Instability of Poppet Valve Circuit*

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This review focuses on instability and self-excited vibrations occurring in poppet valves used in hydraulic systems. First, the causes of the instability are considered and then studies on the local instability based on linear theory are reviewed. Second, some global behaviors of the nonlinear hydraulic systems, i.e., hard self-excitation and chaos, are described, mainly based on the author's studies by numerical simulation.

Key Words: Flow-Induced Vibration, Hydraulics, Poppet Valve, Stability, Non-linear Vibration, Self-Excited Vibration, Chaos, Numerical Simulation

1. Introduction

Poppet valves are extensively used in hydraulic systems as pressure regulating valves, e.g., relief valves, pilot parts of a reducing valves, check valves and cartridge valves, because of the simplicity of their construction and high pressure-sensitivity to valve displacement. It is a familiar fact to engineers in this field that the system is liable to be unstable and often suffers from noisy chatter. Since the instability of the valves is a cause of abnormal performance and a source of noise, it has been studied by many researchers for a long time. Most such studies deal with the local behaviors in the vicinity of steady states of the system. However, the systems have various kinds of nonlinearities, e.g., flow characteristics of valves and orifices, dry friction and collision of the poppet against the seat. Thus, the global behavior is sometimes very different from the local one. However, the problem has rarely been investigated because of the difficulty of analysis due to marked nonlinearities.

In recent years, the situation has been drastically

changed by the development of high-speed computers. That is, it is now possible to analyze the dynamics of systems with complex nonlinearity by making use of numerical simulation, and the change of system responses according to variation of system parameters is easily analyzed.

This review outlines past studies on local stability of hydraulic systems with poppet valves and then describes the course of studies on the mechanism of the instability and the global behavior of poppet valve circuits on the basis of the author's studies by numerical simulation.

Nomenclature

A_o : opening area of orifice
 A_p : cross-sectional area of valve seat ($=\pi d_p^2/4$)
 a_p : equivalent area of poppet
 c_o : discharge coefficient of orifice
 c_p : discharge coefficient of poppet valve
 d_o : diameter of orifice hole
 d_p : diameter of poppet valve seat
 E : mechanical energy of poppet
 ΔE : increment of mechanical energy during a cycle of vibration
 F : axial flow force acting on poppet
 f : perturbation of flow force
 f_L : frequency of pipeline mode

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- f_p : frequency of valve mode
 k : stiffness of poppet spring
 k_x : equivalent stiffness of poppet spring
 m : poppet mass
 P_c : pressure in valve chamber
 P_o : upstream pressure of orifice
 P_s : supply pressure
 p_c : perturbation of chamber pressure
 p_0 : amplitude of chamber pressure
 Q_c : flow rate through poppet valve
 Q_o : flow rate through orifice
 q_c : perturbation of valve flow
 q_o : perturbation of orifice flow
 V_c : volume of plenum chamber
 X : valve lift
 x : perturbation of valve lift
 x_0 : amplitude of valve displacement
 X_i : initially compressed length of suspension spring
 α : half angle of poppet
 β : bulk modulus of working fluid (oil)
 δ : viscous damping coefficient
 ϕ : phase angle of chamber pressure to valve displacement
 ρ : density of working fluid (oil)
 ω : angular frequency of vibration

2. Mechanism of Local Instability of Poppet Valve

Figure 1 shows a typical poppet valve circuit which consists of a poppet, a valve chamber which is considered as a single capacitive system, an orifice and a pipeline to supply working fluid to the valve. The governing equations of the system are given in the Appendix together with their linearized forms, the associated transfer functions and the characteristic equation. In this section, we consider the mechanism of instability of the poppet valve circuit on the basis of the system shown in Fig. 1 from the point of view of mechanical energy transferred to the poppet⁽¹⁾.

Fig. 1 Schematic diagram of poppet valve circuit

From the linearized equation (11) of poppet motion, the following energy equation is derived:

$$\Delta E = \int_{E_1}^{E_2} dE = - \int_{T_1}^{T_2} \delta \dot{x}^2 dt + \int_{T_1}^{T_2} f \dot{x}^2 dt, \quad (1)$$

where $E (= m\dot{x}^2/2 + kx^2/2)$ is the mechanical energy of the poppet, $E_1 = E(T_1)$, $E_2 = E(T_2)$, $T (= T_2 - T_1)$ is the period of the vibration of the poppet and ΔE indicates the incremental change of the mechanical energy of the poppet during a cycle of vibration, which is amplified for and is attenuated for $\Delta E > 0$ and is attenuated for $\Delta E < 0$.

