Design of Miter Conical Involute Gears
Based on Tooth Bearing*

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Miter gears are the most often used of all bevel gears. Now that cutting and
grinding methods of conical involute gears with a cone angle of 45 degrees have been
established, miter conical involute gears may become the most useful of conical
involute gears. However, in the former design of the miter conical involute gear, tooth
bearing has not been taken into consideration; thus, the desired tooth bearing has not
been obtained. In this study, first a new design for miter conical involute gears is
developed, making it possible to locate the tooth bearing at the central part of the
tooth width. Second, test gears are designed and made. Finally, tooth bearing tests are
performed, and the design applicability is proven.

Key Words: Machine Element, Gear, Miter Conical Involute Gear, Tooth Bearing,
Applicability

1. Introduction

In this paper, the miter-type conical involute
gears are called “miter conical gears.”

Miter conical gears can transmit uniform rotation
even when mounting errors exist. For this reason,
they can be used instead of conventional miter gears
in the case where an accurate transmission of
rotational motion is required. In fact, in the gear-
grinding machine fabricated by Okamoto Machine
Tool Works Ltd., miter conical gears have been used
instead of conventional miter gears for the past 10
years. Except for this example, however, there are no
cases where miter conical gears are put to practical
use, to the the author's knowledge.

The author has carried out research and develop-
ment on the conical involute gear for practical use[10], and found that in order to realize the prac-
tical use of these gears, establishment of an appropriate
design and a grinding method is indispensable.

To date, grinding machines which can grind miter
conical gears have been few. Recently, though, a new
grinding method called “infeed grinding” has been
developed, and miter conical gears can now be ground
by a conventional Reishauer-type gear-grinding

Next let us discuss design. In the former design
of the miter conical involute gear, two limits are
taken into account: one is the limit of undercutting
on the toe and the other is the limit of outer circular
thickness which approaches zero on the heel. How-
ever, since tooth bearing has not been taken into
consideration, the desired tooth bearing has not been
obtained. This has been a main factor preventing full-
scale practical use of miter conical gears.

For this reason, the author developed a new
design for miter conical gears which made it possible
to locate the tooth bearing at the central part of the
tooth width. In this paper, the new design is present-
ed, and the test gears made based on this design are
described. Finally, the results of tooth bearing tests
are reported, and the design applicability is proven.
2. Basic Dimensions and Gear Dimensions of Miter Conical Gears

A gear used for miter conical gears is a straight conical involute gear with a generating cone angle of 45°. Figure 1 shows the rack for generating this gear, and a generating rolling cylinder of the gear. Consider the rack which originally generates a spur gear, and next incline its plane of symmetry with respect to the gear axis at the generating cone angle of 45°. This inclined rack generates miter conical gears. The basic dimensions of this gear are:

- normal module of generating rack: \( m \)
- normal pressure angle of generating rack: \( \alpha_n \)
- helix angle of tooth trace in plane of symmetry of generating rack: \( \phi (=0°) \)
- generating cone angle of generating rack: \( \delta (=45°) \)
- number of teeth: \( z \)

Figure 2 shows the tooth surfaces of the gear generated by the rack shown in Fig. 1. The right tooth surface is a left-handed involute helicoid, and the left tooth surface is a right-handed one. The dimensions of this gear are obtained by setting \( \phi = 0° \) in the general equations obtained before\(^{(3)}\). The angle \( \delta \) in these equations takes the value 45° in the case considered here.

Subscripts \( r \) and \( l \) indicate the right and left tooth surfaces, respectively.

The pressure angle in the plane of rotation \( \alpha_r \) is given by

\[
\tan \alpha_r = \tan \alpha_n \cos \delta \quad \text{(1)}
\]

the pitch helix angles \( \beta_r \) and \( \beta_l \) by

\[
\tan \beta_r = - \tan \alpha_n \sin \delta \quad \text{(2)}
\]
\[
\tan \beta_l = \tan \alpha_n \sin \delta \quad \text{(3)}
\]

the base helix angles \( \beta_{sr} \) and \( \beta_{sl} \) by

\[
\tan \beta_{sr} = - \tan \beta_r \cos \alpha_r \quad \text{(4)}
\]
\[
\tan \beta_{sl} = \tan \beta_l \cos \alpha_l \quad \text{(5)}
\]

the module in the plane of rotation \( m_s \) by

\[
m_s = m \quad \text{(6)}
\]

the pitch diameter \( d_p \) and the base diameter \( d_b \) by

\[
d_p = 2m_s = 2m \quad \text{(7)}
\]
\[
d_b = 2m \cos \alpha_r \quad \text{(8)}
\]

3. Design of Gear

3.1 Basic form of gear with full-depth teeth

Although the basic form of the gear was presented in the previous paper\(^{(3)}\), it is summarized again here as the starting point of this research.

