Adaptive Control for Robot Manipulators with Flexible Joints*

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This paper presents an adaptive control scheme for robot manipulators with flexible joints. Our control scheme theoretically guarantees that the tracking error converges to zero and that all the signals in the system remain bounded. We also analyze the robustness of the adaptive controller. To show the capability of the proposed controller, an example of a one-link robot manipulator with one revolute flexible joint is considered. The experimental results are given to demonstrate the effectiveness of this method.

Key Words: Stability, Adaptive Control, Robot, Flexibility, Parameter Linearization

1. Introduction

Most industrial robot manipulators employ DC or AC motors connected in series with high-ratio gear boxes used mainly for speed reduction and torque increase. In some applications, transmission belts or long shafts are also used in the driving system. When high performance at a high speed is required, the flexibility of joints (which is caused by the elasticity of gear teeth, shafts and so on) has to be taken into account in the modeling and the design of the control laws⁽¹⁾. If we assume that the elasticity may be modeled as a linear spring, we obtain a dynamic model of twice the order of one with rigid joints. Moreover, we lose the matching property between the nonlinearity and input in this case⁽²⁾, so the design of control laws is a much more difficult task.

Several advanced control algorithms have been proposed for robot manipulators with flexible joints when the physical parameters are known. These

approaches are based on nonlinear dynamic feedback linearization⁽³⁾, pseudo-linearization technique⁽⁴⁾, and dynamic feedback with sliding mode regulators⁽⁵⁾. A simple PD controller that guarantees the global stability about a reference position has been presented⁽⁶⁾. Singular perturbation method is used, and a velocity control law is designed to stabilize the fast modes of the system⁽²⁾. A global nonlinear tracking control is introduced⁽⁷⁾.

If the physical parameters are not exactly known, however, we have to consider more advanced control schemes, e.g., adaptive or robust controls. As regards the adaptive control, several schemes are proposed^{(8),(9)}, but they are suitable only for robot manipulators with rigid joints. In recent years some research on control for robot manipulators with flexible joints has been carried out⁽¹⁰⁾⁻⁽¹²⁾; however, it is assumed that the stiffness of the joints is known and/or relatively large. In this paper, we present an adaptive control scheme without the assumptions on the joint stiffness as in Refs.(10)-(12). However, in the same way as in Refs.(10)-(12), we also assume here that the link angles and velocities are measurable in addition to the motor angles and velocities. As a

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result, our control scheme guarantees that the tracking error converges to zero and that all the signals of the system remain bounded.

The organization of this paper is as follows: in section 2, the dynamic model of the manipulators with flexible joints, its properties, and the linear parametrization of the model are discussed. The adaptive control law is proposed and its robustness is examined in section 3. In section 4, the experimental results are given. The conclusions are drawn in section 5.

2. Dynamic Model

We consider a manipulator with n revolute joints. The following general assumptions are made regarding the mechanical structure.

[A1.] The flexibility in the joint can be modeled as a linear torsional spring.

[A2.] The kinetic energy of the motor's rotor is due to its own rotation. The rotor/gear inertia is symmetric about its axis of rotation.

Let q_1 represent the $n \times 1$ vector of the positions of the links, and q_2 represent the $n \times 1$ vector of the equivalent motor positions (i.e. $q_{2i} = \theta_{mi}/G_{ri}$, θ_{mi} is the actual position of the i th motor, G_{ri} is the gear ratio for the i th joint); then we define $q = [q_1^t, q_2^t]^t$. For simplification, we use "motor position" by omitting the "equivalent" hereafter. The external torques applied to the joints are represented by the $n \times 1$ vector u . By Lagrangian mechanics under [A1] and [A2], we can obtain the following dynamic model for the manipulator:

$$B_1(q_1)\ddot{q}_1 + C_1(q_1, \dot{q}_1)\dot{q}_1 + G_1(q_1) + D_1\dot{q}_1 + K_1(q_1 - q_2) = 0 \quad (2.1)$$