Here we assume that the valve displacement and the pressure in the valve chamber vary sinusoidally about a steady state as

$$x = x_0 \sin \omega t$$

$$p_c = -p_0 \sin(\omega t - \phi), \quad (2)$$

where x and p_c are perturbations from a steady state and the minus sign before p_0 means that the chamber pressure decreases with an increase in valve displacement.

Substituting Eq. (2) into Eq. (1), we obtain the following equation.

$$\Delta E = -\pi \delta \omega x_0^2 + \pi a_p p_0 x_0 \sin \phi \quad (3)$$

The first term represents energy dissipated by the damping force acting on the poppet. This term is always negative for positive damping. The second term is work done on the poppet by the flow force during a cycle of vibration. If $\phi > 0$, that is, the chamber pressure p_c delays with respect to the valve motion, the mechanical energy in the poppet increases during a cycle of vibration. If $\Delta E > 0$, i.e., the energy intake of the poppet overcomes the dissipation energy, then the total mechanical energy of the poppet increases for every cycle of the vibration. Consequently, the vibration is amplified.

There are a variety of causes of the delay of the chamber pressure depending on circuit construction. For example, if the compressibility of oil in the valve chamber of Fig. 1 is not ignored, the delay of chamber pressure P_c with respect to valve displacement X is given by the formula $\phi = -\tan^{-1}(T\omega) < 0$ for the case of $L=0$ from Eq. (15) in Appendix. Thus, the second term of Eq. (3) becomes always positive. This implies that the delay due to the compressibility of oil can be a factor of instability. The transmission delay due to a long pipeline can also be a factor of instability.

Unless the damping force acting on the poppet is increased actively, it is usually so small that the valve is apt to vibrate excessively. This is the essential reason why the poppet valve circuit is liable to become unstable.

Even for the case in which the effect of the compressibility of oil can be ignored, the valve chamber

Fig. 5 Direct-acting poppet valve circuit

Kasai^{(8),(9)} has investigated the effect of the upstream and the downstream pipeline of the poppet valve on the stability of the circuit and has derived a critical condition for stability. Furthermore, he has argued on the basis of theory and experiment that the circuit becomes unstable due to external periodic disturbances.

Waldling and Johnson^{(10),(11)} have examined the effect of the pipeline downstream of the poppet valve on the stability, which Funk did not consider. In the analysis, they have taken account of the effect of dynamic characteristics of the spring suspending the poppet. As a result, they have shown that the spring dynamics and the downstream pipeline affect the stability much more than the compressibility of the valve chamber, but they have not referred to vibratory modes excited in the unstable system.

All the studies mentioned above have dealt with the unstable phenomena caused by interference between a poppet valve and the other circuit elements. On the other hand, there are several studies in which the instability of the poppet valve circuit is attributed to the unsteady flow force acting on the poppet.

Mathematical models of the poppet motion are usually derived under the assumption that the poppet can move only in the axial direction. However, in conventional use, the poppet is suspended only by a spring. Thus, the motion is not constrained only in the axial direction, and the motion perpendicular to the valve axis and the rotational motion cannot be ignored.

Maeda^{(12),(13)} has indicated the occurrence of self-excited vibrations due to the unsteady transverse flow force. He has theoretically and experimentally shown that when the poppet position deviates laterally from the center position, negative damping force is generated due to momentum change in unsteady flow around the poppet, and that the larger the overlap ratio of the valve and the seat, the more unstable the valve becomes. It is very interesting that this instability is caused by internal flow characteristics of the poppet and not by the coupled motions of the poppet with other circuit elements.

Recently, Maeda has demonstrated a rotational vibration occurring in a automobile braking system

Fig. 6 Flow patterns issuing from poppet valve

caused by the negative damping force⁽¹⁴⁾.

Concerning instability due to internal flow of the poppet, there have been studies by McCloy and McGuigan⁽¹⁵⁾ and Green and Woods⁽¹⁶⁾, who have indicated instabilities caused by the flow around the poppet.

