Free Vibrations of Twisted Cantilevered Thin Plates Having Thickness Varying in Two Directions*

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Free vibrations of twisted cantilevered thin plates having thickness varying in two directions are investigated analytically and experimentally in this study. A numerical procedure based on the Rayleigh-Ritz method is presented using algebraic polynomials as displacement functions. To demonstrate the procedure, nondimensional frequency parameters and mode shapes are analyzed for typical twisted plates. The convergence of frequency parameters is studied and effects of plate thickness variation on vibrating characteristics are discussed. Experimental tests are carried out for three kinds of aluminum specimens. On the basis of comparison between experimentally obtained resonance frequencies and mode shapes and those obtained from the analysis, it appears that the numerical procedure proposed in the present study is useful for analyzing the free vibrations of twisted thin plates with variable thickness.

Key Words: Free Vibration, Vibration of Continuous System, Eigenvalue Analysis, Twisted Thin Plate, Variable Thickness, Rayleigh-Ritz Method, Experimental Verification

1. Introduction

A considerable amount of literature dealing with free vibrations of turbomachinery blades is available, in which the blades are treated as twisted cantilevered thin plates or cylindrical panels with uniform thickness. However, the thickness of the actual blades generally varies in both spanwise and chordwise directions. Therefore, to predict their vibrating behavior it is necessary to investigate vibrations of twisted plates having thickness varying in two directions. The free vibrations of either spanwise or chordwise tapered plates have been studied⁽¹⁾⁻⁽⁴⁾, but no result for twisted plates with thickness varying in two directions has been reported.

The main purpose of the present study is to develop a numerical procedure for analyzing vibrations of twisted thin plates with thickness varying in both spanwise and chordwise directions. The procedure

presented is based on the Rayleigh-Ritz method using algebraic polynomials of coordinate variables as displacement functions.

To demonstrate the procedure, frequency parameters and mode shapes of vibration are computed for typical twisted plates with both spanwise and chordwise tapered thickness. The convergence of frequency parameters is studied with increasing number of subdivisions of the plate and of terms in assumed displacement functions. Then, effects of thickness variation on frequency parameters and mode shapes are discussed.

Finally, experimental tests are carried out for three kinds of specimens fabricated from aluminum blocks. Resonance frequencies and mode shapes obtained are compared with analytical results.

2. Analytical Method

A twisted cantilevered plate with thickness varying in both spanwise and chordwise directions is shown in Fig. 1. The plate is twisted around the x -axis with constant rate of twist k . The length of the plate is a and the width is b . The thickness of the plate at a point $(0, -b/2)$ is t_0 and that on the edges

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Fig. 1 A twisted cantilevered thin plate having thickness varying in two directions

$x=a$ and $y=b/2$ is t_1 .

The principle of virtual work of free vibrations for a twisted thin plate having variable thickness is expressed by

$$\begin{aligned} & \iint_A \left[H\sqrt{g} \left\{ \Omega_x \delta \Omega_x + \Omega_y \delta \Omega_y + \nu (\Omega_x \delta \Omega_y + \Omega_y \delta \Omega_x) \right. \right. \\ & \quad \left. \left. + \frac{1-\nu}{2} \Omega_{xy} \delta \Omega_{xy} \right\} \right. \\ & \quad + D\sqrt{g} \left\{ \Gamma_x \delta \Gamma_x - \Phi_x \delta \Omega_x - \Omega_x \delta \Phi_x + \Gamma_y \delta \Gamma_y - \Phi_y \delta \Omega_y \right. \\ & \quad - \Omega_y \delta \Phi_y + \nu (\Gamma_x \delta \Gamma_y + \Gamma_y \delta \Gamma_x - \Phi_x \delta \Omega_y \\ & \quad - \Omega_y \delta \Phi_x - \Phi_y \delta \Omega_x - \Omega_x \delta \Phi_y) \\ & \quad \left. \left. + \frac{1-\nu}{2} (\Gamma_{xy} \delta \Gamma_{xy} - \Phi_{xy} \delta \Omega_{xy} - \Omega_{xy} \delta \Phi_{xy}) \right\} \right] dx dy \\ & - \iint_A \gamma \omega^2 t \sqrt{g} (gu \delta u + v \delta v + w \delta w) dx dy = 0 \quad (1) \end{aligned}$$