$$J_m\ddot{q}_2 + D_2\dot{q}_2 + K_1(q_2 - q_1) = u \quad (2.2)$$

$B_1(q_1)$ is the $n \times n$ positive symmetric inertia matrix for the "rigid" robot manipulator, i.e., $B_1(q_1) = (B_{ij}(q_1))$, which can be calculated using standard techniques (e.g., Eq. (6.66) in Ref. (13)). $C_1(q_1, \dot{q}_1)$ is an $n \times n$ matrix. $C_1(q_1, \dot{q}_1)\dot{q}_1$ contains coriolis and centripetal forces and torques. K_1 is the positive stiffness matrix, $K_1 = \text{diag}[k_1, \dots, k_n]$. $G_1(q_1)$ is the gravitational torques acting on the manipulator. D_1 and D_2 are the damping ratio matrix of the link and motor, respectively. J_m is the diagonal positive inertia matrix of motor rotors. We can rewrite Eq. (2.1) and Eq. (2.2) as

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + D\dot{q} + Kq = \tau \quad (2.3)$$

where $\tau = [0, u^t]^t$ is a $2n \times 1$ vector. $B(q)$, $C(q, \dot{q})$, $G(q)$, K and D are structured as follows:

$$B(q) = \begin{pmatrix} B_1(q_1) & 0 \\ 0 & J_m \end{pmatrix}; C(q, \dot{q}) = \begin{pmatrix} C_1(q_1, \dot{q}_1) & 0 \\ 0 & 0 \end{pmatrix};$$

$$K = \begin{pmatrix} K_1 & -K_1 \\ -K_1 & K_1 \end{pmatrix}; G(q) = \begin{pmatrix} G_1(q_1) \\ 0 \end{pmatrix};$$

$$D = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}.$$

There are some useful properties of the dynamic model Eq. (2.3) which will be used in the design of the controller.

[P1.] If the elements of $C_1(q_1, \dot{q}_1)$ are defined as

$$C_{1kj} = \sum_{i=1}^n \frac{1}{2} \left\{ \frac{\partial B_{1kj}}{\partial q_{1i}} + \frac{\partial B_{1ki}}{\partial q_{1j}} - \frac{\partial B_{1ij}}{\partial q_{1k}} \right\} \dot{q}_{1i}, \quad (2.4)$$

then the matrix $\dot{B} - 2C$ is skew-symmetric.

[P2.] The linear parametrizability of the dynamic model Eq. (2.3) enables us to write:

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + D\dot{q} + Kq = Y(q, \dot{q}, \ddot{q})P = \tau \quad (2.5)$$

where $Y(q, \dot{q}, \ddot{q}) \in R^{2n \times r}$ is the regressor matrix of known functions of $q, \dot{q}, \ddot{q}$ independent of the physical parameters of the manipulator. $P \in R^r$ is a vector of the constant unknown physical parameters of the manipulator.

Note that for a given manipulator, the size of the vector P , i.e., the value of r , is usually greater than n and can vary with application.

3. Adaptive Control Law Design

Besides the assumptions [A1] and [A2], we must make the following assumptions:

[A3.] The link position q_1 and velocity $\dot{q}_1$ and the motor position q_2 and velocity $\dot{q}_2$ are measurable.

[A4.] The desired trajectory $q_{1d}(t) \in R^{n \times 1}$ is four times differentiable.

To obtain the bounded desired motor and link position, velocity and acceleration, [A4] is necessary. This assumption can be easily met in the design of the desired trajectory.