McCloy and McGuigan⁽¹⁵⁾ have shown, on the basis of their observation of flow issuing from the poppet valve into a confined downstream chamber, that three flow patterns A, B and E are possible out of five flow patterns A, B, C, D and E indicated by Schrenk⁽¹⁷⁾ as shown in Fig. 6, and they have pointed out that vibrations may be induced by the change of flow force accompanying the change of flow patterns.

Green and Woods has listed the following five causes of poppet valve instability⁽¹⁶⁾.

- (1) Instability resulting from coupled motions of the poppet with other circuit elements.
- (2) Instability caused by flow transition from laminar to turbulent with increasing or decreasing flow rate accompanying poppet displacement.
- (3) Instability caused by negative restoring flow force.
- (4) Instability caused by pressure difference between opening and closing of the valve (hysteresis of flow force).
- (5) Vibrations induced by fluctuation of supply pressure.

The instability (1) has already been stated in Refs. (1) - (11). As for (5), we cannot infer whether the vibrations are self-excited vibrations or a kind of forced vibrations, since the frequency of pressure fluctuation, the valve natural frequency and the excited vibration frequency have not been shown.

The instabilities (2), (3) and (4) are said to be unstable phenomena caused by flow force, but the details of the phenomena are unclear, since Green and McGuigan have shown neither the measurement results concerning the flow force, nor the vibratory waveforms.

4. Global Stability of Poppet Valve Circuit

Most studies on the poppet valve have dealt with the local stability, i.e., the stability of steady states as reviewed previously, but few studies have investigated the global behavior of the systems. The poppet valve circuit shown in Fig. 1 frequently exhibits strange behavior, that is, disagreement of theoretical stability boundaries with corresponding experimental results and occurrence of irregular vibrations which are not expected from the local nature of the system. However, no study has been performed to clarify the actual conditions under which the problems occur. Here the author will describe several global behaviors of the poppet valve circuit on the basis of his studies^{(1),(18)-(21)}.

4.1 Local stability of the system shown in Fig. 1

Prior to describing the global nature of the system shown in Fig. 1, the local stability will be mentioned briefly.

Figure 7 shows a stability map plotted using supply pressure P_s and valve displacement, X where pipeline length L is taken as a parameter and the fine solid line represents a static change of valve displacement with supply pressure corresponding to a constant cracking pressure ($P_s = 2$ MPa). For the curve A corresponding to pipeline length, $L = 0$, the system is stable in the small valve displacement region, whereas it becomes unstable in the relatively large valve displacement region. On the other hand, for the case of, $L \neq 0$, the small valve displacement region becomes unstable as seen in the curves B and C and the unstable region gradually spreads out with increasing L . Although the circuit treated here is different from Ichikawa's circuit of Fig. 4, in that it has an orifice between the valve chamber and the pipeline, the stability of the circuit approaches that of Fig. 4 with increasing pipeline length. Green and Woods⁽¹⁶⁾ have also pointed out the instability in the small valve displacement region, but have not referred to the pipeline effect as an instability factor of this region.

Fig. 7 Local stability of the poppet valve circuit shown in Fig. 1

If a steady state is unstable, a self-excited vibration naturally develops from an infinitesimal perturbation, which is called as "soft" self-excited vibration⁽²²⁾. However, in practice, the steady operating points of mechanical systems are stabilized by dry friction. Consequently, self-excited vibration does not occur unless a disturbance of appropriate magnitude is given to the systems.

To determine a stability boundary experimentally, we usually use the two procedures. In one of these, it is examined whether any self-excited vibrations occur by giving an appropriate disturbance to the poppet. In another procedure, the stability boundary is determined from the condition at which the self-excited vibration is attenuated for a changing parameter. Whichever of the two procedures is used, the theoretical stability boundaries for the system shown in Fig. 1 do not agree with experimental ones. The reason is that the global behavior cannot be described by linear theory having a local nature, because the self-excited vibrations having a large amplitude are markedly affected by nonlinearity. Numerical simulation exhibits the power to analyze such a complex system subject to marked nonlinearity.

4.2 Hard self-excited vibration

Figure 8 shows a result of numerical simulation carried out to examine the influence of disturbances on the system behavior.

