Characteristic Evaluation and Comparison of Robotic Mechanisms*
(Evaluation and Comparison Based on Static Characteristics)

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This paper compared and evaluated several kinds of representative robotic mechanisms based on static characteristics. Especially, open-loop mechanisms and two kinds of closed-loop mechanisms which are parallel crank mechanisms were adopted as the representative robotic mechanisms. On the characteristic comparison of the robotic mechanisms, the previous research had been carried out for the condition under which the configurations of the mechanisms were limited to a certain form. However, static characteristics of the robots are greatly changed by the mechanism configurations. Therefore, this paper paid special attention to the changes of each mechanism characteristic by these configurations. Particularly, geometrical characteristics, strength characteristics and characteristics based on input/output displacement of those robotic mechanisms were compared and evaluated. As a result, the superiority or inferiority in the characteristics of these robotic mechanisms was successfully revealed.

**Key Words**: Robotics, Mechanism, Characteristic Mechanism Configuration

1. Introduction

As working capacity of robots is significantly affected by the characteristics of their arm or leg mechanisms, many studies regarding selection of robotic mechanisms have been carried out\(^{(1,2)}\). In particular, several recent studies have been reported on comparison of plural robotic mechanisms\(^{(3),(4)}\). However, these reports mainly compared the robotic mechanisms, assuming that attitude was fixed and the link length ratio of each robotic mechanism was set to a certain value.

Previously, we clarified that characteristics of robotic mechanisms largely depended on their attitudes and link length ratios\(^{(5),(7)}\) and also revealed that robotic mechanisms could be synthesized in order that joint forces and bending moments could be minimized by making clear the relationship between the characteristics of robotic mechanisms and their configurations in advance\(^{(8)}\).

In designing robotic mechanisms by using configurations of mechanisms as design variables, characteristics of the robotic mechanism, which have been properly chosen as the results of comparing with other robotic mechanisms, will largely change with the progress of synthesis, because their characteristics change with mechanism configurations, as described above. Namely, even if a robotic mechanism is chosen because of its superior characteristics on a certain configuration, its characteristics will change in the course of designing. As a result, the characteristics of the synthesized robotic mechanisms can be worse than those which a designer expects. Furthermore, it is sufficiently predicted that the robot which uses other mechanisms can have superior working capacity.

Also in the design of robotic mechanisms, some characteristics may be required to be optimized simultaneously. However, as the range of link length ratios or attitudes is limited in order to optimize a certain characteristic, it is difficult to improve other characteristics. To overcome this problem, designers have to select robotic mechanisms that show good characteristics over a wide range of attitudes and link length

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ratios at first.

We have already shown the relationships between the mechanism configurations and the characteristics of a parallel crank mechanism, which is one of the most representative robotic mechanisms, by using static characteristic charts[6,9]. Static characteristic charts show changes of characteristics by using contour graphs and can make it easy to understand changes of mechanism characteristics with their attitudes. Furthermore, most of the characteristics of arbitrary parallel crank mechanisms usually can be determined from static characteristic charts regarding a certain link length ratio by using coordinate transformation of the working space and transformation equations of characteristic values.

Thus, characteristics of arbitrary mechanisms can be determined by using a few characteristic charts. However the coordinate transformations and the transformation equations have strong non-linearity so that designers may not predict how characteristics of robotic mechanisms will change with large changes of their link length ratios. Therefore, it is almost impossible in designing robotic mechanisms to optimize their shapes while link length ratios are being changed randomly.

As described above, it is necessary to consider the changes of characteristics of robotic mechanisms with their configurations. If they are made clear, it becomes easy to select and synthesize robotic mechanisms by using mechanism configurations as variables.

Thus, in this paper we first reveal the changes of mechanism characteristics with mechanism configurations regarding the representative open-loop and closed-loop mechanisms which are often used as robot arms or robot legs. Furthermore, comparison and evaluation of the robotic mechanisms considering the changes of mechanism configurations are carried out based on those results. As a result, some characteristics of each robotic mechanism are made clear, and useful data for selection and synthesis of robotic mechanisms are revealed.

