Active Flutter Suppression of a Vertical Pipe Conveying Fluid*

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This paper presents a study on the flutter suppression of a vertically cantilevered pipe conveying fluid. The response of the cantilevered pipe is controlled by the torques produced by two pairs of tendons attached to the pipe and two servomotors. The equation of motion is solved by means of the Galerkin method. To reduce the system order, a suboptimal control law is used. The good agreement between the numerical and experimental results shows that this method is effective for the flutter suppression of the vertically cantilevered pipe conveying fluid.

Key Words: Flexible Structure, Self-Excited Vibration, Vibration Control, Flutter, Vibration Coupled with Fluid Motion, Optimal Control

1. Introduction

The flutter of a pipe conveying fluid has been investigated and reported in many studies such as those by Gregory and Paidoussis⁽¹⁾, Rousselet and Herrmann⁽²⁾, and Bajaj and Sethna⁽³⁾. The flutter suppression of the pipe conveying fluid is an important problem in engineering, e.g., in oil pipelines, heat-exchanger pipes and jet cutting nozzles. Recently, Doki and Tani⁽⁴⁾ studied theoretically the feasibility of an active flutter suppression scheme for pipes conveying fluid.

The purpose of the present study is to propose an effective method for the active flutter suppression of a vertically cantilevered pipe conveying fluid. In order to control the response of the cantilevered pipe, two control torques are produced by two pairs of tendons attached to the pipe. The Galerkin method is used to solve the equation of motion. A suboptimal control law is used to reduce the number of system modes. It

is found from both theoretical and experimental results that this method is effective for suppressing the flutter of the pipe conveying fluid.

2. Theoretical Analysis

Let us consider a vertically cantilevered pipe of length L , flexural rigidity EI , mass m_b per unit length and incompressible fluid of mass per unit length flowing through the pipe with a constant velocity V as shown in Fig. 1. A tendon mechanism consists of two servomotors, two pairs of strings and two moment sticks. Two servomotors generate two torques $T_1(t)$ and $T_2(t)$. It is assumed that the motions of the pipe in the y and z directions are not coupled. This is because the motion of the pipe is suppressed by the control before its deflection becomes large. With W_1 and W_2 denoting the deflection of the pipe in the y and z directions, the governing equations of the motion and the boundary conditions of the pipe are given by

$$\begin{aligned} (m_b + m_f) \frac{\partial^2 W_1}{\partial t^2} + m_s \delta(x - x_1) \frac{\partial^2 W_1}{\partial t^2} + E^* I \frac{\partial^5 W_1}{\partial t \partial x^4} \\ + 2m_f V \frac{\partial^2 W_1}{\partial t \partial x} + c \frac{\partial W_1}{\partial t} \\ + EI \frac{\partial^4 W_1}{\partial x^4} + (m_f + m_b) g \frac{\partial W_1}{\partial x} + \left\{ Km_f V^2 \frac{\partial^2 W_1}{\partial x^2} \right. \\ \left. - (m_f + m_b) g (L - x) \right\} \frac{\partial^2 W_1}{\partial x^2} \end{aligned}$$

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Fig. 1 Model of analysis and coordinate system

$$\begin{aligned}
 & -\left\{K_T d^2 \frac{\partial W_1}{\partial x} + T_1(t)\right\} \frac{\partial \delta(x-x_1)}{\partial x} = 0, \quad (1) \\
 & (m_b + m_f) \frac{\partial^2 W_2}{\partial t^2} + m_s \delta(x-x_1) \frac{\partial^2 W_2}{\partial t^2} + E^* I \frac{\partial^5 W_2}{\partial t \partial x^4} \\
 & + 2m_f V \frac{\partial^2 W_2}{\partial t \partial x} + c \frac{\partial W_2}{\partial t} \\
 & + EI \frac{\partial^4 W_2}{\partial x^4} + (m_f + m_b) g \frac{\partial W_2}{\partial x} + \left\{K m_f V^2 \frac{\partial^2 W_2}{\partial x^2} \right. \\
 & \left. - (m_f + m_b) g (L-x) \right\} \frac{\partial^2 W_2}{\partial x^2} \\
 & - \left\{K_T d^2 \frac{\partial W_2}{\partial x} + T_2(t)\right\} \frac{\partial \delta(x-x_1)}{\partial x} = 0, \quad (2) \\
 & W_1 = \frac{\partial W_1}{\partial x} = 0, \quad W_2 = \frac{\partial W_2}{\partial x} = 0 \quad : x=0, \\
 & \frac{\partial^2 W_1}{\partial x^2} = \frac{\partial^3 W_1}{\partial x^3} = 0, \quad \frac{\partial^2 W_2}{\partial x^2} = \frac{\partial^3 W_2}{\partial x^3} = 0 \quad : x=L, \quad (3)
 \end{aligned}$$

