A Study on Structural Optimum Design Based on Qualitative Sensitivities*

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An optimum design method is considered as a method whose results are the most reasonable from various standpoints in the prescribed mathematical formulation. However, its formulations present difficulties in choosing objective functions, setting allowances to constraints and so on, because they are greatly concerned with the results of optimization. In this study, we proposed concepts of qualitative sensitivities and qualitative optimality compared to quantitative concepts, as well as a general algorithm of the qualitative optimization method. We newly defined three kinds of fuzzy language, assigned fuzzy sets to each of them and assigned them to each candidate for discrete design variable. We proposed fuzzy reasoning rules by using these two kinds of fuzzy sets. We also extended the proposed algorithm to multi-objective optimization. Through the numerical example of a single-objective problem, we obtained almost the same solution as in quantitative methods, and efficiencies were confirmed for this case. Also through the example of a multiobjective problem, the new method was shown to have more flexibility in determining the preferred objective functions compared to the conventional method.

Key Words: Design Engineering, Fuzzy Set Theory, Optimum Design, Sensitivity Analysis, Qualitative Sensitivity, Fuzzy Reasoning

1. Introduction

Design is a process of finding the best or a better solution through many decisions and compromises by consideration of many factors of design objects. When we wish to carry out design with the aid of a computer, we need to model those objects, and treat them in a mathematical way. In many cases, these models must be expressed entirely in a quantitative manner. Once we have achieved a quantitative model, we can carry out optimization with these mathematical models. Optimization is the process of finding the most reasonable solution (optimum solution) within the given mathematical models; it is a method for obtaining a result which coincides with that of well-trained designers, so-called experts. However, the results of optimum design greatly depend on the formulation of the problem. Thus such formulations become a heavy burden on the designers. In single objective optimum design, the designer has to choose

one objective function among many factors in the design problem and then set allowances for the other factors as constraints. In order to reduce the burden on the designers in formulation of the problem, it would be natural and reasonable to formulate the problem as a multiobjective optimum design problem. As the solutions of a multiobjective problem are usually not determined uniquely, we have to choose one solution from among them through a multicriteria decision-making process. In this process, we have to grasp the situations of the optimum solutions by local information as trade-off ratios, and we have to make a decision by considering the difference between the marginal rate of substitution and these ratios. These kinds of decisions usually have to be made in quantitative ways, and this presents a new and difficult burden for designers. To solve these problems, we have conducted a series of studies on qualitative optimization, which make it possible to carry out optimization by having a qualitative grasp of the situation. In Refs. (1) and (2), after we obtained trial optimization results by using monotonicity analysis, we added active constraints to the objective function and changed the problem to a 2-objective optimum design problem, obtaining Pareto solutions around the trial

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optimum solution by inferring the desired direction of design variables, using qualitative reasoning. In Ref. (3), we proposed the concept of the qualitative trade-off ratio and developed a method for solving interactive multicriteria decision-making problems. Here, we use fuzzy language to show the optimization situation in order to make it easier for designers to grasp it. In Ref. (4), in order to avoid the combinational explosion in qualitative operation, we used fuzzy language and extended fuzzy rules to each operation to create a better system of inference. However, since it was based on sign logic, it still involved ambiguity. Hence, we need trial optimization to ensure the optimality of the solution. In this study, by comparing the concepts and the methods of qualitative optimization with quantitative optimization, we propose the ideas of qualitative sensitivity and qualitative optimality. To make the inference more precise, and to avoid the need for a trial optimization, we abandon sign logic-based reasoning and use fuzzy reasoning. To obtain qualitative sensitivity, we set three fuzzy languages and set membership values for each fuzzy language. As we assume that the design variable should take the given specified values, we also assign fuzzy sets to each candidate for design variables. We propose a new inference method to infer the new candidates for design variables using these two kinds of fuzzy sets. We also extend the proposed method to a multiobjective optimization. We apply the proposed method to a simple single structural optimum design problem and compare the results with the one obtained by quantitative optimization to examine the effectiveness of the method. Then we apply the method to multiobjective optimum design. Through these numerical examples, it is found that the proposed method can yield almost the same result as the conventional method even with expedient sets of qualitative sensitivities when we can grasp the qualitative sensitivities to some degree of precision. Also, through a numerical example of a multiobjective optimum design problem, the proposed method exhibits much greater ease in accepting information of optimization, and it shows flexibility in transforming these data and making local decisions. Comparing it with the previous method based on sign logic, we find that although the designer has to make a few precise and minute decisions, the new method reduces the number of iterations because of the benefits of a revised inference algorithm.

