which means $\zeta=1$, therefore $a_1=-\{F_s/K_x(\theta_2)\}''$ is the solution of Eq. (4) for any n . This indicates that the tooth surface modification under no load must be concave to compensate the static deflection by F_s , because K_x is convex as shown in Fig. 13.

(3) In the case of $1 < m_p < 2$, calculated examples of U_1 and U_2 are shown in Fig. 11(c). This figure shows that $U_n=0$ ($n=1, 2$) are not realized simultaneously under a certain load when a_1 and $K(\theta_2)$ are given, because the solutions of $U_n=0$ depend on n .

Figure 12 shows combinations of a_1 and T_2 to realize $F_{d1}=0$ ($U_1=0$) under $K(\theta_2)$ given in Table 2. It indicates that the load T_2 which realizes $F_{d1}=0$ increases with the increase of tooth surface modification represented by a_1 . This variation of the load T_2 with a_1 is the reason why the adjustment of tooth surface modification has some effect (good or bad) on gear noise and vibration under a certain load.

Figure 13 shows the variations of the equivalent

Fig. 12 Combinations of a_1 and T_2 to realize $F_{d1}=0$

Fig. 13 Variations of equivalent tooth-pair spring constant and actual contact ratio for different a_1 and T_2

tooth-pair spring constant and the actual contact ratio m_p of the example shown in Fig. 12, which depend on a_1 under a certain load T_2 and cause the variation of the combination of a_1 and T_2 which realizes $F_{d1}=0$.

Figures 2(c) and 4 also show similar results through another example which is verified by the previous experiment.

(4) In the case of $m_p=2$, which means $\zeta=0$, therefore $a_1=-\{F_s/K_y(\theta_2)\}''$ is the solution of Eq. (4) for any n . This means that the tooth surface modification must be convex to compensate the equivalent static deflection, because K_y is concave as shown in Fig. 13.

4.2.2 In the case of $2 < m_p \leq 3$ The static equivalent profile consists of two regions of two-pair mesh and three-pair mesh, therefore $U_n=0$ implies Eq. (5).

$$\begin{aligned} & \{(a_{1z} - a_{1y})\zeta + a_{1y}\} \cos(n\pi\zeta) \\ & - \{(a_{1z} - a_{1y})/n\pi\} \sin(n\pi\zeta) = 0, \end{aligned} \quad (5)$$

where

$$\begin{aligned} a_{1z} &= a_1 + z_s''(\theta_2) = a_1 + \{F_s/K_z(\theta_2)\}'', \\ K_z(\theta_2) &= \frac{K(\theta_2 - 2\theta_{2p}) + K(\theta_2) + K(\theta_2 + 2\theta_{2p})}{1 + [K(\theta_2 - 2\theta_{2p})\{\Delta p(\theta_2) - \Delta p(\theta_2 - 2\theta_{2p})\} \\ & \quad + K(\theta_2 + 2\theta_{2p})\{\Delta p(\theta_2) - \Delta p(\theta_2 + 2\theta_{2p})\}]/F_s}, \end{aligned}$$

$z_s(\theta_2)$: equivalent static deflection in the region of three-pair mesh and

$K_z(\theta_2)$: equivalent tooth-pair spring constant in the region of three-pair mesh.

The solutions of Eq. (5), namely the combinations of a_1 , $K(\theta_2)$ and F_s , are shown by example as follows.

(1) In the case of $2 < m_p < 3$, the calculated examples in Figure 11(c) show that $U_1=0$ cannot be realized, but $U_2=0$ can be realized in the region of $2 < m_p < 3$.

Figure 14 shows the variation of U_1 of Fig. 11(c) with C_p of $K(\theta_2)$. It shows that $U_1=0$ can be realized by changing C_p of $K(\theta_2)$ under the same a_1 , which means that selection of C_p gives larger influence to U_1 in the region of heavy load (or high contact ratio).

Fig. 14 Variation of U_1 for different C_p of $K(\theta_2)$

than in that of light load, because the deflection becomes larger than the tooth surface modification.

(2) In the case of $m_p=3$, which means $\zeta=1$, therefore $a_1=-\{F_s/K_z(\theta_2)\}''$ is the solution of Eq. (5) for any n . This means that the tooth surface modification must be convex to compensate the equivalent static deflection, because K_z is concave as shown in Fig. 13.