2. Basic Characteristics of the Mechanisms

2.1 Basic characteristics of the mechanisms

The robotic mechanisms, which are evaluated and compared with each other in this paper, are shown in Fig. 1. Figure 1(a) shows an open-loop mechanism. Its driving shafts are usually connected to points A and B. Figures 1(b) and (c) show parallel crank mechanisms, which are representative robotic closed-loop mechanisms. Driving shafts of the mechanism shown in Fig. 1(b) are connected to points A and B; others shown in Fig. 1(c) are connected to points B and C. Furthermore, working points of both the mechanisms are points P. In this paper, the mechanism shown in Fig. 1(b) is called “Type-A parallel crank mechanism” or “Type-A mechanism” [abbreviated to “Type-A”] and another, shown in Fig. 1(c), is called “Type-C parallel crank mechanism” or “Type-C mechanism” [abbreviated to “Type-C”]. Regarding those mechanisms, geometrical characteristics as to working space, strength characteristics as to joint forces and bending moments induced in links and characteristics based on input/output displacement of those mechanisms are compared and evaluated.

2.2 Expression method of robotic mechanisms

As reported previously[6], arbitrary configurations of the mechanism shown in Fig. 1(b) can be expressed by using a few representative link length ratios and link angles. In this paper we also express configurations of the robotic mechanisms by a few link length ratios and link angles following the previous report[6]. Furthermore, regarding link length

\[ \text{Fig. 1 Configurations of robotic mechanisms} \]
ratios and link angles of each mechanism, the same symbols are applied when they seem to correspond to each other.

An open-loop mechanism as shown in Fig. 1(a) is expressed by the following variables,
\[
\begin{align*}
L &= l_i + l_h \\
L &= l_i / l_h \\
L &= \theta_i + \theta_h \\
\Delta \theta &= \theta_i - \theta_h
\end{align*}
\]
where \( l_i = PB \), \( l_h = AB \), and \( \theta_h = \angle ABP \).

On an Angle-A mechanism as shown in Fig. 1(b), link lengths are defined as \( l_i = PD \), \( l_h = DE = CB \), \( l_k = AE = CD \), and \( l_k = DE = CB \), and on a Type-C mechanism as shown in Fig. 1(c), these are defined as \( l_i = PD \), \( l_h = DE = BA \), and \( l_k = AE = CD \). Moreover on both the mechanisms, \( \theta_i \) and \( \theta_h \) are defined as shown in Fig. 1(b) and Fig. 1(c), respectively. Based on those definitions, arbitrary configurations of those mechanisms are expressed by the following variables;
\[
\begin{align*}
L &= l_i + l_h + l_k \\
k_i &= l_i / l_h \\
k_h &= (l_h + l_k) / l_i \\
\Delta \theta &= \theta_i - \theta_h
\end{align*}
\]

In this paper, the range of \( \Delta \theta \) is limited by the following equation so that robots cannot take singular attitudes on working.
\[
30^\circ \leq \Delta \theta \leq 150^\circ
\]

3. Evaluation and Comparison of the Mechanisms

As reported previously\(^{(9)}\), the characteristics of the mechanisms were indicated by link length ratios \( k_i \), \( k_h \) and trigonometric functions involving \( \Delta \theta \), \( \theta_h \) (or \( \theta_i \)), which denoted changes of mechanism attitudes. It is almost impossible to compare and evaluate the robotic mechanisms by considering them over the entire range where link length ratios and link angles can be changed, because evaluation and comparison conditions may become infinite.

Here, it is evident that all the mechanisms shown in Fig. 1 show superior movement characteristics at about \( \Delta \theta = 90^\circ \) from the viewpoint of evaluation using pressure angles of mechanisms\(^{(9)}\) and the manipulability measure\(^{(10)}\). Furthermore, robots usually tend to be controlled in order that \( \Delta \theta \) of each leg or arm mechanism may become about \( 90^\circ \). Thus in this paper, evaluation and comparison of the mechanisms are carried out assuming the value of \( \Delta \theta \) as \( 90^\circ \) to focus conditions of evaluation and comparison.

3.1 Geometrical characteristics

3.1.1 Working space In this paper, the space which can be swept by working points of the robotic mechanisms under the condition restricted by Eq.(3) is called the operational space. Also, the space which is swept on working is called the working space. If the total link length \( L \) to sweep the required working space sufficiently can be made shorter, occupied spaces of robot arms or legs can decrease. Therefore, the robotic mechanisms need to be selected in order that their working space is as large as possible, compared with the total link length.