There have been numerous other studies related to the support of the designer in the qualitative way of thinking; for example, surveys by Kurumaya⁽⁵⁾ and Forbus⁽⁶⁾, and studies by Murthy and Addanki⁽⁷⁾ and Mittel and Araya⁽⁸⁾. As for structural optimum design

problems, there have been studies on monotonicity analysis by Papalambros and his colleagues^{(9)–(12)}. In these studies, they assumed that optimum solutions were located at the intersection of active constraints, and they tried to find an active constraints set. After that, they obtained the optimum solution by solving a nonlinear analysis by Newton's method. In the series of studies by Agogino, Cagan and colleagues^{(13)–(17)}, after inferring the active constraints set by using qualitative reasoning, they used symbolic computation to express the formulation of the optimum results. Success in these studies lies in grasping active sets of constraints in qualitative ways and reducing the complicated problem to a simpler one. In Refs. (16) and (17), they applied the ideas to the upper part of a design process. These studies gave a qualitative grasp only in a simplification of the original problem, and they did not use qualitative ways of thinking in revision of design variables in the process of iteration. The reason for this lies in difficulties in avoiding combinational explosion caused by ambiguity of qualitative operations.

2. Quantitative Optimization

In this section, we will briefly explain the quantitative optimization (conventional optimization) in order to compare it with the qualitative optimization which will be explained later. Here, we will choose a gradient projection method (GPM) for a single-objective optimization, and a multicriteria decision-making problem for a multiobjective optimization.

2.1 Algorithm for single-objective optimum design

Generally, a single-objective optimum design problem can be formulated as follows:

$$\min_{x_i \in X_i} f(\mathbf{x}) \quad (1.a)$$

subject to

$$g_j(\mathbf{x}) \leq \hat{g}_j; \quad (j=1, 2, \dots, M) \quad (1.b)$$

where

$$\mathbf{x} = \{x_1, x_2, \dots, x_N\}^T$$

X_i : Feasible design variable set for x_i (Generally, a real number)

$\hat{g}_j$: Allowance of constraint g_j .

In GPM, we obtain the optimum solution through the following revision of each design variable through a number of iterations.

<without active constraints>

$$\mathbf{x}_{\text{new}} = \mathbf{x} - \nabla f \cdot \tau \quad (2)$$

<with active constraints>

$$\mathbf{x}_{\text{new}} = \mathbf{x} - \mathbf{z} \cdot \tau \quad (3)$$

where τ is the step width, ∇f is the sensitivity of the objective function, and $\mathbf{z}$ is the projection vector of the hyperplane composed of active constraints. In

Fig. 1 GPM in design variable space

nonlinear cases, GPM should correct back to the feasible region explained in Fig. 1.

2.2 Conditions of convergence

For convergence, GPM provides two conditions as follows, and it stops iteration when it satisfies either of them.

$$(1) \quad \beta(\mathbf{x}) < \varepsilon \quad (4.a)$$

$$(2) \quad f(\mathbf{x}) \leq f(\mathbf{x}_{\text{new}}) \quad (4.b)$$

where β means the Euclid norm of the projection vector $\mathbf{z}$. The first condition means that it would violate at least one active constraint when we want to revise the objective function. The second one means overshoot, which makes it impossible to revise the objective function.