where $\delta(x)$ is the Dirac delta function, m_s is the mass of the moment sticks attached to the pipe at $x=x_1$, K_T is a spring constant of a tendon, K is a nozzle coefficient of the pipe, c and E^* are the outer and inner viscous damping coefficients, and t is time.

We express the deflection as

$$W_1 = \sum_{m=1}^{\bar{M}} a_m(t) \phi_m(x), \quad W_2 = \sum_{m=1}^{\bar{M}} b_m(t) \phi_m(x) \quad (4)$$

where $a_m(t)$ and $b_m(t)$ are unknown time functions and $\phi_m(x)$ is the eigenfunction of the free vibration of a cantilevered beam. Substituting Eq. (4) into Eqs. (1) and (2) and performing the Galerkin integral, the following equations are obtained.

$$\sum_{m=1}^{\bar{M}} (M_{rm} \ddot{a}_m + D_{rm} \dot{a}_m + K_{rm} a_m) = G_r U_1(t), \quad (5)$$

$$\sum_{m=1}^{\bar{M}} (M_{rm} \ddot{b}_m + D_{rm} \dot{b}_m + K_{rm} b_m) = G_r U_2(t) \quad (6)$$

where dot ($\cdot$) stands for differentiation with respect to time and $U_1(t) = T_1(t)$, $U_2(t) = T_2(t)$. The

coefficients M_{rm} , D_{rm} , K_{rm} and G_r are given as functions of L , EI , E^* , m_f , m_b , c , K , K_T , m_s and x_1 . Since Eqs. (5) and (6) have the same form, we consider only Eq. (5) after this.

In order to apply the optimal regulator theory to the motion control of the pipe, a state variable vector is introduced as

$$\mathbf{X}(t) = [a_1, a_2, \dots, a_{\bar{M}}, \dot{a}_1, \dot{a}_2, \dots, \dot{a}_{\bar{M}}]^T \quad (7)$$

where T indicates the transposed matrix. Discretizing the state variable vector with a sampling period Δt , Eq. (5) is rewritten as the following state equation for the digital control system.

$$\mathbf{X}(k+1) = \mathbf{A}\mathbf{X}(k) + \mathbf{B}U_1(k) \quad (k=1, 2, \dots) \quad (8)$$

where k stands for $k\Delta t$ and

$$\mathbf{A} = e^{\mathbf{A}_c \Delta t} : (2\bar{M} \times 2\bar{M}), \quad \mathbf{B} = (e^{\mathbf{A}_c \Delta t} - \mathbf{I}) \mathbf{A}_c^{-1} \mathbf{B}_c : (2\bar{M} \times 1) \quad (9)$$

$$\begin{aligned}
 \mathbf{A}_c &= \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix}, \quad \mathbf{B}_c = \begin{bmatrix} \mathbf{O} \\ \mathbf{M}^{-1}\mathbf{G} \end{bmatrix}, \\
 \mathbf{M} &= [M_{rm}], \quad \mathbf{D} = [D_{rm}], \quad \mathbf{K} = [K_{rm}], \quad \mathbf{G} = \{G_r\}, \\
 \mathbf{I} &= [\delta_{rm}]. \quad (10)
 \end{aligned}$$

We choose the deflection of the pipe at $x=x_2$, as an output of the system,

$$\mathbf{Y}(k) = \mathbf{C}\mathbf{X}(k) \quad (11)$$

where

$$\mathbf{C} = [\phi_1(x_2), \phi_2(x_2), \dots, \phi_{\bar{M}}(x_2), 0, \dots, 0]. \quad (12)$$