In the design of the controller, we use the following notations:

$$\begin{aligned} e &= q_d - q, \dot{q}_r = \dot{q}_d + \Lambda e \\ s &= \dot{e} + \Lambda e, e_p = P - \hat{P} \end{aligned} \quad (3.1)$$

where $s = [s_1^t, s_2^t] \in R^{2n}$, $q_d = [q_{1d}^t, q_{2d}^t]^t \in R^{2n}$, $e_1 \in R^n$ and $e_2 \in R^n$ are the errors for the link and motor, and e_p is the parameter estimated error. We use the superscript " $\hat{}$ " above the parameter vector to express its estimated value.

By the linear parametrizability, we define

$$\begin{aligned} Y_d(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d)P \\ = B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) + D\dot{q}_r + Kq_d \end{aligned} \quad (3.2)$$

where $Y_d(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d) \in R^{2n \times r}$, and for simplification it can be rewritten as

$$\begin{aligned} Y_d(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d) \\ = \begin{bmatrix} Y_{d1}(q_1, \dot{q}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, q_{2d}) \\ Y_{d2}(q_2, \dot{q}_2, q_{2d}, \dot{q}_{2d}, \ddot{q}_{2d}, q_{1d}) \end{bmatrix} \end{aligned} \quad (3.3)$$

where Y_{d1} and $Y_{d2} \in R^{n \times r}$. Note: $Y_{d1}P$ independent of $\dot{q}_{2d}, \ddot{q}_{2d}$ and $Y_{d2}P$ is also independent of $\dot{q}_{1d}, \ddot{q}_{1d}$.

Now we give the adaptive controller:

$$0 = Y_{d1}\hat{P} + K_{D1}s_1, \quad (3.4)$$

$$u = Y_{d2}\hat{P} + K_{D2}s_2 + \varepsilon(s_2, t)(s_1^t K_{D1}s_1). \quad (3.5)$$

The estimation law for the parameters is presented as follows:

$$\dot{\hat{P}} = M^{-1}Y_{d1}^t s \quad (3.6)$$

where

$$K_D = \begin{pmatrix} K_{D1} & 0 \\ 0 & K_{D2} \end{pmatrix}; \quad \varepsilon(s_2, t) = \frac{s_2}{\|s_2\| + \lambda_1 e^{-\lambda_2 t}}.$$

K_{D1}, K_{D2} are $n \times n$ positive definite diagonal gain matrixes. λ_1 and λ_2 are positive real numbers. The motor's desired trajectory q_{2d} can be calculated by Eq. (3.4). M is a constant positive definite adaptation gain matrix.

The following theorem summarizes the stability property of the system under the adaptive control law Eqs. (3.4), (3.5) and Eq. (3.6).

Theorem 1: Consider the manipulator dynamics Eq. (2.3) under the adaptive control law Eqs. (3.4), (3.5) and Eq. (3.6). Then the tracking error e and $\dot{e}$ converge to zero as $t \rightarrow \infty$, and all the signals in the system remain bounded.

Proof: We substitute Eq. (3.4) and Eq. (3.5) into Eq. (2.3), and obtain the following error dynamics:

$$B\dot{s} + Cs = -(D + K_D)s - Ke - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon(s_2, t)(s_1^t K_{D1}s_1) + Y_{d1}e_p. \quad (3.7)$$

Select a Lyapunov function candidate as

$$V = \frac{1}{2}s^t Bs + \frac{1}{2}e^t Ke + \frac{1}{2}e_p^t M e_p. \quad (3.8)$$

Differentiating Eq. (3.8) using Eq. (2.3), noting $s^t(B - 2C)s = 0$, we can obtain

$$\dot{V} = s^t(B\dot{s} + Cs) + e^t K \dot{e} + e_p^t M \dot{e}_p. \quad (3.9)$$

Substituting the error dynamics Eq. (3.7) into Eq. (3.9), and setting the parameter estimation law as Eq. (3.6), which results in

$$s^t Y_{d1} e_p + e_p^t M \dot{e}_p = 0, \quad (3.10)$$

we have

$$\begin{aligned} \dot{V} &= -s^t(K_D + D)s - e^t \Lambda K e \\ &\quad - s^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon(s_2, t)(s_1^t K_{D1}s_1) \\ &\leq -s^t(K_D + D)s - e^t \Lambda K e. \end{aligned} \quad (3.11)$$