2.2 Algorithm for multiobjective optimum design

Generally, there exist a number of requirements in the real design process; thus it is not easy to choose one objective function from them. In such cases, it would be more rational and natural to formulate the problem as a multiobjective optimization.

$$\min_{\mathbf{x} \in X_i} \mathbf{F}(\mathbf{x}) \quad (5)$$

subject to

$$g_j(\mathbf{x}) \leq \bar{g}_j; \quad (j=1, 2, \dots, M)$$

where

$$\mathbf{x} = \{x_1, x_2, \dots, x_N\}^T$$

$$\mathbf{F}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_L(\mathbf{x})\}^T$$

$\mathbf{F}$ is a vector function of L objective functions. However, in multiobjective optimum design problems, it is difficult to obtain a unique solution. Therefore we need another concept of solution, such as Pareto solutions. When we want to determine one solution from Pareto solutions, there are methods of converting the vector function to a scalar function, one of which is called a multicriteria decision-making problem and is defined as follows.

$$\min_{\mathbf{x} \in X_i} \Phi(\mathbf{F}) \quad (6)$$

subject to

$$g_j(\mathbf{x}) \leq \bar{g}_j; \quad (j=1, 2, \dots, M)$$

where Φ is a preference function. When we can identify the preference function before we carry out optimization, we can easily convert the problem to Eq. (1) and obtain the solution. However, usually it cannot be identified before the optimization. Thus,

the designers have to determine the global preference from local information on preference in each scalar optimization. One such local situation of preference, called the trade-off ratio, can be expressed as

$$\mathbf{m} = \left\{ \frac{\partial f_1}{\partial f_p}, \dots, \frac{\partial f_{p-1}}{\partial f_p}, 1, \frac{\partial f_{p+1}}{\partial f_p}, \dots, \frac{\partial f_L}{\partial f_p} \right\}^T. \quad (7)$$

In the SWT method proposed by Haimes and Hall⁽¹⁸⁾, the designer has to determine the difference between the preferred trade-off ratio and the gradient of the hyperplane defined by the objective functions, and then make the solution closer to the preferred trade-off ratio.

3. Qualitative Optimization

Qualitative optimization in this study means the method of finding an optimum solution by inferring directions of design variables to optimize the objective function, using qualitative sensitivities. In this study, we assume that the design variables should take the discrete values which were set before the optimization (Each domain X_i in Eq. (1) is a given set of discrete values). In practical design problems, the domain of design variables does not always consist of real numbers. For example, in a truss problem, the number of nodes is an integer. In most of the design cross-sectional areas that might be chosen from some industrial standards, and considering material characteristics, a material may be chosen among a definite number of materials, and these values may have specified values and may be discrete numbers. In such a case, design variables might be chosen from given specified values or a database. Thus, even if we assumed that the design variables can only take the specified values, it would not be far from the real design problem. The solutions obtained might be near to the optimum solution among the sets of satisfying solutions.

3.1 Proposed algorithm

The proposed algorithm in this study is as follows.

1) Set desired direction

Set the direction of objective functions to optimize them.

2) Set qualitative sensitivity

Set sensitivities of objective functions and constraints qualitatively. Here "qualitatively" means that we can set them as in sign logic or membership value of each fuzzy language. We do not need to set them distinctly in quantitative ways.

3) Inferring the values of design variables

Inferring the new set of design variables to revise the objective function.

In past studies⁽¹⁾⁻⁽⁴⁾, we inferred the direction of design variables to meet the desired directions of

objective functions and constraints. Then we moved each design variable to the next value in the data base (given specified values) in those directions. In this study, we use qualitative sensitivities expressed in fuzzy languages and infer the next state of design variables by fuzzy reasoning.

In this study, we consider the case where new design variables may be chosen from the given values. It is not such an important problem to calculate ∇f accurately or quantitatively. The main concern lies in knowing the effects of the changes in the design variable x_k on objective function and constraints qualitatively. These effects evaluated qualitatively are called "qualitative sensitivities" and are represented by three kinds of fuzzy language ("small", "normal" and "large"). These languages show the effects by their membership values. The qualitative sensitivity of function $h(x)$ with respect to x_k is defined as

$$QS(h(x), x_k) = \text{Sign} \left(\frac{\partial h}{\partial x_k} \right) \begin{cases} \mu_{\text{small}} \\ \mu_{\text{normal}} \\ \mu_{\text{large}} \end{cases} \quad (8)$$

where $\text{Sign}(\partial h(x)/\partial x_k)$ is the sign of $\partial h(x)/\partial x_k$. This means that we do not need to calculate the exact values of $\partial h(x)/\partial x_k$. We allocate meaning to each language as follows.