In order to realize the digital optimal regulator, a performance index is introduced as follows:

$$J = \frac{1}{2} \sum_{k=0}^{\infty} [\mathbf{X}^T(k) \mathbf{Q} \mathbf{X}(k) + r U_1^2(k)] \quad (13)$$

where $\mathbf{Q}$ and r are the weighting matrix and coefficient about the state variable and the control input, respectively. Minimizing the performance index J , the optimal control of the systems (8) and (11) is sought as follows

$$U_1(k) = -[r + \mathbf{B}^T \mathbf{P} \mathbf{B}]^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A} \mathbf{X}(k) \triangleq -\mathbf{F}_d^o \mathbf{X}(k) \quad (14)$$

where $\mathbf{P}$ is a positive definite matrix and a unique solution of the following Riccati matrix equation.

$$\mathbf{P} = \mathbf{Q} + \mathbf{A}^T \mathbf{P} \mathbf{A} - \mathbf{A}^T \mathbf{P} \mathbf{B} [r + \mathbf{B}^T \mathbf{P} \mathbf{B}]^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A}. \quad (15)$$

Since many vibration modes ϕ_m of the cantilevered beam must be used to analyze the flutter of the pipe conveying fluid with high accuracy, it is necessary to make a reduced-order system even if the time delay of one sampling period is compensated. Therefore, Koust's suboptimization technique⁽⁵⁾ is applied to this optimal control problem. The reduced-order system is given by

$$\begin{aligned}
 \mathbf{X}_d(k+1) &= \mathbf{A}_s \mathbf{X}_d(k) + \mathbf{B}_s U_1(k) \\
 \mathbf{Y}_d(k) &= \mathbf{C} \mathbf{X}(k) \triangleq \mathbf{C}_s \mathbf{X}_d(k) \quad (16)
 \end{aligned}$$

where $\mathbf{A}_s : (2\bar{N} \times 2\bar{N})$, $\mathbf{B}_s : (2\bar{N} \times 1)$, $\bar{N} < \bar{M}$ and

$$\mathbf{X}_d(k) = \mathbf{N} \mathbf{X}(k), \quad \mathbf{N} = \mathbf{C}_s^{-1} \mathbf{C} \quad (17)$$

A suboptimal control law is obtained as follows:

$$\begin{aligned} U_1(k) &= -F_d^o X(k) = -H_d X_d(k), \\ F_d^o &= H_d N, \\ H_d &= F^o N^T (N N^T)^{-1}. \end{aligned} \quad (18)$$

Since it is impossible to measure directly all of the state variables with sensor, a minimal-order observer is designed by means of the Goppinas method, to estimate the state variables which are not detected by the sensor. Furthermore, since the digital computer requires time to compute the control input, the compensation of the time delay is taken into consideration. Finally, we obtain

$$\begin{aligned} U_1(k) &= -H_d A_s \bar{X}_d(k-1) - H_d B_s U_1(k-1), \\ \bar{X}_d(k) &= \bar{D} \omega(k) + \bar{H} Y_d(k), \\ \omega(k) &= \bar{A} \omega(k-1) + \bar{K} Y_d(k-1) + \bar{B} U_1(k-1) \end{aligned} \quad (19)$$

where $\bar{D}$, $\bar{H}$, $\bar{A}$, $\bar{K}$ and $\bar{B}$ are composed of A_s , B_s and Q .

3. Experiment

A schematic diagram of the experimental layout is shown in Fig. 2. The long flexible pipe conveying water was hung vertically and clamped at its upper end. The pipe and two pairs of tendons were made of flexible plastic. The moment sticks were made of aluminum and attached to the pipe at $x_1=0.3$ m. The configuration and material properties of the pipe, stick and tendon are listed in Table 1. The torques of the two motors were determined by two input voltages taking only the resistance of armatures of the motors into consideration. The torque constant and the resistance of armatures of the motors were 0.025 Nm/A and 3.1 Ω , respectively. The deflections in the y and z directions of the pipe were measured at $x_2=0.2$ m by two laser sensors. Two output voltages of the laser sensors were converted into two digital voltages with AD converters and then sent to the digital computer at the sampling period of $\Delta t=20$ ms. According to the control regulator of Eq. (18), the outputs from the sensors were processed and the control inputs $U_1(k)$ and $U_2(k)$ were determined on line by the computer. The control inputs were conver-

ted into analog voltages and then supplied to the two motors. On the other hand, the optimal feedback gain matrices and the observer gain matrices were determined previously off line and preserved in the memory of the computer to be used later on line.