Since $(K_D + D)$ and K are positive definite, we have $\dot{V} \leq 0$. This indicates that s and e converge to zero as $t \rightarrow \infty$ and that the parameter estimated error e_p is bounded. In the calculation of control input u in Eq. (3.5), we need $q_{2d}, \dot{q}_{2d}$ and $\ddot{q}_{2d}$, which are bounded (shown in Remark 2). We conclude that the tracking errors (e and $\dot{e}$) converge to zero as $t \rightarrow \infty$ and that all the signals in the system remain bounded.

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Remark 1. In practice, we may simplify the algorithm by not estimating *a priori* known parameters (for example, the inertia and damping ratio of the motors can be known from their specifications). We

can decompose the parameters P into $[P_u^t, P_k^t]^t : P_u \in R^a (a \leq r)$ are the unknown parameters; $P_k \in R^{(r-a)}$ are known. Correspondingly, $Y_d = [Y_{du}^t, Y_{dk}^t]^t$. The control laws Eqs. (3.4), (3.5) and Eq. (3.6) can be written as

$$\begin{aligned} 0 &= Y_{du1}\hat{P}_u + Y_{dk1}P_k + K_{D1}s_1 \\ u &= Y_{du2}\hat{P}_u + Y_{dk2}P_k + K_{D2}s_2 + \varepsilon(s_2, t)(s_1^t K_{D1}s_1) \\ \dot{\hat{P}}_u &= M_u^{-1}Y_{du1}^t s. \end{aligned}$$

With e_{pu} and M_u in the place of e_p and M in the Lyapunov function (3.8), we obtain

$$\dot{V} \leq -s^t(K_D + D)s - e^t \Lambda K e.$$

We can conclude that Theorem 1 also holds in this case.

Remark 2. Now we describe the property of the boundedness of $q_{2d}, \dot{q}_{2d}$ and $\ddot{q}_{2d}$. The control input Eq. (3.5) involves $q_{2d}, \dot{q}_{2d}$ and $\ddot{q}_{2d}$, which can be calculated by Eq. (3.4). From Eq. (3.8) and Eq. (3.11), we know that $e, \dot{e}$ and e_p are bounded. Eq. (3.4) can be written as

$$q_{2d} = q_{1d} + \tilde{K}_1^{-1}(\tilde{Y}_{d1}\tilde{P} + K_{D1}s_1), \quad (3.12)$$

where $\tilde{P}$ is defined as one part of $\hat{P}$, except for the parameter of the stiffness. As one part of $Y_{d1}, \tilde{Y}_{d1}$ corresponds to $\tilde{P}$. From the estimation law Eq. (3.6) and Eq. (3.3), $\tilde{Y}_{d1}\tilde{P}$ is a bounded function of $q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, q_1$ and $\dot{q}_1$, so q_{2d} and $\dot{q}_{2d}$ are bounded. By using Eq. (2.1) we know that $\ddot{q}_1$ exists and is bounded. Differentiating Eq. (3.12), we can obtain

$$\begin{aligned} \dot{q}_{2d} &= \dot{q}_{1d} + \tilde{K}_1^{-1}(\dot{\tilde{Y}}_{d1}\tilde{P} \\ &\quad + \tilde{Y}_{d1}\dot{\tilde{P}} + K_{D1}\dot{s}_1 + \dot{\tilde{K}}_1(q_{1d} - q_{2d})). \end{aligned}$$