In the case of $m_p>3$, the solutions of $U_n=0$ are obtained in the same manner.

$M_p=1$ has no practical meaning, but it is noteworthy that $m_p=2$ and 3 (integers) can realize $F_{dn}=0$ simultaneously for any n , when combinations of a_1 , $K(\theta_2)$ and F_s are chosen properly.

4.3 Combinations of a_1 , $K(\theta_2)$ and F_s to realize $U_n=0$ under any load

The combinations to realize $U_n=0$ discussed in the previous section are effective only under a specified load. In this section, the feasibility of realizing $U_n=0$ under any load, which means that of $U_n=0$, is theoretically examined.

4.3.1 Conditions to realize $U_n=0$

(1) Conditions for a_1 and $K(\theta_2)$ to realize $U_n=0$
Equations (4) and (5) are always valid under any load (any ζ in this case) on the following condition.

$$a_{1x}=a_{1y}=a_{1z}=0,$$

which is

$$\left. \begin{aligned} a_1 &= -\{F_s/K_x(\theta_2)\}'' \\ &= -\{F_s/K_y(\theta_2)\}'' \\ &= -\{F_s/K_z(\theta_2)\}'' \end{aligned} \right\} \quad (6)$$

If $a_1 \neq 0$, then the equivalent tooth-pair spring constants K_y and K_z depend on load F_s , therefore Eq. (6) is not always valid under a load. If $a_1=0$, then K_y and K_z depend not on F_s , but on θ_2 only, therefore Eq. (6) is always valid, when $K(\theta_2)$ (C_p in this case) can be chosen to satisfy $\{1/K_x(\theta_2)\}''=\{1/K_y(\theta_2)\}''=\{1/K_z(\theta_2)\}''=0$. Therefore, conditions for a_1 and $K(\theta_2)$ to realize $U_n=0$ are as follows.

$$\left. \begin{aligned} a_1 &= 0 \\ \{1/K_x(\theta_2)\}'' &= \{1/K_y(\theta_2)\}'' = \{1/K_z(\theta_2)\}'' = 0 \end{aligned} \right\}, \quad (7)$$

Equation (7) means that $U_n=0$ can be realized under the conditions that tooth surfaces of the gear pair are not modified ($a_1=0$) and the equivalent tooth-pair spring constant curves during one period, such as those for K_x and K_y or K_y and K_z , are parallel (horizontal) to the θ_2 -axis.

(2) Conditions for the equivalent tooth-pair spring constant curves to be parallel (horizontal) to the θ_2 -axis

When $a_1=0$, the equivalent spring constants are given as follows.

$$K_x(\theta_2)=K(\theta_2)$$

(region of one-pair mesh)

$$K_y(\theta_2)=K_x(\theta_2)+K(\theta_2-2\theta_{2p})$$

(region of two-pair mesh)

$$K_z(\theta_2)=K_y(\theta_2)+K(\theta_2+2\theta_{2p})$$

(region of three-pair mesh)

$$K_w(\theta_2)=K_z(\theta_2)+K(\theta_2-4\theta_{2p})$$

(region of four-pair mesh)

The equivalent tooth-pair spring constant curves during one period vary with the change of the region of mesh, such as those for K_x and K_y or K_y and K_z , which are different theoretically and unsmooth or discontinuous at the intersection, so that they cannot theoretically be horizontal lines, but can be made approximately horizontal as follows.

(a) By choosing the contact ratio as an integer, the equivalent tooth-pair spring constant curve is composed of that for K_x , K_y or K_z only, therefore C_p should be selected so that it is as horizontal as possible. For example,

$$\left. \begin{aligned} K_x(0) &= K_x(\theta_{2p}) & (m_{p0}=1-0) \\ K_y(0) &= K_y(\theta_{2p}) & (m_{p0}=2-0) \\ K_z(0) &= K_z(\theta_{2p}) & (m_{p0}=3-0) \end{aligned} \right\} \quad (8)$$

where $m_p=3-0$, for example, means 2.999..., because the equivalent tooth-pair spring constant without tooth surface modification is discontinuous at the point of transfer from the three-pair mesh to four-pair mesh. Other m_{p0} s have the same meaning.