where $H = Et/(1-\nu^2)$, $D = Et^3/\{12(1-\nu^2)\}$, E is Young's modulus, ν is Poisson's ratio, γ is the density of plate material, ω is the angular frequency, t is the thickness of plate at an arbitrary point, u and v are displacements in the base vector directions on the middle surface, w is the displacement normal to the middle surface and $g = 1 + k^2 y^2$. Functions Ω_x , Ω_y , Ω_{xy} , Γ_x , Γ_y , Γ_{xy} , Φ_x , Φ_y and Φ_{xy} in Eq.(1) are given as follows:

$$\begin{aligned} \Omega_x &= \frac{\partial u}{\partial x} + \frac{k^2 y}{g} v, \quad \Omega_y = \frac{\partial v}{\partial y}, \\ \Omega_{xy} &= \sqrt{g} \frac{\partial u}{\partial y} + \frac{1}{\sqrt{g}} \frac{\partial v}{\partial x} - \frac{2k}{g} w, \\ \Gamma_x &= \frac{1}{g} \frac{\partial^2 w}{\partial x^2} - \frac{k}{\sqrt{g}} \frac{\partial u}{\partial y} + \frac{k}{g\sqrt{g}} \frac{\partial v}{\partial x} + \frac{k^2 y}{g} \frac{\partial w}{\partial y} \\ & \quad + \frac{k^2}{g^2} w, \\ \Gamma_y &= \frac{\partial^2 w}{\partial y^2} + \frac{k}{\sqrt{g}} \frac{\partial u}{\partial y} - \frac{k}{g\sqrt{g}} \frac{\partial v}{\partial x} + \frac{k^2}{g^2} w, \\ \Gamma_{xy} &= 2 \left(\frac{1}{\sqrt{g}} \frac{\partial^2 w}{\partial x \partial y} - \frac{2k^3 y}{g^2} v - \frac{k^2 y}{g\sqrt{g}} \frac{\partial w}{\partial x} \right), \\ \Phi_x &= \frac{k}{g} \left(\frac{k}{g} \frac{\partial v}{\partial y} + \frac{1}{\sqrt{g}} \frac{\partial^2 w}{\partial x \partial y} - \frac{k^2 y}{g\sqrt{g}} \frac{\partial w}{\partial x} \right), \\ \Phi_y &= \frac{k}{g} \left(\frac{k}{g} \frac{\partial u}{\partial x} - \frac{k^3 y}{g^2} v + \frac{1}{\sqrt{g}} \frac{\partial^2 w}{\partial x \partial y} - \frac{k^2 y}{g\sqrt{g}} \frac{\partial w}{\partial x} \right), \\ \Phi_{xy} &= \frac{k}{g} \left(\frac{1}{g} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{k^2 y}{g} \frac{\partial w}{\partial y} \right) \quad (2) \end{aligned}$$

Since a numerical procedure presented in this paper is based on the Rayleigh-Ritz method, non-

dimensional displacements, U , V and W , should be assumed by functions which satisfy the fixed boundary conditions at the edge $X=0$, namely $U=0$, $V=0$, $W=0$ and $\partial W/\partial X=0$. The following algebraic polynomials of X and Y are used in this study.

$$\begin{aligned} U &= \frac{u}{a} = \sum_{i=1}^{n_i} \sum_{j=0}^{n_j} a_{ij} X^i Y^j, \quad V = \frac{v}{a} = \sum_{p=1}^{n_p} \sum_{q=0}^{n_q} b_{pq} X^p Y^q \\ W &= \frac{w}{a} = \sum_{r=2}^{n_r} \sum_{s=0}^{n_s} c_{rs} X^r Y^s \end{aligned} \quad (3)$$

where a_{ij} , b_{pq} and c_{rs} are unknown coefficients, and $X = x/a$ and $Y = y/a$.

