Deformation Mechanism of Hot Finish Rolling of Plastic Gears*

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In the present study, the deformation mechanism of hot finish rolling of plastic gears is discussed. This mechanism is important for developing a simulation program for this finishing process. For this purpose, indentation tests of rolling rollers were performed so as to show the relationship between the plastic deformation and various conditions. As a result, it was found that the plastic deformation depends strongly on the Hertzian contact width between rollers. In hot finish rolling, the Hertzian contact width between tooth flanks of die wheel and work gear is generally not so large that the plastic deformation causes changes in the work tooth form. Hence, it can be concluded that the changes in the work tooth form in the hot finish rolling process are not induced by the plastic deformation of the work tooth flank. Thus, we calculated the temperature distribution in the direction of tooth thickness by the finite element analysis of heat conduction. According to the result of this calculation, a melted layer of work tooth flank material exists on the tooth surface. Consequently, the changes in the work tooth form in the hot finish rolling process are induced by the flow of the melted layer.

Key Words: Gear, Helical Gear, Plastic Gear, Hot Finish Rolling, Deformation Mechanism

1. Introduction

Hot finish rolling has been proposed as a simple finishing process for injection-molded plastic gears⁽¹⁾. In this process, plastic work gears are finished by meshing with a heated die wheel under very low applied load. For practical use, it is necessary to obtain general information about selection of the optimum operating conditions for each material used for plastic gears. However, there are numerous kinds of materials for plastic gears, so it is almost impossible to determine such information through experimental investigations only. Moreover, specifications of the work gear are manifold, so the information may not be applied for general use.

For this purpose, we need a computer simulation

program which can well reproduce the hot finish rolling process. It will enable the easy selection of the operating conditions, and will reduce the cost of producing plastic gears of high quality. In order to develop such a program, it is necessary to determine the deformation mechanism in the hot finish rolling process and to construct its model for the simulation. However, hot finish rolling is a new finishing process for plastic gears, so there are many unknowns in the deformation mechanism.

Thus, in the present study, indentation tests of two rolling rollers are performed so as to reveal relationships between the plastic deformation of work tooth flank and various operating conditions, e.g., tooth flank temperature of die wheel, applied load for rolling, and rolling time. One of the rollers is made of copper and corresponds to the die wheel. The other is made of the acetal equivalent of Duracon and corresponds to the work gear. The results obtained from these experiments show that changes in the work tooth form during the hot finish rolling process of plastic gears are not induced by plastic deformation due to the applied load for rolling, though this is a deformation mechanism in the conventional roll

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finishing of steel gears. Then, finite element analyses of heat conduction in the region near the work tooth flank are performed so as to reveal the state of the flank during the hot finish rolling process. According to results obtained from these experiments and analyses, a hypothesis can be proposed for the deformation mechanism in the hot finish rolling. In the hypothesis, the changes in the work tooth form are induced by a material flow in a melted layer on the work tooth flank. The hypothesis will be useful in constructing a model of the mechanism to simulate the hot finish rolling process by computer.

2. Indentation Tests of Rolling Rollers

With regard to the deformation mechanism in the hot finish rolling of plastic gears, it may be intuitively surmised that a plastic deformation due to the applied load for rolling induces the changes in the work tooth form, due to the thermoplasticity of the work gear material. This is because the mechanism corresponds to that in roll finishing of steel gears. In order to reveal the effects of various operating conditions on the plastic deformation, indentation tests of rolling rollers were carried out. In these tests, a heated copper roller, which corresponds to the die wheel, is pressed against a plastic roller, which corresponds to the work gear, and each roller rotates with or without sliding.

The hot finish rolling rig designed and fabricated by the authors and others⁽¹⁾ has been provided with a mechanism for altering the combination of rotational speeds of the rollers. Figure 1 shows the schematic features of the test rig.

The experiments were performed under the following procedures. A copper roller, 1, is heated by four pipe heaters set into its rim, and the tooth flank temperature can be controlled by varying the current applied through a slip ring, 4. After the temperature reaches a required level and is held constant, the copper roller is driven by an electric controlled induction motor, 3, at any required rotational speed. At the

same time, a plastic roller, 2, also rotates at any required rotational speed through gears 6 and 7. The combination of the gear ratio and the roller radii is altered to yield a contact state of rollers with both rolling and sliding. A load for indentation is applied through a weight, 8. The plastic roller completes one revolution after contact with the copper roller, and the indentation test is over.