small: Means that $h(x)$ will not increase very much even when we update x_k in the direction of $\text{Sign}(\partial h(x)/\partial x_k)$

normal: Means that $h(x)$ will increase when we update x_k in the direction of $\text{Sign}(\partial h(x)/\partial x_k)$

large: Means that $h(x)$ will increase greatly when we update x_k in the direction of $\text{Sign}(\partial h(x)/\partial x_k)$

Allocating these meanings, we can determine membership values in some quantitative ways. If one is a well-trained designer, one can determine these values adequately from experience or intuition. In fuzzy reasoning, we use two kinds of fuzzy sets: one is a fuzzy set of qualitative sensitivity and the other is a fuzzy set of design variable. We will set a membership function to each fuzzy set expediently as follows. As a type of membership function, we can use any function which is nondecreasing in its left-hand side and nonincreasing in its right-hand side by its definition, but we expediently use the function in Eq. (9) due to its simplicity.

$$M_{\text{term}}(x) = \begin{cases} 1 - \left| \frac{x-m}{w} \right| & \text{for } M_{\text{term}}(x) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

where M_{term} denotes the membership function of fuzzy languages and design variables. For example, M_{small} means the membership function of fuzzy language "small" and M_{ican} is that of the discrete value of design variable x_{ican} . According to Eq. (9), we have to determine the value m (representing number)

which makes the membership 1 and width w in that fuzzy set. In this study, we will determine each fuzzy set of qualitative sensitivities and design variables in line with the following concept;

small: Basically, it does not move. Even if it moves, it does so at most to the next discrete value beyond the current one.

normal: Basically, it will move to the next discrete value in the direction $\text{Sign}(\partial h/\partial x_k)$. It can also move to the second next one.

large: Basically, it will move to the second or third next discrete value in the direction $\text{Sign}(\partial h/\partial x_k)$. However, we have to set the range of values a little wider so as to include some uncertainties of the qualitative sensitivity

x_k : Of such width as to effect its influence on each side of the range up to the next discrete value.

In this study, we set the membership function to each qualitative sensitivity in Eq. (10.a, b, c).

$$\text{small: } m = x_k, w = 1.5 \text{ width} \quad (10.a)$$

$$\text{normal: } m = x_k \pm \text{width}, w = 2 \text{ width} \quad (10.b)$$

$$\text{large: } m = x_k \pm \text{width}, w = 2.5 \text{ width} \quad (10.c)$$

where x_k represents the current value of design variables and "width" represents a standard unit increment among its data base. We set it to twice of the mean value of the width of the given data base. As for design variables, we set the membership function of each discrete given value x_i as in Eq. (11).

$$x_i: m = x_i, w = 2 \text{ width} \quad (11)$$

In the case where there is no active constraint, we assume intersection of two fuzzy sets: fuzzy sets composed of qualitative sensitivities and those of design variables. Then superior to this intersection set will be the degree of influence of the fuzzy language on the design variable. Then we add a multiple of the degree of influence of each fuzzy language to the design variable and qualitative sensitivity, and set this value as the reasoning (Ex) of the design variable.

$$Ex_f(x_{\text{ican}}) = \sum_i \mu_i \times \sup_{x \in R} \{ \min M_i(x), M_{\text{ican}}(x) \} \quad (12)$$

where i can be small, normal and large. For example, in Fig. 2, the reasoning of the next state ("dominant") will require adding a membership value in point a times μ_{small} and membership value in point b times μ_{normal} . We choose each new design variable so as to have the maximum of reasoning in this study.

In the case where there is at least one active constraint, we infer the new design variables as follows: We will call the new design variable explained in the preceding paragraph x_{ldom} and assume that constraint g_b violates the allowance to that constraint. Then we can find sets of design variables that constraint g_b will correct back within the constraint

Fig. 2 Example of fuzzy reasoning

after we set the qualitative sensitivity of constraint g_b to design variable $x_{i\text{dom}}$, and we can calculate the reasoning to each discrete design variable under consideration of the influence of constraint g_b .

$$Ex_g(x_{i\text{can}}) = \sum_i \mu_i \times \sup_{x \in R} \{\min M_i(x), M_{i\text{can}}(x)\} \quad (13)$$

We assume that Eq. (12) gives the reasoning of the decreasing objective function and Eq. (13) is the reasoning of the correct-back from the violated constraint. We will think of the infimum of these two examples of reasoning as follows.

$$Ex_{fg}(x_{i\text{can}}) = \inf\{Ex_f(x_{i\text{can}}), Ex_g(x_{i\text{can}})\} \quad (14)$$

Then we try to consider the next state of design variables from higher orders of these examples of reasoning until it comes to satisfy the violated constraint. Equation (14) can be easily extended to the case with several constraints in the same manner.