Substituting Eq.(3) into Eq.(1) and integrating over the plate area, a characteristic equation of free vibration is derived. Frequency parameters λ and corresponding mode vectors are computed from the characteristic equation in the usual way for eigenvalue problems. In the derivation of the characteristic equation, the double integrals of strain and kinetic energies are evaluated by approximate integral method, dividing the plate into rectangular elements as shown in Appendix A.

The frequency parameter λ is defined in this study as

$$\lambda^2 = \frac{\gamma t_0 a^4 \omega^2}{D_0} \quad (4)$$

where $D_0 = Et_0^3/\{12(1-\nu^2)\}$.

3. Analytical Results

3.1 Convergence study of λ with increasing number of subdivisions of the plate

Since the energy integrations are computed approximately by dividing the plate into rectangular elements, the convergence of λ with the increase of number of elements is studied for the plate with $b/a=1$, $b/t_0=20$, the twisted angle of plate tip of 60° and the thickness parameter defined in Eq.(6) of $\alpha=0.8$.

The thickness at an arbitrary point of the plate is given by

$$\frac{t}{t_0} = 1 - \alpha \left\{ X + (1-X) \left(\frac{1}{2} + \frac{a}{b} Y \right) \right\} \quad (5)$$

where α is the thickness parameter given as

$$\alpha = 1 - \frac{t_1}{t_0}. \quad (6)$$

Table 1 shows frequency parameters for the first ten modes obtained by using algebraic polynomials with 45 terms (See Appendix B). In the table, $n \times n$ indicates the number of subdivisions of the plate in X and Y directions.

It is seen in the table that very good convergence of the frequency parameters is obtained by dividing the plate into $25(5 \times 5)$ rectangular elements. The following analysis, therefore, is carried out using 5×5 subdivision of the plate.

Table 1 Convergence of λ with increasing number of subdivisions of the plate

Num. of div. No.	3×3	4×4	5×5
1	2.5721	2.5600	2.5563
2	8.2189	8.2138	8.2126
3	16.626	16.443	16.393
4	20.131	20.064	20.043
5	23.086	22.963	22.923
6	26.431	26.403	26.391
7	31.871	31.860	31.849
8	36.305	36.340	36.335
9	38.167	38.180	38.180
10	39.783	39.773	39.768

$b/a=1$, $b/t=20$, $K=60^\circ$, $\alpha=0.8$,
Num. of terms: 45-45-45

Table 2 Convergence of λ with increasing number of terms in assumed displacement functions

Num. of terms No.	28-28-28	36-36-36	45-45-45
1	2.5593	2.5574	2.5563
2	8.2295	8.2168	8.2126
3	16.746	16.448	16.393
4	20.258	20.127	20.043
5	23.171	23.021	22.923
6	27.235	26.620	26.391
7	33.793	32.394	31.849
8	38.065	36.849	36.335
9	41.032	38.883	38.180
10	42.148	40.328	39.768

$b/a=1$, $b/t=20$, $K=60^\circ$, $\alpha=0.8$,
Num. of div.: 5x5

Table 3 Frequency parameters of twisted plates having thickness varying in two directions

K α No.	0°			30°			60°		
	0	0.4	0.8	0	0.4	0.8	0	0.4	0.8
1	3.475	2.895	2.457	3.407	2.870	2.679	3.235	2.723	2.556
2	8.517	6.403	4.356	14.45	13.65	9.785	14.29	11.08	8.213
3	21.30	15.36	8.311	18.76	14.21	11.94	21.86	20.94	16.39
4	27.20	18.72	9.921	27.22	20.22	15.50	28.09	23.25	20.04
5	30.99	22.04	13.72	34.90	27.42	19.48	41.08	34.22	22.92
6	43.58	46.52	53.88	46.50	39.25	22.25	45.80	35.09	26.39
7	54.21	37.43	18.28	56.35	40.83	24.46	55.70	45.42	31.85
8	61.30	41.51	20.16	60.82	43.61	27.54	63.35	46.98	36.33
9	64.19	43.10	20.65	63.97	51.42	32.52	67.54	54.21	38.18
10	71.08	50.87	29.56	70.86	53.40	35.41	69.60	60.79	39.77

$b/a=1$, $b/t_0=20$, Num. of div.: 5x5, Num. of terms: 45-45-45

3.2 Convergence study of λ with increasing number of terms in assumed displacement functions

Convergence studies are performed to obtain reasonable values of frequency parameters with increasing number of terms of the algebraic polynomials for the same plate analyzed in the previous sections. The results are summarized in Table 2, in which $n-n-n$ indicate the number of terms in assumed displacement functions U , V and W (See Appendix B).