The shaft, 5, of the copper roller is made of ceramic to prevent the heat from being lost, and the plastic roller is fixed on its shaft by fasteners 9 and 10 during the test to prevent alignment error due to clearance between roller and its shaft from affecting the measurement of roller dimensions. The acetal equivalent of Duracon has been selected for the material of the plastic rollers, and silicon grease has been used for lubricant to prevent the material of the plastic roller from sticking to the copper roller.

In order to evaluate the amount of plastic deformation induced on the surface of the plastic roller by the indentation test, the radius of the roller should be measured before and after the test. The decrease in the radius corresponds to the amount of plastic deformation. The plastic roller is mounted on the center-type chucks, and its radius is measured by the electronic micrometer. In order to take into consideration inevitable variations in the radius of the plastic roller induced in the manufacturing process, the measurements are performed at four points every 90 degrees in the circumferential direction, and the mean value of the measured data is used for the evaluation of the plastic deformation.

In the indentation test of rolling rollers, the following parameters can be expected to affect the amount D of the plastic deformation induced on the surface of the plastic roller: surface temperature T_c of the copper roller, applied load W for the indentation, rolling velocity V_r , sliding velocity V_s , radius R_c of relative curvature, and width B of the roller. In the present paper, the rolling velocity corresponds to the peripheral velocity of the copper roller, and the sliding velocity corresponds to the relative peripheral velocity of the copper roller to the plastic one. The ratio of the sliding velocity to the rolling velocity, i.e., the specific sliding, is used for one of the parameters. In order to vary the radius of relative curvature and the width of the roller, nine kinds of plastic rollers with different radii and widths, and one kind of copper roller have been prepared. The dimensions of these rollers are listed in Table 1. In order to vary the rolling velocity and the specific sliding, change in the voltage applied to the induction motor is used to vary the rotational speed of the copper roller, and the gear ratio of the gear pair transmitting the rotation from

Fig. 1 Indentation test rig of rolling rollers

the copper roller to the plastic one is also varied.

As the first step of investigations, the effects of the applied load, the radius of relative curvature, and the width of the roller on the plastic deformation were discussed; then it was revealed that these effects can be evaluated by a calculated value of the Hertzian contact width without any consideration of change in the material properties of the plastic roller surface.

Figure 2 shows the relationship between the amount D of the plastic deformation and the Hertzian contact width b . It was obtained by the indentation tests of the rolling rollers. The experiments were performed under the conditions that the surface temperature of the copper roller is 510 K, the rolling velocity is 13 mm/s, the specific sliding is 0.0, and the applied load is 30 or 70 N. Plots in Fig. 2 show the measured data, and a solid curve shows its approximation. This approximate equation is derived as follows. The amount D of the plastic deformation must always be positive and must be zero when the Hertzian contact width b vanishes. Hence, as an example of the approximate equations,

$$D = a_1 \{ \exp(a_2 b^4) - 1 \} \quad (1)$$

was selected. Then, two coefficients a_1 and a_2 were

Table 1 Dimensions of copper and plastic rollers

	Copper roller	Plastic roller
Diameter (mm)	80	40,60,80
Width (mm)	30	5,10,15

Table 2 Test operating conditions

Temperature of copper roller (K)	510,530,550
Applied load (N)	30,70
Rolling velocity (mm/s)	13,15,17
Specific sliding	-0.5,0.0,+0.5

Fig. 2 Relationship between Hertzian contact width and plastic deformation
(Copper roller temperature: 510 K, Rolling velocity: 13 mm/s, Specific sliding: 0)

calculated by the least square method. When the exponent of the Hertzian contact width b was set at 4, deviation of the measured data from the calculated values from the approximate equation became the smallest in the results using various integers for the exponent. Hence, the value of the exponent was determined as 4. The same approximation will be used in the latter part of the present paper. Though this equation may not be a sufficient approximation for calculation of the plastic deformation of the work tooth flank by the hot finish rolling in the computer simulation, it can elucidate the effect of the Hertzian contact width on the plastic deformation. It might seem strange that some measured data were negative in the region of the small Hertzian contact width. This means that the radius of the plastic roller before indentation is smaller than that after indentation. These anomalous results reflect the low accuracies of the plastic rollers. The eccentricity of the roller somewhat exceeds 0.01 mm, and the parallelism also exceeds 0.01 mm. Hence, it is difficult to measure the plastic deformation when it is relatively small. However, we will discuss these results only qualitatively, so these anomalous data may be regarded as zero.