The working space \( S \) of each robotic mechanism can be expressed as the following equation by using the link length ratios and the link angles defined at Eq. (1) and (2), respectively. Here, subscripts 'O', 'A' and 'C' denote characteristic values regarding open-loop, Type-A and Type-C mechanisms, respectively.
\[
S_O = S_A = \frac{2\sqrt{3} k_h k_i}{(1 + k_h)} L^2 \pi
\]
\[
S_C = \frac{2\sqrt{3} k_i k_k}{(1 + k_l)(1 + k_h)} L^2 \pi
\]

As the above equations show, the changes of working spaces \( S \) with total link lengths of open-loop and Type-A mechanisms are equal and depend on the link length ratio \( k_h \). Moreover for the Type-C, \( S_c \) depends on both \( k_i \) and \( k_h \).

Assuming that the total link length \( L \) of each mechanism is equally defined as the unit length, comparison and evaluation as to working spaces are performed. The changes of \( S \) with link length ratios \( k_i \) and \( k_h \) of those mechanisms are shown in Fig. 2. Figure 2(a) shows the changes of both \( S_O \) and \( S_A \).
with \( k_b \) and Fig. 2(b) shows the change of \( S_C \) with \( k_b \), and \( k_b \) by using a contour graph.

As known from Fig. 2, both \( S_0 \) and \( S_\alpha \) have the maximum values at \( k_b = 1 \). \( S_C \) has the maximum value at \( k_b = 1 \), assuming that \( k_1 \) is constant, and also tends to increase with increasing \( k_b \).

Next, the results of comparison of those mechanism characteristics as to working spaces are denoted. It is known from comparison based on Fig. 2 that \( S_0 \) and \( S_\alpha \) are larger than \( S_C \) over the wide range of \( k_b \). Compared with other mechanisms, the working spaces of Type-C become fairly small with decreasing \( k_1 \). Namely from the viewpoint of the working spaces, both open-loop and Type-A mechanisms are superior to Type-C. Furthermore, \( k_b \) of all mechanisms should be set at 1. Moreover \( k_b \) of Type-C should be large.

### 3.1.2 Occupied space index

As described in 3.1.1, the ratio of a working space \( S \) to a total link length \( L \) can be larger so that the essential length of \( L \) to sweep the required working space can become short. However, if the closed-loop area of a mechanism is large, interference is often induced and the working capacity of the robot may become worse.

Previously, we have proposed the occupied space index to evaluate such a characteristic. The occupied space index is defined as the ratio of a closed loop area to the square of the distance between points \( O \) and \( A \) shown in Fig. 1; interference is apt to occur with the increasing value of the occupied space index.

The occupied space index \( \Lambda \) of each mechanism is expressed by the following equations, respectively.

\[
\Lambda_0 = 0 \quad (6)
\]

\[
\Lambda_\alpha = \frac{k_2}{(k_1 + 1)(k_2^2 + 1)} \quad (7)
\]

\[
\Lambda_C = \frac{k_2(k_1 + 1)}{k_1 k_2^2 + (k_2 + 1)} \quad (8)
\]

Here \( \Delta \theta = 90^\circ \), as described above.

The changes of \( \Lambda \) with \( k_1 \) and \( k_2 \) are revealed in Fig. 3 by means of a contour graph. As known from Fig. 3 on Type-A and Type-C, \( \Lambda \) decreases with increasing \( k_1 \), and if \( k_1 \) is constant, \( \Lambda \) reaches the maximum at about \( k_0 = 1 \). As known from the comparison of these mechanisms, \( \Lambda_C \) is larger than \( \Lambda_\alpha \) and \( \Lambda_0 \) in the wide range of both \( k_1 \) and \( k_2 \), and especially at \( k_1 < 1 \), \( \Lambda_C \) is considerably larger than others.

Considering the results of comparison regarding working spaces and occupied spaces, working spaces of Type-C are smaller than those of other robotic mechanisms and also ratios of closed loop areas to working spaces are larger than those of others. Thus, the geometrical characteristics of Type-C are inferior to those of open-loop and Type-A mechanisms.