As can be seen in Table 2, the frequency parameters converge monotonically from above and the algebraic polynomials with 45 terms should be used to obtain convergent frequency parameters for the first ten modes.

3.3 Vibratory characteristics of typical twisted thin plates with both spanwise and chordwise tapered thickness

In the present section, effects of thickness variation on frequency parameters and mode shapes are investigated for plates with $a/b=1$, $b/t_0=20$, and $K=0^\circ$, 30° and 60° .

Table 3 shows the first ten frequency parameters and Fig. 2 depicts the variation of λ against the thick-

ness parameter α . The frequency parameters of flexible vibration for untwisted plates decrease monotonically with increasing thickness parameter as shown in Fig. 2(a). It is also seen in this figure that the frequency parameters of in-plane vibration increase considerably with thickness parameter. In contrast, flexible and in-plane vibrations are coupled for the twisted plate and therefore the variation curves of λ do not cross each other, as shown in Figs. 2(b), (c). The trends of decreasing λ with the increase of α are maintained for twisted plates, but the influence of in-plane vibration is also observed for plates having relatively small pretwist, namely $K=30^\circ$.

The changes of mode shape with increasing thickness parameter are shown in Fig. 3 by means of nodal lines for the plate with $K=60^\circ$. The first two modes are influenced little by the change of thickness parameter. However higher modes of vibration are affected strongly so that mode shapes become complicated as shown in Fig. 3. This finding indicates that the terms of assumed displacement functions should be carefully chosen for plates with large taper.

Fig. 2 Variation of frequency parameter with thickness parameter
 $b/a=1$, $b/t_0=20$, Num. of div.: 5×5 , Num. of terms: 45-45-45

Fig. 3 Variation of mode shape with thickness parameter
 $b/a=1$, $b/t_0=20$, $K=60^\circ$, Num. of div.: 5×5 , Num. of terms: 45-45-45

4. Experimental Tests and Comparison with Analytical Results

4.1 Test specimens and test procedures

Three kinds of test specimen are fabricated from aluminum alloy blocks (Young's modulus of 71 GPa, density of 2.8 and Poisson's ratio of 0.33) using a three-dimensional modeling machine. The geometrical dimensions of specimens are given in Table 4. The thickness of specimens is measured at 121 points as shown in the table. On the front surface of specimens a very thin layer is formed using white acrylic paint in order to measure mode shapes and strain gauges are pasted on the rear surface of the specimens to detect the resonance.

A specimen bolted in steel blocks is acoustically excited by a siren from the rear of the specimen. Oscillation frequencies at the maximum output of the

Table 4 Geometrical dimensions of specimens (mm)

(a) $a=102.0$, $b=50.0$, $K=0^\circ$

t	1	2	3	4	5	6	7	8	9	10	11
1	0.95	0.96	0.96	0.97	0.97	0.97	0.98	0.99	0.99	1.00	1.00
2	1.04	1.04	1.03	1.03	1.03	1.03	1.03	1.02	1.02	1.02	1.02
3	1.13	1.12	1.10	1.08	1.08	1.07	1.05	1.05	1.04	1.03	1.02
4	1.25	1.21	1.18	1.17	1.14	1.13	1.11	1.08	1.08	1.05	1.03
5	1.31	1.28	1.26	1.24	1.21	1.18	1.14	1.10	1.02	1.05	1.02
6	1.41	1.38	1.34	1.31	1.27	1.23	1.18	1.14	1.11	1.06	1.03
7	1.05	1.45	1.41	1.36	1.31	1.27	1.23	1.16	1.11	1.06	1.03
8	1.59	1.54	1.49	1.43	1.38	1.32	1.27	1.21	1.14	1.10	1.04
9	1.68	1.62	1.56	1.50	1.44	1.37	1.30	1.23	1.16	1.10	1.04
10	1.77	1.71	1.62	1.56	1.49	1.41	1.34	1.27	1.20	1.12	1.04
11	1.84	1.83	1.79	1.73	1.62	1.51	1.40	1.35	1.28	1.22	1.10

strain gauge pasted on the plate are measured as resonance frequencies of the specimen. Mode shapes at resonance are obtained by forming a hologram on a photographic plate, using the time-average method of holographic interferometry.