4. Results of Experiment and Simulation

In the present study, 8 eigenfunctions were used to determine the flutter velocity with high accuracy. The 8th order system ($\bar{M}=8$) was reduced to the 3rd order system ($N=3$) to calculate the control input.

Table 2 lists the theoretical and experimental values of natural frequency. The theoretical value of the first mode agrees well with the experimental one. Table 3 lists the theoretical and experimental values of the flutter velocity. The theoretical value almost agrees with the experimental one, being slightly larger than the latter.

The response curves of the flutter in the case with the flow speed $V=4.7$ m/s and without control are shown in Fig. 3. It can be seen that the deflection amplitude increases gradually with time.

Figure 4 shows the response curves obtained by simulation and experiment in the case with control. Figure 5 shows the control inputs of simulation and experiment. In these cases, the weighting matrix and coefficient were chosen as $Q_{ij}(i=j=1, 9)=200$ and $r=1000$, respectively. The initial disturbance was given

Table 1 Values of Parameters

Length of Pipe	$L=1(\text{m})$
Outer Diameter of Pipe	$d_o = 9.4(\text{mm})$
Inner Diameter of Pipe	$d_i = 8.2(\text{mm})$
Young's Modulus	$E = 1.3 \times 10^3(\text{N/m}^2)$
Inner Damping Coefficient	$E^* = 6.0 \times 10^5(\text{N} \cdot \text{s/m}^2)$
Outer Damping Coefficient	$c = 0.004(1/5)$
Length of Moment Stick	$d = 30.0(\text{mm})$
Mass of Moment Stick	$m_s = 10.9 \times 10^{-3}(\text{kg})$
Spring Constant of Tendon	$K_T = 340(\text{N/m})$
Mass per Unit Length of Pipe	$m_b = 1.6 \times 10^{-2}(\text{kg/m})$
Mass per Unit Length of Fluid	$m_f = 5.3 \times 10^{-2}(\text{kg/m})$
Nozzle Parameter	$K = 1.9$

Fig. 2 Schematic diagram of the experimental layout

Table 2 Natural frequencies of the pipe

Mode Number	1st	2nd
Calculated Value	0.69(Hz)	2.22(Hz)
Experimental Value	0.66(Hz)	1.98(Hz)

Table 3 Critical flow velocity of flutter

	V_{cr}
Calculated Value	4.66(m/s)
Experimental Value	4.35(m/s)

Fig. 3 Flutter response curves of the pipe without control : $V=4.7(\text{m/s})$

Fig. 4 Response curves of the pipe with control : $V=4.7(\text{m/s})$, $r=1000$

Fig. 5 Control input : $V=4.7(\text{m/s})$, $r=1000$

at the tip of the pipe under control. From these figures, it can be seen that the experimental results agree well with the simulation ones, and that the flutter can be suppressed effectively by means of the present method.

5. Conclusion

The experiment and the simulation of the active flutter suppression of the pipe conveying water were performed by using the tendon mechanism and the optimal regulator theory with suboptimization technique. The present actuator and control method are effective for the flutter suppression of the pipe conveying water.

References

- (1) Gregory, R.W. and Paidoussis, M.P., Unstable Oscillation of tubular cantilevers conveying fluid,

- I. Theory ; II. Experiments, Proc. Royal Society (London), Vol. 293 (1966), p. 512 and p. 528.
- (2) Rousselet, J. and Herrmann, G., Dynamic Behavior of Continuous Cantilevered Pipes Conveying Fluid Near Critical Velocities, ASME J. Applied Mechanics, Vol. 48 (1981), p. 943.
- (3) Bajaj, A.K. and Sethna, P.R., Flow Induced Bifurcations in Three - Dimensional Oscillatory Motions of Continuous Tubes, SIAM J. Applied Mathematics, Vol. 44 (1984), p. 270.
- (4) Doki, H. and Tani, J., Active Control of Cantilevered Pipes Conveying Fluid, Proc. Int. Conf. on Computational Mechanics, G. Yagawa, ed., p. VI-47, Springer Verlag.
- (5) Koust, R.L., Suboptimal Control of Linear Time-Invariant Systems Subject to Control Structure Constraints, IEEE Transactions, Automation and Control, Vol. 15 (1970), p. 557.