### 3.1.3 Relative space index

As the distance between an operational space and a robot body becomes shorter, workpieces held at the working point tend to interfere with the robot body; as it becomes longer, the occupied space of the robot may become larger. In order to design robots suitable for a required operation and a working environment, it is important to consider the distances between operational space and robot body. In order to evaluate such a geometrical characteristic, this paper proposes a relative space index which is defined by the following equation:

\[
\Gamma = \frac{r_{\text{min}}^2}{S} \quad (9)
\]

where \( r_{\text{min}} \) denotes the minimum distance between points \( P \) and \( O \) shown in Fig. 1 at \( \Delta \theta = 30^\circ \); \( S \) denotes the size of operational space. The sizes of \( \Gamma \) correspond to the distances between operational spaces and robot bodies, assuming that the sizes of operational spaces are equal. \( \Gamma_0, \Gamma_\alpha \) and \( \Gamma_C \) of those mechanisms shown in Fig. 1 are defined by the following equations, respectively.

\[
\Gamma_0 = \Gamma_\alpha = \frac{1 + k_2^2 - k_1 \sqrt{3}}{2 \sqrt{3} k_2 \pi} \quad (10)
\]

\[
\Gamma_C = \frac{k_1^2 k_2 + (1 + k_1)^2 - k_1 k_2 (1 + k_1) \sqrt{3}}{2 \sqrt{3} k_1 k_2 (1 + k_1) \pi} \quad (11)
\]

As known from the above equations, the changes of \( \Gamma_0 \) and \( \Gamma_\alpha \) with link length ratios are equal; they depend
on $k_2$ as well as $S_o$ and $S_a$. Furthermore, $\Gamma_c$ depends on both $k_1$ and $k_2$.

Figure 4(a) shows the changes of both $\Gamma_o$ and $\Gamma_a$ with $k_2$; Fig. 4(b) shows the changes of $\Gamma_c$ with $k_1$, and $k_2$ using a contour graph.

As known from the above results, the relative space indices of open-loop and Type-A mechanisms have the minimum at $k_2=1$ and also at $k_2<1$ they increase rapidly. $\Gamma_c$ has the minimum at the value where $k_2$ is defined by the following equation.

$$k_2 = \frac{(1+k_1)}{k_1}$$  \hspace{1cm} (12)

It is known from the comparison of those robotic mechanisms as to occupied space indices, that $\Gamma_o$ and $\Gamma_a$ are larger than $\Gamma_c$ in the wide range of their link length ratios. Namely, open-loop and Type-A mechanisms are convenient robotic mechanisms for robots in which the operational space is required to have a long distance from the body. On the contrary, Type-C is applicable for robots in which the operational space has a short distance from the body.

3.2 Strength characteristics

If robotic mechanisms are designed in order that bending moments and joint forces may be minimized, masses of their moving parts can be reduced and their working capacity can be improved. In this paragraph, comparison and evaluation of these robotic mechanisms regarding the strength characteristics, joint forces $F$ and bending moments $M$, are carried out.

3.2.1 Joint forces When the load $P_F$ is applied to working points of the robotic mechanisms in the positive direction of $y$-axis, a joint force $F$ produced at each joint is expressed by the following equations, respectively. Parameters used here have been defined in paragraph 2.2. On the following equations as to $F$ (and $M$), the first subscript denotes the kind of mechanism, as in 3.1; the second subscript denotes joint positions defined in Fig. 1.

$$F_{\theta_0} = P_F$$  \hspace{1cm} (13)

$$F_{\theta_1} = (1+k_1) \frac{\cos \theta_0}{\sin (\Delta \theta)} P_F$$  \hspace{1cm} (14)

$$F_{\theta_2} = \sqrt{F_{\theta_1}^2 + F_{\theta_2}^2 - 2P_F \cdot F_{\theta_2} \sin \theta_0}$$  \hspace{1cm} (15)

$$F_{\theta_3} = k_2 \frac{\cos \theta_2}{\sin (\Delta \theta)} P_F$$  \hspace{1cm} (16)

$$F_{\theta_4} = \sqrt{F_{\theta_3}^2 + F_{\theta_4}^2 - 2P_F \cdot F_{\theta_4} \sin \theta_0}$$  \hspace{1cm} (17)

The above equations reveal that joint forces $F$ of every mechanism are independent of $k_2$.