With reference to Fig. 2, the effect of the radius of relative curvature on the plastic deformation is hardly noticeable. As mentioned above, the effect can be evaluated only by the Hertzian contact width. It is found that the increase in the Hertzian contact width increases the amount of the plastic deformation.

As the next step of the investigations, the experiments were performed under the same conditions as those of Fig. 2 except for the specific sliding of +0.5 and -0.5 so as to reveal its effect on the plastic deformation. Figure 3 shows the same relationship as shown in Fig. 2 using the specific sliding as a parameter. The effect of the sliding is again hardly noticeable. Thus, the approximation was performed using

Fig. 3 Effect of specific sliding on plastic deformation
(Copper roller temperature: 510 K, Rolling velocity: 13 mm/s)

all measured data without distinction of the specific sliding. A solid curve in Fig. 3 shows the approximation.

In order to investigate the effect of the rolling velocity, the experiments were carried out under the same conditions as those of Fig. 2 except for the rolling velocity of 13, 15, and 17 mm/s. Figure 4 shows the relationship between the Hertzian contact width and the plastic deformation, using the rolling velocity as a parameter. In these experiments, some physical restrictions have prevented the rolling velocity from varying over a wide range. When the rolling velocity is too large, the induced plastic deformation is very small, so its accurate measurement is almost impossible. In contrast, when the rolling velocity is too small, insufficiency of motor torque prevents the smooth rotation of rollers. Hence, the rolling velocity can be varied in a small range; then the effect of the rolling velocity was not so noticeable. However, we believe the effect of the rolling velocity must be non-negligible. Thus, the approximations were performed separately with respect to the rolling velocity. Three curves in Fig. 4 show the approximations. As shown in Fig. 4, large rolling velocity reduces the plastic deformation.

In the last step of the investigations, the experiments were performed under the same conditions as those of Fig. 2 except that the copper roller temperatures of 510, 530, 550 K were used so as to reveal the effect of temperature on the plastic deformation. Figure 5 shows the relationship between the Hertzian contact width and the plastic deformation, using the copper roller temperature as a parameter. As shown in Fig. 5, the high temperature of the copper roller yields large plastic deformation.

Consequently, the amount of the plastic deformation increases with the increase in the Hertzian contact width, the decrease in the rolling velocity, and the

increase in the surface temperature of the copper roller. In other words, the amount of the plastic deformation increases with the increase in the heat supplied to the plastic roller. The large Hertzian contact width means a large area of heat supply, and the low rolling velocity means a long heating time.

Referring to Figs. 2 through 5, it is also found that the plastic deformation can hardly be induced when the Hertzian contact width is less than 0.4 mm under the test operating conditions used here.

3. Plastic Deformation in Hot Finish Rolling

According to the results mentioned in section 2, the plastic deformation of the work tooth flank in the hot finish rolling of plastic gears depends strongly on the Hertzian width of contact between die and work tooth flanks. Thus, the Hertzian contact width during the hot finish rolling process was calculated, using the computer simulation program⁽²⁾ for roll finishing of steel gears which has been developed by the authors and others. In order to take into consideration the difference between materials, compliances used in the simulation were converted by the following equations. For the compliance for the bending and the shearing deflection,

$$K_{bc} = K_{bs} \frac{E_s}{E_c} \text{ and } K_{bp} = K_{bs} \frac{E_s}{E_p}, \quad (2)$$

where K_b is the compliance and E is the Young's modulus, and suffix c means copper, p means plastic, and s means steel gear. For the compliance for the