The response for the velocity disturbance smaller than a critical value converges to a stable steady state. On the other hand, the response for the velocity disturbance larger than the critical value grows into a self-excited vibration. This sort of self-excited vibrations occurring at a stable steady state due to large disturbances is called "hard" self-excitation⁽¹⁾, which

(a) Damped oscillation

$L = 0.6$ m, $P_{si} = 2.0$ MPa, $P_s = 5.49$ MPa, $X_0 = 0.25$ mm

(b) "Hard" self-excited oscillation

Fig. 8 Responses for hard self-excited vibrations

corresponds to a situation in which a stable limit cycle encircles an unstable limit cycle which contains a stable equilibrium point⁽²²⁾.

By taking this phenomenon into consideration, the self-excited region is considerably widened in comparison with the locally unstable region as shown in Fig. 9. The most noticeable characteristic of the hard self-excitation is that once the self-excited vibration occurs, the vibration does not stop until a far lower supply pressure than the cracking pressure is reached.

Figure 10 shows the change of amplitude of vibrations with supply pressure P_s obtained by numerical simulation and the corresponding experimental results, where the cracking pressure $P_{st}(=2.0 \text{ MPa})$ is indicated by a chain line and the hatched areas indicate locally unstable regions. As seen in Fig. 10, the self-excited vibration occurs also in the stable region and continues until $P_s=1.1 \text{ MPa}$.

Figure 11 shows a stability map (a) for the system shown in Fig. 1 and the change of the natural frequencies (b) along a static valve displacement

Fig. 9 Self-excited region (locally unstable region + hard self-excited region)

Fig. 10 Amplitude of self-excited vibration

curve plotted as a dotted line in (a). The symbols $\circ$ and $\bullet$ are frequencies of self-excited vibrations. There are two separate unstable regions AB and CD corresponding to two modes of self-excited vibrations, i.e., the pipeline mode with the frequency f_L in the small displacement region AB and the valve mode with the frequency f_p in the relatively large displacement region CD.

The self-excited vibration of the valve mode has been recently shown⁽¹⁹⁾ to be caused by asymmetry of the static characteristics of chamber pressure P_c with respect to valve displacement X . A steady point A on the static relation of P_c to X shown in Fig. 12 is stable from the stability criterion Eq. (4) derived from Eq. (13) for, $L=0$, because the pressure sensitivity to X , $|\partial P_c / \partial X|_0$, is low at the point A where the valve displacement is relatively large.

$$2\zeta_p\omega_p(\omega_p^2 T + \frac{1}{T} + 2\zeta_p\omega_p^2) > \frac{a_p}{m} \left| \frac{\partial P_c}{\partial X} \right|_0 \quad (4)$$

As seen in Fig. 12, the vibratory waveform of P_c in response to the sinusoidal change of X about the steady state markedly distorts and the apparent center of the vibration shifts to the upper side P_{ce} . This implies that the steady point A as a center of vibration shifts to an apparent center at the smaller valve opening corresponding to P_{ce} . Consequently, $|\partial P_c / \partial X|_0$ in Eq. (4) drastically increases for a large

(a) Local stability ($P_{st}=2.0 \text{ MPa}$)

(b) Frequencies of valve mode and pipeline mode

Fig. 11 Local stability and frequencies of valve mode and pipeline mode

Fig. 12 Static characteristics of poppet valve

Fig. 13 Modal frequencies of direct-acting poppet valve circuit and frequencies of self-excited vibrations ($\circ$ = period-one, $\bullet$ = period-two, $\square$ = period-three, $\bigcirc$ = period-four, ∇ = period-five, $\blacksquare$ = period-six, $\ominus$ = period-eight, $\blacktriangle$ = almost periodic, $\triangle$ = chaotic, ϕ = intermittent chaotic, $\odot$ = damped vibration)

disturbance and the system becomes unstable.

The mechanism of the hard self-excitation of the pipeline mode still remains unclear at present.

4.3 Chaotic vibrations

In hydraulic circuits, we often use a direct-acting poppet valve which is directly connected with a pipeline and not separated by an orifice from the pipeline as shown in Fig. 5. The stability of the circuit is considerably degraded by the direct interference of the motion between the poppet valve and the oil column in the pipeline and sometimes irregular vibrations are observed in the circuit. This section describes the instability and the irregular vibrations induced in the circuit⁽²⁰⁾.