For $\dot{\tilde{Y}}_{d1}\tilde{P}$ and $\tilde{Y}_{d1}\dot{\tilde{P}}$ are bounded functions of $q_1, \dot{q}_1, \ddot{q}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}$ and $q_{1d}^{(3)}$, and $\dot{\tilde{K}}_1$ is bounded. Therefore $\dot{q}_{2d}$ and $\ddot{q}_{2d}$ are also bounded. The same procedure can be repeated for $q_{1d}^{(3)}$ and $\ddot{q}_{2d}$ by differentiating (2.1) and (3.12). Therefore the control laws Eqs. (3.4), (3.5) and Eq. (3.6) involve the measurement of $q_1, \dot{q}_1, \ddot{q}_1$ and $\ddot{q}_2$ and calculation of $q_{2d}, \dot{q}_{2d}, \ddot{q}_{2d}$, which are all bounded. We will later show the calculation of $\ddot{q}_{2d}$ for a one-link manipulator with a flexible joint.

Remark 3. Without the term $\varepsilon(s_2, t)(s_1^t K_{D1}s_1)$ in Eq. (3.5), we can also prove the stability of the system, as done in the proof of Theorem 1. To increase the effect of the link signal feedback, adding a term $\varepsilon(s_2, t)(s_1^t K_{D1}s_1)$, we can obtain a better performance by a suitable choice of λ_1 and λ_2 .

Analysis of robustness

In practice, there are some unmodeled effects, for example, the nonlinear effects of flexible joints and friction. In the flexible joints, the stiffnesses of joints are variable rather than constant, even though they can be approximated to a linear spring with small errors. Friction has a significant effect on the motion of the manipulator. Exact description of the nonlinear friction is very difficult. Some research has studied

adaptive⁽¹⁴⁾ and nonadaptive⁽¹⁵⁾ friction compensation in robot manipulators.

Considering the nonlinear effects of the flexibility, friction and other unmodeled disturbances, we assume that the dynamic model Eq. (2.3) can be rewritten as

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + Kq + D\dot{q} = \tau + \tau_d, \quad (3.13)$$

where τ_d is an unmodeled disturbance which is bounded as

$$\|\tau_d\| \leq d_0 + d_1\|\dot{e}\| + d_2\|e\|, \quad (3.14)$$

for some positive constants d_0, d_1 and d_2 .

Now we analyze the stability of the system controlled by the adaptive control laws Eqs. (3.4), (3.5) and Eq. (3.6) under the unmodeled effect τ_d . We have the following error dynamics:

$$B\dot{s} + Cs = -(D + K_D)s - Ke - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon(s_2, t)(s_1^t K_{D1} s_1) + Y_d e_p - \tau_d. \quad (3.15)$$

For simplification, we chose Λ to be a diagonal matrix. We select a Lyapunov function candidate as

$$V_2 = \frac{1}{2}s^t B s + \frac{1}{2}e^t K e + \frac{1}{2}e_p^t M e_p + e^t \Lambda (K_D + D)e. \quad (3.16)$$

Differentiating Eq. (3.16) we obtain

$$\begin{aligned} \dot{V}_2 = & s^t B \dot{s} + \frac{1}{2}s^t \dot{B} s + e^t K \dot{e} + e_p^t M \dot{e}_p \\ & + 2e^t \Lambda (K_D + D)\dot{e}. \end{aligned} \quad (3.17)$$

In the same way as what has been used in the proof of Theorem 1, simplifying $\dot{V}_2$ using Eq. (3.15) gives:

$$\begin{aligned} \dot{V}_2 = & -\dot{e}^t (K_D + D)\dot{e} - e^t (\Lambda K + \Lambda^2 (K_D + D))e \\ & - s^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon(s_2, t)(s_1^t K_{D1} s_1) + s^t Y_d e_p + e_p^t M \dot{e}_p - s^t \tau_d. \end{aligned} \quad (3.18)$$