Figure 3 shows the rack and the basic form of the gear in the axial plane. Here, \( h_0 = 2m \). Dimensions of the gear are as follows\(^{(3)}\).

![Fig. 1 Rack for generating straight conical involute gears, and the generating rolling cylinder of the gear](image1)

![Fig. 2 Tooth surfaces of straight conical involute gear](image2)

![Fig. 3 Rack and basic form of gear in axial plane](image3)
\[
\begin{align*}
B_{00} &= -mx_{a} \cot \delta \\
B_{0a} &= mx_{a} \cot \delta \\
B_{0b} &= m(x_{a} - x_{u}) \cot \delta \\
B_{at} &= B_{0a} + 2m \sin \delta \\
B_{as} &= B_{0a} + 4m \sin \delta \\
b_{a} &= m(x_{a} \cosec \delta + \tan \delta) \\
b_{b} &= m(-x_{a} \cosec \delta + \tan \delta) \\
d_{a} &= mz \\
d_{s} &= mz \cos^{2} \alpha_{s} \\
d_{a} &= m(z \cos^{2} \alpha_{a} + 4 \cos \delta) \\
d_{s} &= m(z + 2x_{a} + 2 \sec \delta) \\
d_{a} &= m(z + 2x_{a} + 2 \sec \delta - 4 \cos \delta) \\
R_{a} &= (mz/2) \sec \delta \\
R_{s} &= R_{a} + m 
\end{align*}
\]

where \(x_{a}\) is an addendum modification coefficient of the limit of undercutting on the toe and \(x_{s}\) is that of the limit of the outer circular thickness, which approaches zero on the heel. These are expressed by the following equations:\(^9\)

\[
\begin{align*}
x_{a} &= \sec \delta - (z/2) \sin^{2} \alpha_{a} \\
x_{s} &= ((\pi/4)B_{a} \cot \alpha_{s} - B_{s} \sec \delta)/(B - B_{s}) \\
z &= 2((\pi/4) \cot \alpha_{s} - \sec \delta)/(B - B_{s}) \\
B &= (\inv a_{as} - \inv a_{s}) \cot a_{s} \\
B_{s} &= (\cos \alpha_{a}/\cos \alpha_{as}) - 1 
\end{align*}
\]

Equation (11) expresses the relation of \(x_{s}\) to \(z\) with pressure angle \(a_{as}\) as a parameter.

3.2 Basic form of gear with stub teeth

For this gear, let the depth of involute teeth \(h_{b}\) be \(h_{b} = 2ym\) (0 < \(y\) < 1). The following equations are obtained corresponding to the above Eqs. (9) to (11), where \(y\) is the coefficient of tooth depth.

\[
\begin{align*}
B_{00} &= -mx_{a} \cot \delta \\
B_{0a} &= mx_{a} \cot \delta \\
B_{0b} &= m(x_{a} - x_{u}) \cot \delta \\
B_{at} &= B_{0a} + 2ym \sin \delta \\
B_{as} &= B_{0a} + 4ym \sin \delta \\
b_{a} &= m(x_{a} \cosec \delta + y \tan \delta) \\
b_{b} &= m(-x_{a} \cosec \delta + y \tan \delta) \\
d_{a} &= mz \\
d_{s} &= mz \cos^{2} \alpha_{s} \\
d_{a} &= m(z \cos^{2} \alpha_{a} + 4 \cos \delta) \\
d_{s} &= m(z + 2x_{a} + 2y \sec \delta) \\
d_{a} &= m(z + 2x_{a} + 2y \sec \delta - 4 \cos \delta) \\
R_{a} &= (mz/2) \sec \delta \\
R_{s} &= R_{a} + ym 
\end{align*}
\]

4. Setting Condition of Tooth Bearing Position and Contact Ratio

4.1 Setting condition of tooth bearing position

In Fig. 3, the straight line \(PO\) indicates the plane perpendicular to the paper surface. This is called the plane of contact, because when a pair of gears generated by this rack is conjugated, the point of contact is located only in this plane\(^5\). Thus, intersection of the tooth surface and the plane of contact forms a path of contact on the tooth surface.