Comparison and evaluation regarding joint forces are revealed assuming that the unit load is applied to the working point of each robotic mechanism. As known from the above equations, the joint forces depend on $\theta_0$ (or $\theta_2$), and $\theta_1$ (or $\theta_3$) changes from $0^\circ$ to $360^\circ$ as the mechanism attitudes and the loading directions change. Generally, it is difficult to evaluate and compare those mechanisms by considering the whole range of $\theta_0$ (or $\theta_2$). However, the range of trigonometric functions is within $\pm 1$ so that the change of joint forces with change in trigonometric functions may be predicted easily. Furthermore, $\theta_0$ (or $\theta_2$) takes arbitrary values unlike $\Delta \theta$. Thus in this paper, comparison and evaluation are carried out, assuming that $\Delta \theta = 90^\circ$ and $\theta_0$ (or $\theta_2$) is determined in order to maximize $F$. [For example, $\theta_0$ is set to $0^\circ$ in Eq.(16).]

The above results are simple, so their details are omitted and the summary will be described later. As known from the above equations, the maximum joint forces of Type-A and Type-C increase linearly with $k_i$. Namely, their joint forces may become $(k_i+1)$ times as large as loads acting on working points. On the contrary, joint forces of an open-loop mechanism are always equal to $P_F$ acting on working points. Namely as $k_i$ becomes larger, the joint forces induced in the closed-loop mechanisms may become considerably larger than those of open-loop mechanisms. However, it must not be said that the strength characteristics of open loop mechanisms are superior to those of closed-loop mechanisms. Namely, by using static characteristic charts, the attitudes of robotic
closed-loop mechanisms can be determined in order to minimize their joint forces. As a result, the joint forces of the closed-loop mechanisms can be smaller than those of open-loop mechanisms.

### 3.2.2 Bending moments

Bending moment $M$ produced at each joint of the robotic mechanisms by the load $P_F$ acting on their working points in the positive direction of the $y$-axis is expressed by the following equations respectively.

$$M_{oa} = \frac{k_2}{1 + k_2} \cos (\theta_a + \theta_b) |P_F| L$$  \hspace{1cm} (18)

$$M_{ad} = \frac{k_2 k_0}{(1 + k_i)(1 + k_0)} |\cos (\theta_b)| |P_F| L$$  \hspace{1cm} (19)

$$M_{cd} = \frac{k_2 k_0}{(1 + k_i)(1 + k_0)} |\cos (\theta_a)| |P_F| L$$  \hspace{1cm} (20)

Assuming that $P_F$ and $L$ are the unit load and the unit length respectively, comparison and evaluation regarding bending moments are carried out. Furthermore, $\theta_a$ (or $\theta_b$) is determined in order to maximize $M$, assuming that $\Delta \theta = 90^\circ$, as in 3.2.1. Then, as known from Eqs. (19) and (20), the change of $M_{ad}$ with $k_i$ and $k_0$ is equal to that of $M_{cd}$ with $k_i$ and $k_0$.

Figure 5(a) shows the relationship between $M_{oa}$ and $k_2$; Fig. 5(b) shows the changes of both $M_{ad}$ and $M_{cd}$ with $k_i$ and $k_0$. As known from Fig. 5(a), the maximum value of $M_{oa}$ at $\Delta \theta = 90^\circ$ increases with increasing $k_0$. Furthermore, as known from Fig. 5(b), both $M_{oa}$ and $M_{cd}$ rapidly increase with increasing $k_1$ and $k_2$.

Comparison of those mechanisms regarding bending moments reveals that $M_{oa}$ is larger than both $M_{ad}$ and $M_{cd}$ in the wide range of $k_0$. Therefore, large bending moments are often induced in the robot arm or leg, which consists of open-loop mechanisms. As described in 3.2.1, robotic open-loop mechanisms may not be as seriously affected by joint forces as robotic closed-loop mechanisms. However, bending moments may become considerably larger so that masses of the moving parts of the robots using open-loop mechanisms cannot be reduced from the viewpoint of strength design.

The following results are obtained from the above comparison regarding joint forces and bending moments. From the viewpoint of strength design, the closed-loop mechanisms shown in Figs. 1(b) and 1(c) are good mechanisms, so the masses of the moving parts can be reduced. However, their working attitudes have to be determined carefully in order to minimize loads induced in their joints and links by using the static characteristic charts.