From Eq. (3.14) and Eq. (3.1), we have

$$\begin{aligned} \|s^t \tau_d\| \leq & (\|\dot{e}\| + \|\Lambda\|\|e\|)(d_0 + d_1\|\dot{e}\| + d_2\|e\|) \\ = & d_0\|\dot{e}\| + d_1\|\dot{e}\|^2 + c_0\|e\| + c_1\|e\|^2 + c_2\|e\|\|\dot{e}\| \end{aligned} \quad (3.19)$$

where $c_0 = d_0\|\Lambda\|$, $c_1 = d_2\|\Lambda\|$ and $c_2 = d_2 + d_1\|\Lambda\|$. Using the estimation law for parameters Eq. (3.6), we obtain

$$\begin{aligned} \dot{V}_2 \leq & -\dot{e}^t K_D \dot{e} - e^t \Lambda^2 K_D e + \|s^t \tau_d\| \\ \leq & -a_1\|\dot{e}\|^2 - a_2\|e\|^2 + c_2\|e\|\|\dot{e}\| + d_0\|\dot{e}\| + c_0\|e\| \end{aligned} \quad (3.20)$$

where $a_1 = \lambda_k - d_1$, $a_2 = \lambda_k \lambda_A^2 - c_1$ and λ_k, λ_A are the smallest eigenvalues of matrices K_D and Λ . A further manipulation of Eq. (3.20) results in

$$\begin{aligned} \dot{V}_2 \leq & -\frac{c_2}{2}(\|\dot{e}\| - \|e\|)^2 - p_1(\|\dot{e}\| - d_0/(2p_1))^2 \\ & - p_2(\|e\| - c_0/(2p_2))^2 + \lambda \leq -p_1(\|\dot{e}\| - d_0/(2p_1))^2 \\ & - p_2(\|e\| - c_0/(2p_2))^2 + \lambda \end{aligned} \quad (3.21)$$

where

$$\begin{aligned} p_1 = & \lambda_k - d_1 - (d_2 + d_1\|\Lambda\|)/2 \\ p_2 = & \lambda_k \lambda_A^2 - d_1\|\Lambda\| - (d_2 + d_1\|\Lambda\|)/2 \\ \lambda = & (d_0^2/p_1 + c_0^2\|\Lambda\|^2/p_2)/4. \end{aligned}$$

If the smallest eigenvalue of the control gain matrix

K_D is sufficiently large, we have $p_1 \geq 0$ and $p_2 \geq 0$. Hence, from Eq. (3.21), we can define an ellipse in a two-dimensional Euclidian space $\|\dot{e}\| - \|e\|$, i.e.,

$$p_1(\|\dot{e}\| - d_0/(2p_1))^2 + p_2(\|e\| - c_0/(2p_2))^2 - \lambda = 0$$

within which the set $\dot{V}_2 \geq 0$. In other words, $\dot{V}_2 < 0$ whenever the norms of the tracking errors $(\|e\|, \|\dot{e}\|)$ are outside this ellipse. From the above discussion, the tracking errors of the system, under the unmodeled effect τ_d , are determined by the control gain matrix K_D and the bounds on τ_d , namely, d_0, d_1 and d_2 .

4. Experimental Results

We will analyze the performance of the proposed control scheme through experiments using a one-link manipulator with a flexible joint (shown in Figure 1). The link is driven by a DC motor through a gear box. The dynamic model of the manipulator is given by the following equation.

$$\begin{pmatrix} J_1 & 0 \\ 0 & J_2 \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} + \begin{pmatrix} K_s & -K_s \\ -K_s & K_s \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix},$$

where J_1, J_2 are the inertias, and D_1, D_2 are the damping ratios of the link and motor, K_s is the stiffness of the joint, $q_2 = \theta_m/G_r$ (θ_m is the actual position of the motor) and G_r is the gear ratio of the gear box ($G_r = 53.5$). A linear parametrized expression in terms of a suitable selected set of parameters, as mentioned in section 2, is obtained as