4.2 Test results and comparison with analytical results

The resonance frequencies obtained from the experimental test f_e are given in Table 5 in comparison with the analytical ones f_a . The agreement

Table 4 Geometrical dimensions of specimens (mm) (Continued)

(b)
 $\alpha=103.7$, $b=50.0$, $K=30^\circ$

t	1	2	3	4	5	6	7	8	9	10	11
1	0.85	0.87	0.90	0.90	0.90	0.92	0.91	0.90	0.90	0.88	0.87
2	0.93	0.94	0.95	0.97	0.95	0.96	0.94	0.94	0.91	0.89	0.86
3	1.02	1.02	1.02	1.03	1.02	1.01	1.00	0.97	0.94	0.91	0.87
4	1.12	1.11	1.10	1.09	1.06	1.04	1.02	0.99	0.96	0.91	0.86
5	1.21	1.18	1.16	1.14	1.12	1.10	1.08	1.03	1.00	0.94	0.89
6	1.31	1.28	1.24	1.22	1.19	1.18	1.11	1.05	1.00	0.94	0.89
7	1.40	1.37	1.33	1.28	1.24	1.19	1.14	1.10	1.03	0.96	0.89
8	1.52	1.46	1.41	1.36	1.31	1.28	1.19	1.13	1.05	0.98	0.89
9	1.62	1.56	1.50	1.43	1.38	1.30	1.23	1.15	1.08	0.98	0.90
10	1.71	1.66	1.60	1.51	1.44	1.34	1.26	1.18	1.09	0.99	0.89
11	1.81	1.80	1.74	1.67	1.60	1.52	1.41	1.31	1.18	1.14	1.04

(c)
 $\alpha=102.4$, $b=50.0$, $K=60^\circ$

t	1	2	3	4	5	6	7	8	9	10	11
1	0.72	0.76	0.86	0.91	0.94	0.96	0.96	0.96	0.92	0.87	0.83
2	0.76	0.84	0.91	0.96	1.00	1.01	1.00	0.98	0.96	0.91	0.87
3	0.87	0.88	0.95	1.00	1.04	1.05	1.04	1.03	0.98	0.93	0.87
4	0.94	0.99	1.03	1.10	1.09	1.10	1.08	1.05	1.00	0.93	0.85
5	1.02	1.07	1.12	1.18	1.16	1.15	1.14	1.10	1.04	0.97	0.88
6	1.13	1.19	1.21	1.23	1.26	1.22	1.19	1.13	1.07	0.98	0.88
7	1.23	1.25	1.31	1.30	1.28	1.26	1.22	1.17	1.08	0.99	0.89
8	1.34	1.37	1.38	1.37	1.36	1.33	1.27	1.20	1.10	1.00	0.89
9	1.42	1.45	1.45	1.44	1.42	1.38	1.30	1.22	1.12	1.00	0.87
10	1.52	1.53	1.53	1.52	1.48	1.41	1.35	1.24	1.23	1.01	0.89
11	1.70	1.72	1.75	1.64	1.56	1.50	1.41	1.34	1.29	1.24	1.13

Table 5 Comparison of experimental and analytical resonance frequencies (Hz)

K No.	0°			30°			60°		
	f_e	f_a	E(%)	f_e	f_a	E(%)	f_e	f_a	E(%)
1	112	124	+1.6	118	117	-0.8	116	109	-6.0
2	479	478	-0.2	551	562	+2.0	447	465	+4.0
3	644	665	+3.3	984	962	-2.2	1386	1452	+4.8
4	1352	1398	+3.4	1527	1552	+1.6	1698	1662	-2.1
5	1674	1764	+5.4	2356	2325	-1.3	2733	2656	-2.8
6	2382	2604	+9.3	2494	2450	-1.8	2805	2945	+5.0
7	2542	2706	+6.5	2957	2997	+1.4	3839	3818	-0.5
8	3237	3384	+4.5	3396	3375	-0.6	4201	4318	+2.8

f_e =Experimental, f_a =Analytical, $E=(f_a/f_e-1)\times 100$

between experimental and analytical resonance frequencies is good (the maximum difference between the two is +9.3% at the sixth mode of the untwisted specimen). The difference is mainly due to measuring errors of plate thickness and of resonance frequency.