3.2 Idea of qualitative optimality

The quantitative optimality can easily be assumed as follows.

Optimality (quantitative)

If there is no possibility of improving the objective function or if it violates at least one constraint upon updating of design variables x_{opt} in any direction which seems to improve objective function, x_{opt} is the local optimum.

Analogous to this optimality, the qualitative optimality is proposed as follows.

Qualitative Optimality

Considering an improvement to the next state from the current state, if we cannot find any other states which will not violate at least one constraint qualitatively, then the current state is the qualitative local optimum.

3.3 Algorithm for multiobjective optimization

When we want to use the proposed method explained previously, we should try to reduce (or increase) objective functions alternatively as follows: First, we will calculate the reasoning for each objec-

Table 1 Multiplying rules of fuzzy languages

multiply	small	normal	large
small	very small	small	normal
normal	small	normal	large
large	normal	large	very large

tive function in Eq. (12) and find their infimum for find the reasonings to each design variable.

$$Ex_F(x_{i\text{can}}) = \inf\{Ex_{f1}, \dots, Ex_{fL}\} \quad (15)$$

From the many sets of design variables, a set of design variables which have maximum values of the obtained reasonings will be the next new set of design variables. When we cannot reduce all objective functions simultaneously, which means when we have obtained one of the Pareto solutions, we have to consider the trade-off relationships among objective functions. In this study, we will use the qualitative trade-off ratio proposed in the previous studies, and we use a fuzzy language expression for the qualitative trade-off ratio; when the objective function f_p is a nominal objective function, we can express the qualitative trade-off ratio as

$$m = \{TFL_{1,\dots}, TFL_{p-1,\text{plus}} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}, TFL_{p-1,\dots}, TFL_L\}^T \quad (16)$$

Here, the designer must set the fuzzy language in the direction in which he or she wants the objective function f_p to move in order to reduce the preference function which he or she has implicitly. If we call this fuzzy language FL_p , the desired direction of objective functions can be obtained by multiplying the qualitative trade-off ratio by the fuzzy language.

$$dF = m \otimes FL_p \quad (17)$$

where $\otimes$ is a multiplying rule in fuzzy language. Each of the desired directions obtained from the above equation will be given by dF_i . In inferring the new design variables, we will calculate qualitative direction by multiplying the desired direction and qualitative sensitivity and obtain

$$Q_{\text{dir}}(f_j, x_i) = QS(f_j, x_j) \otimes dF_j. \quad (18)$$

Using Q_{dir} we can infer the new design variables in the same way as in the use of Eq. (15).

Finally, we will set the multiplying rules of fuzzy language in the following rules expediently. The meaning of fuzzy language follows in Table 1, and the membership value will be determined by multiplying the membership value of each fuzzy language and summing up the same languages. When there is any rhetorical element "very", we expediently multiply 1.2 (>1.0) until the membership value becomes 1.0 to emphasize its meaning.

4. Numerical Examples

4.1 Example of application to single-objective optimum design

Let us now treat a simple optimization of a 3-bar truss (Fig. 3) as an example, in order to show the efficiency, we compare the results obtained by the proposed method with those obtained by quantitative optimization.