$$\begin{aligned} Y(q, \dot{q}, \ddot{q})P = & \begin{pmatrix} -v & \ddot{q}_1 & \dot{q}_1 & 0 & 0 \\ v & 0 & 0 & \ddot{q}_2 & \dot{q}_2 \end{pmatrix} \begin{pmatrix} K_s \\ J_1 \\ D_1 \\ J_2 \\ D_2 \end{pmatrix} \\ = & \begin{pmatrix} 0 \\ u \end{pmatrix} \end{aligned}$$

where $v = (q_2 - q_1)$. By the definition of Eq. (3.2), we have

Fig. 1 The one-link manipulator with flexible joint

$$Y_d(q, \dot{q}, q_d, \dot{q}_r, \ddot{q}_r)P = \begin{pmatrix} Y_{d1} \\ Y_{d2} \end{pmatrix} P$$

$$= \begin{pmatrix} -v_d & \ddot{q}_{r1} & \dot{q}_{r1} & 0 & 0 \\ v_d & 0 & 0 & \ddot{q}_{r2} & \dot{q}_{r2} \end{pmatrix} \begin{pmatrix} K_s \\ J_1 \\ D_1 \\ J_2 \\ D_2 \end{pmatrix}$$

where $v_d = (q_{2d} - q_{1d})$. As described in section 3, q_{2d} can be calculated by Eq. (3.12), i.e.,

$$q_{2d} = q_{d1} + \hat{K}_s^{-1}(\ddot{q}_{r1}\hat{J}_1 + \dot{q}_{r1}\hat{D}_1 + K_{D1}\dot{s}_1).$$

Differentiating Eq. (3.12), we can obtain

$$\begin{aligned} \dot{q}_{2d} &= \dot{q}_{d1} + \hat{K}_s^{-1}(\dot{q}_{r1}^{(3)}\hat{J}_1 + \ddot{q}_{r1}\hat{D}_1 + \dot{q}_{r1}\hat{J}_1 \\ &\quad + \dot{q}_{r1}\hat{D}_1 + \hat{K}_s(q_{1d} - q_{2d}) + K_{D1}\dot{s}_1), \\ \ddot{q}_{2d} &= \ddot{q}_{d1} + \hat{K}_s^{-1}(\ddot{q}_{r1}^{(4)}\hat{J}_1 + \dot{q}_{r1}^{(3)}\hat{D}_1 + 2\dot{q}_{r1}^{(3)}\hat{J}_1 \\ &\quad + 2\dot{q}_{r1}\hat{D}_1 + \ddot{q}_{r1}\hat{J}_1 + \dot{q}_{r1}\hat{D}_1 \\ &\quad + 2\hat{K}_s(\dot{q}_{1d} - \dot{q}_{2d}) + \hat{K}_s(q_{1d} - q_{2d}) + K_{D1}\dot{s}_1). \end{aligned}$$

In the calculation of $\dot{q}_{2d}$ and $\ddot{q}_{2d}$, we have to use $\ddot{q}_1$ and $\ddot{q}_1^{(3)}$, which are bounded as mentioned in Remark 2. To avoid complications in calculating $\dot{q}_{2d}$ and $\ddot{q}_{2d}$, we use a digital differentiation in the experiments.

For simplification in adaptive control, as mentioned in Remark 1, we assume that $P_k = [J_2, D_2]^t$ is the known parameter group which can be obtained from the motor specification, and that $P_u = [K_s, J_1,$

$D_1]^t$ is the unknown parameter group. The actual parameters of the manipulator, shown in Table 1, are from the motor specification and identified off-line.

In order to analyze the performance of our control scheme, the speed pattern of the desired trajectory is chosen as:

$$\dot{q}_{1d} = -140t^3(t-T)^3/T^3, \quad 0 \leq t \leq T.$$

The accelerating time T is 0.08 s (the desired trajectory is shown in Fig. 2). The purpose of the experiments is to demonstrate the stability and the performance characteristics inferred from the theoretical development.