### 3.3 Characteristics based on input/output displacement

In this paragraph, static torque of the moving shafts to hold the attitudes of those mechanisms and the static compliance which leads to displacement error of their working points are compared and evaluated as the characteristics based on input/output displacement.

#### 3.3.1 Holding torque

In this paper, the static torque of the moving shafts required to hold the attitudes of robotic mechanisms with the load acting on their working points is called holding torque. In designing robots in which the working speed is low and a heavy load acts on the working point, the application of the robotic mechanisms to minimize holding torque can decrease the work of the driving system and improve the static driving characteristics efficiently. Furthermore if the robotic mechanisms are designed in order that the input torques of each driving shaft may be equal, so that the same actuator can be used, the maximum accelerations induced at each driving shaft may be equal. As a result, the dynamic characteristics of the robots are expected to be improved.

Holding torque $T_{oa}$ required at each driving shaft of the robotic mechanisms by the load $P_F$ acting on their working points in the positive direction of the $y$-axis is expressed by the following equations respectively.

$$T_{oa} = \frac{1}{1 + k_2} |k_2 \cos (\theta_a + \theta_b) - \cos \theta_b| |P_F| L$$  \hspace{1cm} (21)
In the above equations, the first subscript of $T$ denotes the kind of mechanism as well as the joint force $F$, and the second subscript 'A' or 'B' denotes the torque required by the driving link, of which angle displacement is indicated by $\theta_a$ or $\theta_b$ in Fig. 1, respectively.

Comparison and evaluation regarding holding torque are carried out, assuming that $P_F$ and $L$ are the unit load and the unit length, respectively. Figure 6 shows the changes of $T$ with $k_i$ and $k_2$. The values of $\theta_a$ and $\theta_b$ are determined as in 3.2; then the contour graphs for $T_{\theta A}$ and $T_{\theta B}$ are the same, and those for $T_{\theta A}$ and $T_{\theta B}$ are the same too.

Figure 6 (a) shows the changes of the maximum values of $T_{\theta A}$ and $T_{\theta B}$ with $k_2$ at $\Delta \theta = 90^\circ$. As known from Fig. 6(a), the differences between $T_{\theta A}$ and $T_{\theta B}$ are relatively large at $k_2 < 1$. However at $k_2 \geq 1$, both $T_{\theta A}$ and $T_{\theta B}$ increase gradually with increasing $k_2$, and then the differences decrease. Namely, if the open-loop mechanisms at $k_2 > 2$ are used as robotic mechanisms, the required maximum torque of each driving shaft becomes almost equal. Figure 6(b) shows the relationship between the holding torque of Type-A and $k_2$. As known from the figure, the required maximum torques of both the driving shafts are equal at $k_2 = 1$. However, the differences between them become larger rapidly as $k_2$ separates from 1. Figure 6(c) shows the change in $T_{\theta A}$ with $k_i$ and $k_2$; Fig. 6(d) shows the change in $T_{\theta B}$ with $k_2$. As the figures show, $T_{\theta A}$ increases with both increasing $k_i$ and $k_2$; $T_{\theta B}$ decreases with increasing $k_2$.

As known from the above results, by using Type-C mechanisms as robotic mechanisms of which $k_i$ is small and $k_2$ is larger than 1, the robot can be designed in order that the required torques of both the driving shafts may be almost equal and small. Even if open-loop mechanisms are applied to robotic mechanisms, the required torques of both the driving shafts can easily become almost equal, too. However, those values may become larger than those of Type-A and Type-C. The use of Type-A as robotic mechanisms increases the difference in required torque between the two driving shafts, and these values may become larger than those of Type-C. Thus, Type-A is
inferior to the others as to holding torque.

3.3.2 Displacement error of working point

Ordinarily in robots, the elastic displacement of reduction mechanisms and actuators due to the load acting on the working point is larger than that of links\(^{12}\). Namely, the displacement error occurring at working points is almost due to the compliance of input ends. The working point displacement error of those robotic mechanisms is expressed by the following equations, respectively. In the equations, \( \Delta x \) and \( \Delta y \) denote working point displacement errors in the direction of \( X \) and \( Y \)-axis, respectively, and the second subscript denotes the kind of mechanism.