Fig. 4 Effect of rolling velocity on plastic deformation (Copper roller temperature: 510 K, Specific sliding: 0)

Fig. 5 Effect of copper roller temperature on plastic deformation (Rolling velocity: 13 mm/s, Specific sliding: 0)

contact approach, on the basis of Hertz's theory of contact,

$$K_c = K_{cs} \sqrt{\frac{1-\nu_c^2}{E_c} + \frac{1-\nu_p^2}{E_p}} / \sqrt{2 \frac{1-\nu_s^2}{E_s}}, \quad (3)$$

where K_c is the compliance for copper-plastic contact, K_{cs} is the compliance for steel-steel contact, and ν is Poisson's ratio. The Hertzian contact width was calculated by a Weber-Banaschek equation⁽³⁾.

Figure 6 shows the calculated results of the Hertzian contact width at the center of the face width of the work gear in the finish rolling process in which the applied load is 50 N. Specifications of die wheel and work gear are listed in Table 3. The applied load in the hot finish rolling of plastic gears is set at a much lower level than that in roll finishing of steel gears. As shown in Fig. 6, therefore, the Hertzian contact width has a very small value. This estimation of the Hertzian contact width may not be correct, but is only approximate. However, the order of the calculated Hertzian contact width is much lower than the minimum level of 0.04 mm for inducing the plastic deformation, which was mentioned in section 2. As a result, the changes in the work tooth form in the hot finish rolling of plastic gears cannot be induced by the plastic deformation due to the applied load for the rolling. This is the deformation mechanism in the roll finishing of steel gears, but not in the hot finish rolling of plastic gears.

Table 3 Specifications of die wheel and work gear

	Die wheel	Work gear
Tooth form	Normal	
Module (mm)	1.5	
Pres. angle (deg)	20	
Number of teeth	53	30
Prof. shift coef.	0	0

Fig. 6 Calculated results of Hertzian contact width

4. Deformation Mechanism in Hot Finish Rolling

As mentioned in section 3, the plastic deformation due to die tooth flank indentation hardly contributes to the changes in the work tooth form in hot finish rolling. Thus, in order to investigate the state of the work tooth flank during the hot finish rolling process, the temperature distribution in the region near the work tooth flank was calculated by the finite element method.

Strictly speaking, the problem to be solved is the three-dimensional problem of unsteady heat transfer with phase change. The purpose of the analysis performed here is to reveal the deformation mechanism in the hot finish rolling of plastic gears, not to simulate the process. Hence, the problem may be simplified so as to avoid the complexities. Thus, the phase change was neglected and the thermal properties of the work gear material, the acetal, were set constant. In addition, a heat source moves very fast on the work tooth flank in this problem, so the change in the temperature distribution will be limited in the region near the surface. The previous analysis reported in Ref. (1) has confirmed this fact. For this reason, the problem is simplified to be two-dimensional, and the part of the sector shown in Fig. 7 is selected as the analytical domain. This simplification can also dispel the problems with the memory capacity of the computer and the calculation time. The sector radius of the analytical domain is equal to the radius of curvature of the work gear at a reference point P. A central angle of the sector is so selected that its chord is equal to the Hertzian contact width. The thickness of the analytical domain in the radial direction is 100 μm . For the boundary conditions, the outer temperature to the boundary Γ_a corresponds to that of the die tooth flank, and the other boundaries are regarded as the adiabatic ones. For the initial

Fig. 7 Analytical domain for analysis of heat conduction

condition, the temperature is equal to 300 K over the whole analytical domain. The heating duration in meshing once corresponds to the time required for the geometrical contact point of tooth flanks to move over the distance equal to the Hertzian contact width, i.e., for the work gear to rotate through the angle equal to the central angle Ψ of the sector. Hence, the total heating duration can be calculated from the rolling time, while the heat radiation from tooth flank is neglected when the tooth does not mesh with the die tooth.

Figure 8 shows the distribution of the temperature in the direction of work tooth thickness at the pitch point after a hot finish rolling process in which the tooth flank temperature of the die wheel is 500 K, the applied load is 50 N, the rotational speed is 30 rpm, and the rolling time is 10 s. The Hertzian contact width reaches 0.16 mm, with a heating duration of 6 ms; i.e., the total heating duration is 30 ms. The radius of curvature of the work gear at the pitch point is 7.45 mm.