Experiment 1. We use the control law with the adaptation mechanism switched off (i.e., the adaptation gain matrix $M_u^{-1} = 0$). The estimated parameters are set at their actual values (i.e., $\hat{P}(0) = P$, the nonadaptive controller-1). From the control result (shown in Fig. 3), we can see that the angle of the link follows the desired trajectory quite well with a small vibration.

Experiment 2. We also use the control law with the adaptation mechanism switched off. However, the estimated parameters are not set at their actual values: i.e., the estimated parameters are set as

$$\hat{P}_u(0) = [6000.0, 0.0, 0.0]^t.$$

It is reasonable that $\hat{J}_1(0)$ and $\hat{D}_1(0)$ are set as zeros and stiffness of joint $\hat{K}_s(0)$ is set as a large value (almost the same as for the rigid joint), assuming that the parameters are unknown. In the nonadaptive controller-2 (i.e., $\hat{P} \neq P$) the tracking error and vibration become quite large (shown in Fig. 4).

Experiment 3. We use the control law with the adaptation mechanism switch (i.e., the adaptive controller) on. The initial estimated parameters $\hat{P}(0)$ are set to the same values as those in experiment 2. For the adaptive controller, the adaptation gain matrix

Table 1 The actual parameters of the manipulator

Parameter	Value	unit
K_s	1461	(kgcm/deg)
J_1	0.11	(kgcms ² /deg)
D_1	0.04	(kgcms/deg)
J_2	1.498×10^{-3}	(kgcms ² /deg)
D_2	2.247×10^{-2}	(kgcms/deg)

Fig. 2 The desired trajectory q_{1d}

Fig. 3 The link tracking error obtained by nonadaptive controller-1

Fig. 4 The link tracking error obtained by nonadaptive controller-2

Fig. 5 The link tracking error obtained by adaptive controller

Fig. 6 The histories of estimated parameters obtained by adaptive controller

M_u , and the constant gain matrixes Λ and K_D are chosen as diagonal (i.e., $M_u^{-1} = \text{diag}(0.5, 0.3, 0.3)$, $\Lambda = 10I$, $K_D = 40I$).

Comparing with the nonadaptive controller-1 and 2, from the result of the adaptive controller (shown in Fig. 5), we know that:

1. Initially, the tracking error is almost the same as that of the nonadaptive controller-2. However, it finally becomes smaller with the parameter adaptation driven by the tracking error, which is almost the same as that of the nonadaptive controller-1.

2. Because of the adaptive control, the vibration of the link is quite small, even though parameter uncertainty initially exists.

3. As expected, the estimated parameters in the adaptive controller are driven by tracking error, and the parameter estimated errors are bounded. The changes of estimated parameters (shown in Fig. 6) are seen to be very smooth, as expected from the integrator structure of the adaptive law. This smoothness is desirable for preventing adaptation from exciting the vibration modes of the manipulator with flexible joints.

4. The adaptive control law derived in section 3 predicts that the tracking errors of the system converge to zero, whereas actually the tracking errors do not exactly converge to zero, as seen in the experimental results. This phenomenon is due to the unmodeled disturbance which is inherent in the experiments but ignored in the theoretical analysis. As analyzed in section 3, the tracking errors are determined by the control gain and the bounds of the unmodeled disturbance.

5. Conclusions

This paper has presented an adaptive control scheme applicable to robot manipulators with flexible joints. The asymptotic stability is ensured when all the parameters of the system are unknown. The link position and velocity tracking errors have been shown to converge to zero theoretically, and all the signals in the system remain bounded. The robustness of the control system was analyzed in the presence of the bounded unmodeled disturbance. The experimental results have shown the effectiveness of the proposed adaptive control scheme. The proposed controller uses not only the information of the motors but also that of the links. However, there are some practical cases where the link information is not available. In further research, we would like to study new control schemes which use only the information on the motors.

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