Figure 4 depicts mode shapes of specimens. In this figure the upper photographs are modes from the hologram obtained in the experimental test and the lower are analytical ones in which bold lines express nodal lines and others are contour lines of displacement W . The analytical modes agree very well with the experimental ones.

A good agreement exists between the experimental results and the analytical ones for both resonance frequencies and mode shapes.

5. Conclusions

The vibrating characteristics of twisted cantilevered thin plates having thickness varying in two directions are studied analytically and experimentally. The numerical procedure based on the Rayleigh-Ritz method is proposed using the algebraic polynomials as the displacement functions and the

Fig. 4 Mode shapes of specimens

approximate integral method of dividing the plate into rectangular elements. The results obtained are summarized as follows:

(1) The frequency parameters of flexible vibration for untwisted plates with $b/a=1$, $b/t_0=20$ and both spanwise and chordwise tapered thickness decrease monotonically with the increase of thickness parameter, whereas those of in-plane vibration increase. These decreasing trends are generally maintained for twisted plates, but influences of the in-plane vibration are observed for plates of relatively small pretwist.

(2) The first two mode shapes are little affected by increasing thickness parameter. However, the thickness parameter affects the higher mode shapes strongly.

(3) The resonance frequencies and mode shapes obtained from the analysis agree very well with the experimental results. It can be concluded that the numerical procedure presented in this paper is applicable for analyzing the free vibrations of twisted thin plates with thickness varying in both spanwise and chordwise directions.

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Appendix

(A) Double integrals of the strain and kinetic energies are evaluated approximately by dividing the plate into rectangular elements as shown in Fig. A 1. The double integrals can be generally expressed by

$$\iint_A f(X, Y) X^n Y^m dX dY = \sum_{i=1}^n \iint_{A_i} f_i(X, Y) X^n Y^m dX dY \quad (\text{A } 1)$$

where $f(X, Y)$ is a function of the variables X and Y , $f_i(X, Y)$ ($i=1, \sim, n$) is an approximate function defined in the i -th element for $f(X, Y)$, and n is the number of elements. The function is expressed by

$$f_i(X, Y) = d_1 + d_2 X + d_3 Y + d_4 X^2 + d_5 XY + d_6 Y^2 + d_7 XY^2 + d_8 X^2 Y + d_9 X^2 Y^2 \quad (\text{A } 2)$$

where d_i ($i=1, \sim, 9$) are coefficients determined from values of $f(X, Y)$ at the nine points of the i -th element as shown in Fig. A 1.

Substituting the function $f_i(X, Y)$ obtained from Eq. (A 2) into Eq. (A 1), the approximate values of the double integral can be calculated.

Fig. A1 Subdivision of the plate (3×3)

U, V									Terms
1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	
2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7		
3.0	3.1	3.2	3.3	3.4	3.5	3.6			
4.0	4.1	4.2	4.3	4.4	4.5				
5.0	5.1	5.2	5.3	5.4					
6.0	6.1	6.2	6.3						
7.0	7.1	7.2							
8.0	8.1								
9.0

45

36

28

W									Terms
2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	
3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7		
4.0	4.1	4.2	4.3	4.4	4.5	4.6			
5.0	5.1	5.2	5.3	5.4	5.5				
6.0	6.1	6.2	6.3	6.4					
7.0	7.1	7.2	7.3						
8.0	8.1	8.2							
9.0	9.1								
10.0									

Table B1 Terms in assumed displacement functions

(B) The terms in algebraic polynomials used as the displacement functions U , V and W , in the analysis are shown in Table B 1. In the table numbers i, j indicate $X^i Y^j$.

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