Figure 13 shows the change of the natural frequencies of the system and the frequencies of self-excited vibrations theoretically calculated by numerical simulation. The curves $[f_1]$, $[f_2]$, $[f_3]$... represent pipeline mode frequencies corresponding to the imaginary parts of the characteristic roots of the system, which are approximately equal to the natural fre-

quencies of an oil tube of which one end is open and the other end is closed. The curve $[f_0]$ at the bottom left corner corresponds to the valve mode vibration which is always stable regardless of pipeline length.

The stability of the system is indicated by two horizontal broken lines, between which every mode becomes unstable. With increasing pipeline length, every pipeline mode becomes unstable successively from the lower frequency mode and becomes stabilized again when the frequency reaches the lower critical line.

In this system, several vibratory modes become unstable simultaneously for the same pipeline length and the same operating condition. With increasing pipeline length, the number of unstable vibratory modes increases. However, not all the unstable modes are necessarily excited in the system, but only a few modes selectively occur. The problem of what modes can occur under a given condition is difficult to solve analytically because of marked nonlinearity. Thus, the governing equations including the collision condition of the poppet against its seat have been solved by numerical simulation.

Vibrations occurring in the system are classified and indicated by symbols $\circ \sim \phi$ in Fig. 13. The symbol $\circ$ means the fundamental and the higher modes of vibrations with frequencies f_i ($i=1, 2, 3, \dots$) and the symbols $\bullet \sim \ominus$ indicate the sub-harmonic vibrations of modes $[f_i]$ with frequencies f_i/n ($n=2, 3, 4, 5, 6, 8$). $\blacktriangle$ represent almost periodic vibration.

Subharmonic vibrations and almost-periodic vibrations are familiar in nonlinear systems. However, it is crucial to note that irregular vibrations (chaotic vibrations) shown by the symbols $\triangle$ and ϕ occur in the region indicated by A, B, B' and C. Although these chaotic vibrations shown in Fig. 13 were obtained by numerical simulation, the results have been confirmed to be valid qualitatively by an experiment⁽²⁰⁾.

In region A, only mode $[f_1]$ is unstable and the self-excited vibration transforms into Feigenbaum-type chaotic vibration through period-doubling bifurcation with increasing L as a parameter.

In region B, the three modes $[f_1]$, $[f_2]$ and $[f_3]$ are unstable and the almost periodic vibration composed of the two unstable modes $[f_1]$ and $[f_2]$ changes into Lorenz type chaotic vibration. The waveforms of the almost periodic vibration and the chaotic vibration are shown in Fig. 14 and Fig. 15 together with their power spectra.

In region B', the same kind of chaotic vibration as in the region B occurs.

In region C, a vibration with beating waveform occurs whose power spectrum indicates side bands on

Fig. 14 Vibratory waveform and frequency spectrum of almost periodic vibration

Fig. 15 Vibratory waveform and frequency spectrum of Lorenz-type chaotic vibration

Fig. 16 Vibratory waveform and frequency spectrum of intermittent-type chaotic vibration

both sides of a primary peak of the mode $[f_2]$ and its higher mode peaks as shown in Fig. 16. The vibration is regarded as a kind of intermittent chaos.

The mathematical model used in the numerical simulation is deterministic and does not include any statistical factors. The poppet always collides with the valve seat whenever the chaotic vibrations occur. This implies that the occurrence of the chaotic vibrations may be related to the collision.

Fig. 17 Period-doubling bifurcation diagram for the poppet valve circuit in Fig. 1

Recently, the author⁽²¹⁾ has shown by numerical simulation that a chaotic vibration with a frequency of pipeline mode occurs in the system shown in Fig. 1 having a relatively short pipeline. The chaos is of typical Feigenbaum-type which appears at a limit of period-doubling bifurcations as seen in Fig. 17. The chaotic vibration is considered to be closely related to the chaos occurring in region A of Fig. 13.

Among various self-excited vibrations, simple periodic vibrations produce sound close to pure tones, but almost periodic and chaotic vibrations produce unpleasant noisy tones. Thus, prevention of these vibrations is important also from the viewpoint of noise reduction.

5. Conclusions

Studies on the local instability of poppet valve circuits have been reviewed and the global behavior has been described on the basis of the author's studies by numerical simulation. Since the poppet valve motion is affected by various nonlinearities, the complexity of the dynamic behavior is greater than was at first expected. In particular, valve motions with small valve lift considerably affect the instability as pointed out by Green and Woods. In order to clarify the problem, it is necessary to take into consideration the intermittent opening and closing of the valve and the hysteresis of the flow force acting on the poppet valve besides valve collision. So far, there has been no study taking these factors into consideration.