\[
\Delta x_0 = \frac{CP_1L_1^2}{(1+k_1)^3} \left[ \frac{1}{2} \left( \frac{2k_2^2 \sin 2(\theta_A + \theta_h) + \sin 2 \theta_A}{2k_2 \sin (2 \theta_A + \theta_h)} \right) \right] + (k_1 + 1)^2 \sin 2 \theta_a \] (31)

\[
\Delta y_0 = \frac{CP_1L_1^2}{(1+k_1)^3} \left[ \frac{1}{2} \left( k_2^2 \cos (\theta_A + \theta_h) - \cos \theta_A \right) \right] + k_2^2 \cos^2 (\theta_A + \theta_h) \] (27)

\[
\Delta x_0 = \frac{CP_1L_1^2}{(1+k_1)^3} \left[ \frac{1}{2} \left( \sin 2 \theta_A + k_2 \sin 2 \theta_h \right) \right] \] (28)

\[
\Delta y_0 = \frac{CP_1L_1^2}{(1+k_1)^3} \left[ \frac{1}{2} \left( \cos^2 \theta_A + k_2 \cos^2 \theta_h \right) \right] \] (29)

\[
\Delta x_c = \frac{CP_1L_1^2}{(1+k_1)^3(1+k_2)^3} \left[ \frac{1}{2} \left( k_2 \sin 2 \theta_A \right) \right] \] (30)

\[
\Delta y_c = \frac{1}{(1+k_1)^3} \] (33)

\[
\Delta y_c = \frac{k_2^2 \sin^2 (\theta_A + \theta_h)}{(1+k_1)^3(1+k_2)^3} \] (34)

Figures 7(d) and (e) show the results obtained under the conditions of Eq.(33) and Eq.(34), respectively. As the figures show, the working point displacement error of open-loop and Type-A mechanisms on \( \Delta \theta = 90^\circ \) becomes the minimum at \( k_2 \approx 0.6 \) and \( k_1 = 1 \), respectively. On Type-C, \( \Delta y_c \) increases with increasing \( k_1 \) and \( k_2 \), and in particular \( \Delta y_c \)
changes rapidly with changing $k_t$. Furthermore, $\Delta v_c$ decreases with increasing $k_t$ under the condition expressed in Eq. (34).

As known from the evaluation of those mechanisms, working point displacement errors of Type-C are less than those of other mechanisms in the wide range of link length ratios. Although the working point displacement errors become smaller than those of Type-A at $k_t \geq 1$, the appropriate range of $k_t$ is limited. Thus, even if $k_t$ is determined in order to minimize working point displacement error, it may be difficult to optimize other characteristics by changing link length ratios. In open-loop mechanisms, the working point displacement errors are larger than those of closed-loop mechanisms in the whole range of link length ratio, and in particular at $k_t \geq 1$ they become fairly large. Namely Type-C is the mechanism suitable for the robots which are required to work precisely, and open-mechanisms are not.

4. Conclusions

(1) Geometrical characteristics of open-loop mechanisms are superior to those of closed-loop mechanisms. However, bending moments induced in their links and input torque required to hold their attitudes and displacement errors occurring at the working points of open-loop mechanisms are larger than those of closed-loop mechanisms. Thus, open-loop mechanisms are robotic arm or leg mechanisms suitable for robots which need to be compact and are required to sweep a large operational space with light load acting on their working points.

(2) The characteristics based on input/output displacement and strength characteristics of Type-C parallel crank mechanisms are superior to those of open-loop and Type-A mechanisms. The robots use these mechanisms as their arms or legs so that displacement errors of the working points and masses of moving parts can be reduced. However, geometrical characteristics of Type-C mechanisms are inferior to those of other mechanisms. When this mechanism is applied to the robots which are required to sweep large operational spaces, its link length may become longer. As a result, bending moments induced in its links may become larger than those of the other mechanisms. In designing a robot which is required to reduce masses of moving parts and to sweep a large operational space, the application of Type-C mechanisms must be considered carefully.

(3) Strength and input/output displacement characteristics of Type-A parallel crank mechanisms are superior to those of open-loop mechanisms, and their geometrical characteristics are superior to those of Type-C. Namely, from the viewpoint of static characteristics, Type-A mechanisms are robotic mechanisms suitable for the robots in which various types of work are required. However, Type-A mechanisms are not used as frequent as Type-C and open-loop mechanisms at present. It is worthwhile to consider the application of Type-A mechanisms as robotic arm or leg mechanisms.

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