Here, the optimization problem can be expressed as follows.

$$\text{Minimize } M(\mathbf{x}) = 2\sqrt{2}\rho x_1 + \rho x_2 \quad (19.a)$$

subject to constraints:

$$g_1: u(\mathbf{x}) = \frac{P}{Ex_1} \leq u_a \quad (19.b)$$

$$g_2: v(\mathbf{x}) = \frac{P}{E(x_1 + \sqrt{2}x_2)} \leq v_a \quad (19.c)$$

$$g_3: \sigma_{\text{ten}}(\mathbf{x}) = \max \left\{ \frac{P(\sqrt{2}x_1 + x_2)}{x_1(\sqrt{2}x_1 + 2x_2)}, \frac{P}{x_1 + \sqrt{2}x_2} \right\} \leq \sigma_{\text{tena}} \quad (19.d)$$

$$g_4: \sigma_{\text{com}}(\mathbf{x}) = \frac{Px_2}{x_1(x_1 + \sqrt{2}x_2)} \leq \sigma_{\text{coma}} \quad (19.e)$$

where

P : load ($=1.00 \times 10^3$ [N])

E : Young's modulus ($=2.05 \times 10^{11}$ [Pa])

ρ : density ($=7.86 \times 10^3$ [kg/m³])

u_a : allowance displacement for x direction ($=2.50 \times 10^{-6}$ [m])

v_a : allowance displacement for y direction ($=1.00 \times 10^{-6}$ [m])

σ_{tena} : allowance tensile stress ($=3.00 \times 10^5$ [Pa])

σ_{coma} : allowance compressive stress ($=5.00 \times 10^5$ [Pa])

x_1, x_2 : design variables (Cross-sectional areas of ① and ②, respectively) [m²]

We use the following equations to evaluate qualitative sensitivities from obtained quantitative sensitivities.

$$y = \left| \frac{\partial h / \partial x_k \times \text{width}}{h(\mathbf{x})} \right| \quad (20.a)$$

$$\mu_{\text{small}}(y) = \begin{cases} 1 - 20y & (\text{for } 0 \leq y \leq 0.05) \\ 0 & (\text{otherwise}) \end{cases} \quad (20.b)$$

Fig. 3 3-bar truss

$$\mu_{\text{normal}}(y) = \begin{cases} 20y & (\text{for } 0 \leq y \leq 0.05) \\ 1 - \frac{20}{3}(y - 0.20) & (\text{for } 0.05 \leq y \leq 0.20) \\ 0 & (\text{otherwise}) \end{cases} \quad (20.c)$$

$$\mu_{\text{large}}(y) = \begin{cases} 0 & (\text{for } 0 \leq y \leq 0.05) \\ \frac{20}{3}(y - 0.05) & (\text{for } 0.05 \leq y \leq 0.20) \\ 1 & (\text{otherwise}) \end{cases} \quad (20.d)$$

Equation (20.a) was determined expediently by expecting the influence when the design variable moves to the next specified value after normalizing the quantitative sensitivity. Equations (20.b~d) were determined to meet the concepts of each fuzzy language. We set the design variables using values from the JIS standard (JIS G 3444). The process of qualitative optimization is shown in Fig. 4. Table 2 and Fig. 5 show the comparison of optimization results.

Figure 5 shows the vicinity of the optimum solution, and the magnified view in the circle shows the relationships between discrete values and the optimum solution of the quantitative method in detail. The black dot shows the current state, which also

Table 2 Comparison of the optimum results

contents	quantitative method	initial condition	proposed method
x_1 [m ²]	2.53×10^{-3}	9.800×10^{-3}	2.516×10^{-3}
x_2 [m ²]	1.66×10^{-3}	9.800×10^{-3}	1.707×10^{-3}
Mass [kg]	6.93×10	2.95×10^2	6.94×10
g_1 [m]	1.93×10^{-6}	5.00×10^{-7}	1.94×10^{-6}
g_2 [m]	1.00×10^{-6}	2.06×10^{-7}	9.89×10^{-7}
g_3 [Pa]	3.00×10^5	7.22×10^4	3.00×10^5
g_4 [Pa]	1.34×10^5	4.23×10^4	1.38×10^5

Fig. 4 Process of optimization

Fig. 5 Optimization result

becomes the optimum state, as explained below. The hatched dots show the states without increasing objective function. From this figure, we can see that it will be impossible to improve the situation without violating at least one constraint. Thus, the current state is the optimum state from the qualitative optimal standpoint. In this problem, there exists a local minimum at the intersection of constraints g_1 and g_3 . However, the proposed method can circumvent the local minimum because the proposed fuzzy reasoning does not have the limitation that the new design variable can only move to the next discrete value in the given domain, as in the previous studies.