In addition, future study on poppet valve dynamics should take into consideration the coupling effect of the valve with the other components composing circuits, e.g., pumps, actuators and pipelines.

Appendix 1. Governing equations

In derivation of the governing equations of a poppet valve circuit, we assume the pipeline length to be zero and the pressure upstream of the orifice to be P_s .

The equation of motion of the poppet is

$$m \frac{d^2 X}{dt^2} + \delta \frac{dX}{dt} + kX + kX_i = F. \quad (5)$$

The flow force acting on the poppet is expressed as

$$F = A_p P_c (1 - 4c_p \frac{X}{d_p} \sin 2\alpha). \quad (6)$$

The continuity equation of the valve chamber is

$$\left(\frac{V_c}{\beta}\right) \frac{dP_c}{dt} = Q_o - Q_c. \quad (7)$$

The equation of motion of the fluid column in the pipeline, approximated by a lumped parameter model, is

$$\left(\frac{\rho L}{A_L}\right) \frac{dQ_o}{dt} = P_s - P_o. \quad (8)$$

The flow characteristic of the poppet valve is

$$Q_c = c_p \pi d_p X \sin \alpha \sqrt{\frac{2}{\rho} P_c}. \quad (9)$$

The flow characteristic of the orifice is

$$Q_o = c_o A_o \sqrt{\frac{2}{\rho} (P_o - P_c)}. \quad (10)$$

Appendix 2. Linearized equations and transfer functions

We obtain the following linearized equations from Eqs. (4) - (8) at a certain steady state.

$$\begin{aligned} m\ddot{x} + \delta\dot{x} + k_x X &= a_p p_c \\ C\dot{p}_c &= q_o - q_c \\ I\dot{q}_o &= -p_0 \\ q_c &= c_x x + c_p p_c \\ q_o &= -c_v (p_0 - p_c) \end{aligned} \quad (11)$$

where $C (= V_c/\beta)$ is the capacitance of the valve chamber, $I (= (\rho L)/A_L)$ the inertance of the pipeline, $c_x = (\partial Q_c/\partial X)_0$, $c_p = (\partial Q_c/\partial P_c)_0$, $c_v = (\partial Q_o/\partial (P_o - P_c))_0$, $k_x = k - (\partial F/\partial X)_0$, $a_p = [\partial F/\partial P_c]_0$ and the subscript 0 indicates the steady state. From the above equations, the following transfer functions are derived.

$$\begin{aligned} \frac{X(s)}{P_c(s)} &= \frac{a_p/m}{s^2 + 2\zeta_p \omega_p s + \omega_p^2}, \\ \frac{P_c(s)}{X(s)} &= -\frac{(I c_v s + 1) c_x / (c_v C I)}{s^2 + 2\zeta_L \omega_L s + \omega_L^2} \end{aligned} \quad (12)$$

The characteristic equation of the system is

$$s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0, \quad (13)$$

where

$$\begin{aligned} A_0 &= \omega_L^2 \omega_p^2 + \omega_L^2 (a_p/m) |(\partial P_c/\partial X)_0|, \\ A_1 &= 2\zeta_L \omega_L \omega_p^2 + 2\zeta_p \omega_p \omega_L^2 + a_p c_x / (mC), \\ A_2 &= \omega_L^2 + \omega_p^2 + (2\zeta_p \omega_p)(2\zeta_L \omega_L), \\ A_3 &= 2\zeta_L \omega_L + 2\zeta_p \omega_p, \\ \omega_p^2 &= k_x/m, \omega_L^2 = (1 + c_p/c_v)/(CI), \\ 2\zeta_p \omega_p &= \delta/m, 2\zeta_L \omega_L = c_p/C + 1/(I c_v), \end{aligned} \quad (14)$$

$$|(\partial P_c/\partial X)_0| = c_x / (c_v + c_p).$$

For the case $L=0$, or the pipeline inertance $I=0$, the transfer function P_c/X in Eq. (11) is reduced to

$$\frac{P_c(s)}{X(s)} = -\frac{K}{Ts + 1}, \quad (15)$$

where $K = c_x / (c_v + c_p)$ and $T = C / (c_v + c_p)$.

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