(b) By choosing a large contact ratio, the difference between two equivalent spring constant curves becomes small, such that $K_w(\theta_2) \approx K_z(\theta_2)$, therefore C_p should be selected so that they are as horizontal as possible. For example,

$$\left. \begin{aligned} K_x(0) &= K_y(\theta_{2p}) & (1 < m_{p0} < 2) \\ K_z(0) &= K_y(\theta_{2p}) & (2 < m_{p0} < 3) \\ K_z(0) &= K_w(\theta_{2p}) & (3 < m_{p0} < 4) \end{aligned} \right\} \quad (9)$$

To make equivalent tooth-pair spring constant curves horizontal, it is important to choose an adequate combination of m_{p0} and C_p . The increase of m_{p0} without taking account of C_p does not always realize the decrease of the variation of the equivalent tooth-pair spring constant.

(3) Feasibility of the dimensions to realize combinations of m_{p0} and C_p

M_{p0} and C_p might be given by facewidth b , helix angle ψ and the transverse dimensions D_m of a gear pair as follows.

$$m_{p0}=m_{p0}(D_m, b, \psi)$$

$$C_p=C_p(D_m, b, \psi)$$

To realize a given combination of m_{p0} and C_p , an adequate combination of b and ψ must be selected with D_m as a parameter.

4.3.2 Calculated examples to realize $U_n=0$

Figure 15 shows calculated examples to realize $U_n=0$. Under the conditions of $m_{p0}=2.995$ and $a_1=0$, C_p

Fig. 15 Calculated examples to realize $F_{d1}=0$ under any load

Fig. 16 Equivalent tooth-pair spring constants in Fig. 15

Fig. 17 Calculated example to minimize F_{d1} for practical use

$= -635$ is obtained using Eq. (8). It shows that $C_p = -635$ gives smaller F_{d1} than $C_p = -850$ and that $m_{p0} = 2.995$ yields smaller F_{d1} than $m_{p0} = 3.583$ in the region of $C_p = -850 \sim -635$. To realize $U_n \equiv 0$ under $m_{p0} =$

3.583 , C_p must be calculated using Eq. (9).

Figure 16 shows the equivalent tooth-pair spring constants of the calculated examples shown in Fig. 15, where the variation at $C_p = -635$ becomes smaller than that at $C_p = -850$. This figure indicates that the variation of the equivalent tooth-pair spring constant at $m_{p0} = 3.583$ and $C_p = -635$ (shown by broken curve) becomes larger and results in larger F_{d1} than that at $m_{p0} = 2.995$, because $m_{p0} = 3.583$ includes the region of a four-pair mesh which increases variation. These examples do not perfectly realize $U_n \equiv 0$, because Eqs. (8) and (9) are approximate equations and the equivalent tooth-pair spring constant curves are not sufficiently horizontal as shown in Fig. 16.

Figure 17 shows the minimized F_{d1} , where $a_1 = 273 \mu\text{m/rad}$ is chosen for practical use, which means that the tooth surfaces have a $1 \mu\text{m}$ modification (convex) relatively in the profile and lengthwise directions. It shows that F_{d1} of tooth surfaces modified for practical use can also be reduced markedly by choosing an adequate combination of m_{p0} and C_p .

5. Conclusions

The characteristics of the amplitude F_{dn} of the dynamic incremental tooth load which is generated by tooth surface modification and static deflection are verified by experiment and a gear design method to minimize the F_{dn} is discussed theoretically as follows.

(1) F_{dn} is a function of two variables, velocity and operating load, with the parameter of tooth surface modification, when the dimensions and material of a gear pair are given. The characteristics of load, velocity and tooth surface modification are clarified and verified by experiment.

(2) F_{dn} can be used as an index of the dynamic performance of a gear pair, such as noise, vibration and dynamic load, and a design method for a gear pair to realize the index $F_{dn} = 0$ is discussed theoretically, which clarifies that there are combinations of tooth surface modification a_1 , tooth-pair spring constant $K(\theta_2)$ and static load F_s which minimize F_{dn} and that, in the gear pair which realizes $F_{dn} = 0$ under any load, the tooth surface is not modified and the equivalent tooth-pair spring constant does not vary. In other words, designing a gear pair to show good dynamic performance under any load means realizing an equivalent tooth-pair spring constant with variation as small as possible by choosing the optimal dimensions and realizing tooth surface modification as small as possible by choosing proper manufacturing processes.

Clarifying the relationship between the tooth-pair spring constant and the gear dimensions theoretically and experimentally is one of the most important problems which must be solved to realize a gear

design method for better dynamic performance.

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