From the results explained in the preceding paragraph, though the formulations of Eq. (6) and Eq. (12), which play important roles in fuzzy reasoning, are obtained in expedient ways, we are able to obtain an optimum solution very close to the quantitative optimum solution by the proposed method. The reason why it does not completely coincide with the quantitative results lies in the assumption of the design variable. It shows that it would be possible to carry out optimization based on qualitative sensitivities, where we could determine sensitivities even through a qualitative grasp. In other words, it shows one of the possibilities of qualitative optimizations.

In this example, qualitative sensitivities were given precisely because of the quantitative background. However, when we give these values intuitively, some error may occur in their settlements. However, in the proposed method, we think that fuzziness occurs in both the qualitative sensitivities and design variables, and hence we can allow some errors in estimating the qualitative behavior of objective functions and so on. If we do not have confidence in settlements, we should not set the membership value of "small" to zero.

Table 3 Optimization results

contents	initial condition	obtained Pareto	obtained solution
x_1 [m ²]	9.800×10^{-3}	2.272×10^{-3}	2.272×10^{-3}
x_2 [m ²]	9.800×10^{-3}	2.173×10^{-2}	6.519×10^{-3}
Mass [kg]	2.95×10^2	2.21×10^2	1.02×10^2
v [m]	2.06×10^{-7}	1.48×10^{-6}	4.25×10^{-6}
g_1 [m]	5.00×10^{-7}	2.15×10^{-7}	2.15×10^{-7}
g_3 [Pa]	7.22×10^4	2.35×10^5	2.64×10^5
g_4 [Pa]	4.23×10^4	2.90×10^5	2.48×10^5

Fig. 6 Process of multi-objective optimization

4.2 Example of application to multiobjective optimum design

As an example of application to multiobjective optimum design, we add constraint g_2 of the previous example of the 3-bar truss to the objective function and retain all the other conditions. Table 3 and Fig. 6 show the results of the optimum design carried out using the proposed method.

"Obtained Pareto" in Table 3 was the result obtained by reducing both objective functions simultaneously. "Obtained solution" was the result obtained by following the decision of the designer shown in Fig. 6. After we have obtained the relationship of the trade-off, we check its Pareto optimality by trying to find a better solution by using Eq. (15).

From this numerical example, we have showed that it is possible to carry out a multicriteria decision-making problem even with a qualitative grasp. In the process of optimization, we can make a slight change in the preference situation during the iteration, and we can then obtain a better inference to the designers' implicit preference than by the previous method based on sign logic. This makes it easier for the designer to accept the information of optimization in each interaction. It reduces the burden on the designer by allowing him or her to give his or her preference

qualitatively rather than giving it strictly quantitatively, and the number of iterations is reduced in comparison with the previous method. The effectiveness of the proposed method has been confirmed in one example.

5. Conclusions

1) The concept and method of qualitative optimum design using qualitative sensitivities were proposed. We compared the proposed concepts and quantitative optimization and confirmed its effectiveness. We proposed qualitative optimality for criteria of optimality.

2) We assigned three fuzzy languages and gave membership values to each of them as qualitative sensitivities rather than the sign logic previously proposed. We also assigned a fuzzy set to each candidate for design variables, and proposed a new inference algorithm using fuzzy reasoning by these two kinds of fuzzy sets.

3) We extended the method to a multiobjective optimization problem and developed its application.

4) We applied the proposed method to a 3-bar truss optimization problem. In the single optimum design case, we obtained almost the same solution to the conventional quantitative optimization as if we had been able to estimate qualitative sensitivities to some degree of precision and partially confirmed the method's effectiveness. In the multiobjective optimum design problem, it was found to have more flexibility in accepting and transforming optimum design information than the conventional method, and it can reduce the burden on the designer.

5) A key issue in the proposed method for carrying out optimization lies in grasping qualitative sensitivity. The inference of new design variables becomes quite precise compared to the previous studies; thus we do not need to carry out trial optimization to assure optimality. This makes it possible to carry out optimization with a qualitative formulation. In this study, we focus our attention on the convergence and the research effectiveness of the proposed method, but in the future, we will apply the method to optimization of the upper part of a design (for example, an optimum layout problem).

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