We have confirmed that resistance to the contact deformation of the acetal vanishes when the temperature exceeds 440 K⁽⁴⁾. As shown in Fig. 8, it can be found that the temperature exceeds 440 K within the depth of 30 μm . An increase in the temperature at the depth of 100 μm can also be observed. This may be due to the selected boundary conditions, in which all boundaries except for the heated one are adiabatic. Hence, the calculated distribution of the temperature may be somewhat higher than the real one. However, it can be expected that a melted layer of the work tooth flank material will be induced with thickness of about 20 or 30 μm . In addition, the tooth contact of the involute gears includes both rolling and sliding. From these facts, the following hypothesis can be proposed for the deformation mechanism in the hot finish rolling. The heat supplied through the die tooth flank induces formation of a very thin melted layer on the work tooth flank. Then, the highly viscous fluid of

the melted material flows in the layer. This is due to the sliding between tooth flanks of the die wheel and the work gear. As a result, the work gear changes its tooth forms.

Taking into consideration the above hypothesis for the deformation mechanism in the hot finish rolling, we discuss the results mentioned in section 2 again. This hypothesis can explain why a decrease in the radius of the plastic roller cannot be observed when sliding between the rollers is not applied. The sliding is uniformly given in the circumferential direction, so material flow in the melted layer is also uniformly induced on the surface of the plastic roller. Then, the uniform flow cannot contribute to the deformation of the roller. In the experiments on hot finish rolling being performed by the authors, it has been confirmed that there is almost no difference between tooth thicknesses before and after the finishing although the obtained results have not yet been discussed in detail. Consequently, it is a reasonable hypothesis for the deformation mechanism in the hot finish rolling of plastic gears that the material flow in the melted layer on the work tooth flank caused by the sliding between tooth flanks of die wheel and work gear induces changes in the work tooth form.

Therefore, the limitation on correction of tooth form errors arising in the hot finish rolling depends heavily on the thickness of the melted layer. Thus, we investigated the change in the thickness of the melted layer due to the change in the Hertzian contact width. Figure 9 shows the relationship between the thickness of the melted layer and the Hertzian contact width, using the radius of curvature as a parameter. As mentioned above, the thickness of the melted layer can be evaluated in such a manner that the temperature distribution in the direction of the work tooth thickness is calculated, and then the region where the temperature exceeds 440 K is regarded as the melted layer. The calculations were performed under the same conditions as those in Fig. 8 except for the

Fig. 8 Temperature distribution near tooth flank

Fig. 9 Relationship between Hertzian contact width and depth of melted layer

applied load. In order to vary the Hertzian contact width, the applied load is changed. As shown in Fig. 9, the increase in the Hertzian contact width increases the heating duration; then the thickness of the melted layer becomes larger. The decrease in the radius of curvature also increases the duration, because the rotational angle of the work gear becomes larger even if the Hertzian contact width has the same value. As a result, the decrease in the radius of curvature also increases the thickness of the melted layer. From these results, the correctable tooth form error can be estimated at about 20 μm .

5. Conclusions

In order to reveal the deformation mechanism in the hot finish rolling of plastic gears, indentation tests of rolling rollers and finite element analyses of heat conduction were performed to obtain the following results.

(1) Plastic deformation based on the thermoplasticity of the material of the plastic roller is hardly induced when the calculated Hertzian contact width is less than 0.4 mm.

(2) In hot finish rolling, the calculated Hertzian width of contact between tooth flanks of die wheel and work gear is less than 0.2 mm. Hence, the plastic deformation cannot contribute to the changes in the work tooth form.

(3) During the hot finish rolling process, a melted layer of the material is induced on the work tooth flank. The material flow in the melted layer due to the sliding between tooth flanks contributes heavily to the changes in the work tooth form.

These results indicate that, in order to develop a computer simulation program for the hot finish rolling

of plastic gears, the material flow in the melted layer should be analyzed, and a simple model for the deformation mechanism based on the flow should be constructed. Such problems will be discussed in the near future.

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