In this study, the following condition is given as the setting condition of the tooth bearing position.

\[
b_{s} = b_{u} \tag{15}\]

Substituting Eq. (12) into Eq. (15), we obtain Eq. (16).

\[
x_{s} = -x_{u} \tag{16}\]

In the following, let the value of \(y\) satisfy 0 < \(y\) < 1, which includes a gear with full-depth teeth. In Eqs. (13) and (14), addendum modification coefficients \(x_{a}\) and \(x_{s}\) are expressed in terms of parameters \(\alpha_{s}\), \(z\) and \(y\). Figure 4 shows \(x - x_{a}\) and \(x - x_{s}\) curves when \(y = 1\), and Fig. 5 shows the same when \(y = 0.9\).

In Fig. 4, the condition \(x_{s} = -x_{u}\) holds true at \(z = 57\) when \(\alpha_{s} = 25^\circ\), at \(z = 34\) when \(\alpha_{s} = 28^\circ\), and at \(z = 23\) when \(\alpha_{s} = 30^\circ\). Thus values of both \(z\) and \(\alpha_{s}\) satisfying this condition are obtained, and their relation is shown by the curve for \(y = 1\) in Fig. 6. That is to say, the condition \(b_{s} = b_{u}\) holds for this curve. The curves for \(y = 0.9, 0.8, \text{ and } 0.7\) are obtained in the same way. Here numerical calculations were performed for each
value of $\alpha_0 = 20^\circ, 21^\circ, \ldots, 29^\circ, 30^\circ$.

4.2 Contact ratio

Figure 7 shows a pair of miter conical gears and the plane of contact. The path of contact between tooth surfaces $l_c$ is in this plane. The intersection of the plane of contact and the face cone forms an ellipse, but in this section, the ellipse is approximated by a circle with radius $R_s$ in the vicinity of the intersection of this ellipse and the path of contact. Furthermore, the intersection of the cone in contact with the straight line OP and the plane of contact in Fig. 7 also forms an ellipse. Here the radius of curva-

ture of this ellipse at point $P_0$ is denoted by $R_s$, and the radius $R_a$ is described by $R_a = R_s + m$. Thus the length of action can be described in the plane of contact, as shown in Fig. 8. The contact ratio is obtained by the following equation as $l_c = \pi m \cos \alpha_0$.

$$\epsilon = \frac{P_1 P_2}{l_c}$$

(17)

Substituting Eq. (12) into this equation, we obtain the following equation, expressing contact ratio $\epsilon$.

$$\epsilon = \frac{\sqrt{(z+2y \cos \alpha_0)^2 - (z \cos \alpha_0)^2 - z \sin \alpha_0}}{\pi \cos \alpha_0 \cos \delta}$$

(18)

Figure 9 shows plots of numerically calculated contact ratio $\epsilon$.

5. Production of Gears and Tooth Bearing Tests

All trial gears were made by means of table sliding taper hobbing. Table 1 shows the basic dimensions of trial gears, the coefficient of tooth depth $y$, and contact ratio $\epsilon$. Gear No. 1 was designed based on the conventional design method, and gear Nos. 2 and 3 are designed based on the new method. Figure 10 shows the tooth bearing of gear No. 1 and Fig. 11 shows that of gear No. 2. Figure 12 shows sketches of the tooth bearings. Here, the values in parentheses are the design goals, while those without parentheses are the measured values. For all three trial gears used here, the measured values show good agreement with

<table>
<thead>
<tr>
<th>No.</th>
<th>$m$</th>
<th>$\alpha_0$</th>
<th>$\delta$</th>
<th>$z$</th>
<th>$y$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>20°</td>
<td>0°</td>
<td>45°</td>
<td>34</td>
<td>1.75</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>28°</td>
<td>0°</td>
<td>45°</td>
<td>34</td>
<td>1.44</td>
</tr>
</tbody>
</table>
| 3   | 3   | 30°        | 0°       | 45° | 34 | 0.9        | 1.26

Fig. 6 $z-a_0$ curves satisfying $b_0 = b_a$

Fig. 7 Miter conical gears and plane of contact

Fig. 8 Length of action in plane of contact $P_1 P_2$

Fig. 9 Contact ratio $\epsilon$

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computed ones.

With variation of parameters such as pressure angle and tooth depth, this design allowed location of the tooth bearing at the central part of the tooth width, and this proves the applicability of this design.

6. Conclusions

(1) A new design for miter conical gears is developed, making it possible to locate the tooth bearing at the central part of the tooth width. With the conventional design, we could estimate the position of the tooth bearing in advance but could not control it. However, with this design, we can control the position of tooth bearing, locating it always at the central part of the tooth width. As a result, we were able to optimally design the basic form of miter conical gears.

(2) The development of this design, along with the infeed grinding method of straight conical gears developed before, enables the full-scale practical use of miter conical gears. Performance test results will be presented in the near future.

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References


