Basic Study of the Measurement of 2-D Surface Profile*

(1st report : Transfer Function and Resolution)

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Generally, a large-scale 2-D profile is obtained from the combination of a series of 1-D profiles measured by 1-D differential methods. These methods have the disadvantage that the results are affected by rolling error of the scanning stages. This paper presents some 2-D differential methods which can eliminate the datum errors including the translation error in the longitudinal direction and the pitching and rolling errors, and evaluates their basic characteristics in detail. The analysis shows that the asymmetric 2-D mixed method is superior to the others with respect to the spatial frequency resolution and the longitudinal resolution.

Key Words: Software Datum, Datum Error, 2-D Profile Measurement, Transfer Function, Resolution, 2-D Differential Method

1. Introduction

Usually, there are two methods to measure the 2-dimensional surface profile. The first is the method using optical interferometers such as the Fizeau interferometer method^{(1),(2)} and the Twyman-Green interferometer method⁽³⁾ which profit from the advantages of analyzing the interference fringes. The other is the scanning method⁽⁴⁾. The former has high resolution and accuracy of a fraction of the wavelength of light, but its measurement area is limited by the illumination area of the light. Also in some cases, it is not easy to obtain a standard gauge with sufficient accuracy, especially when the specimen is very large, with high accuracy and high aspheric ratio. Therefore it is not suitable for large-specimen measurement. The latter uses probes to scan the measured surface, and the measuring range is limited only by the strokes of the scanning stage. However, the accuracy is strongly

affected by datum errors of the scanning stage. Therefore, numerous differential multi-point methods have been developed for canceling datum errors, such as the 2-point method^{(5),(6)}, the 3-point method^{(6),(7)}, the differential autocollimation method⁽⁸⁾ and the mixed method^{(9),(10)}. In these methods, several displacement or angular sensors are used, and differential calculation of their output eliminates datum errors of the scanning stage. They are successfully used in the measurement of straightness and sectional surface profiles.

When a 2-D surface profile is measured, the profile is usually obtained by the combination of a series of sectional profiles⁽¹¹⁾. In this way, the translation error in the longitudinal direction and the pitching error are removed, but rolling error remains and causes combination errors in the measurement results. Recently a 2-D least squares serial two-point method⁽¹²⁾ was reported. By this method the 2-D surface profile can be measured by direct 2-D scanning, but the pitching error and the rolling error are not taken into consideration because it is simply the extension of the 1-D two-point method for straightness measurement. Therefore some kind of 2-D

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software datum must be constructed so that all of the 2-D datum errors of the scanning stage can be removed. It is also necessary that the basic characteristics of the 2-D scanning method, such as the spatial frequency resolution and the longitudinal resolution, be clarified to provide a basic reference for the selection of the measurement method and the design of the measuring head.

This paper presents several 2-D differential methods by which the 2-D datum errors can be removed for direct measurement of the 2-D surface profile. Their basic characteristics are considered and some valuable conclusions are obtained.

2. Measurement Method and Transfer Function

First, in the coordinate system shown in Fig. 1 we make some assumptions as follows: the measured profile is $z=f(x, y)$ and the measuring head is placed at the typical point $p(x, y)$; four probes (displacement sensor DS or angular sensor AS) are fixed to the measuring head and the coordinates where the probes will be placed are $A(x-d_{xA}, y-d_{yA})$, $B(x-d_{xB}, y+d_{yB})$, $C(x+d_{xC}, y-d_{yC})$ and $D(x+d_{xD}, y+d_{yD})$ which yield $d_c=d_{xA}+d_{xC}$ and $d_b=d_{yA}+d_{yB}$; the measuring head scans the measured surface two dimensionally and the translation error of the measuring head in the z direction is $z_p(x, y)$ and the pitching error and rolling error are $e_{px}(x, y)$ and $e_{py}(x, y)$ respectively.

2.1 2-D four point method

The measuring head of this kind is formed by four displacement sensors placed at A, B, C and D shown in Fig. 1. When the probe scans the measured surface the output of each displacement sensor can be expressed as

$$m_A = f_A + e_z + d_{yA} \cdot e_{px} + d_{xA} \cdot e_{py} \quad (1)$$

$$m_B = f_B + e_z - d_{yB} \cdot e_{px} + d_{xB} \cdot e_{py} \quad (2)$$

$$m_C = f_C + e_z + d_{yC} \cdot e_{px} - d_{xC} \cdot e_{py} \quad (3)$$

$$m_D = f_D + e_z - d_{yD} \cdot e_{px} - d_{xD} \cdot e_{py} \quad (4)$$

In order to cancel the datum errors of the scanning

Fig. 1 Coordinate system

stage, the following differential calculation is carried out

$$m(x, y) = m_A + m_D - m_B - m_C \\ = f_A(x, y) + f_D(x, y) - f_B(x, y) - f_C(x, y). \quad (5)$$

The Fourier transform of Eq. (5) gives

$$M(j\omega_1, j\omega_2) = F(j\omega_1, j\omega_2) \cdot H(j\omega_1, j\omega_2), \quad (6)$$

where $F(j\omega_1, j\omega_2)$ and $M(j\omega_1, j\omega_2)$ are the Fourier transform of $f(x, y)$ and $m(x, y)$, respectively, and $H(j\omega_1, j\omega_2)$ is called the transfer function or the frequency response of the method given by

$$H(j\omega_1, j\omega_2) \\ = \cos(\omega_2 d_{yA} + \omega_1 d_{xA}) - \cos(\omega_1 d_{xC} - \omega_2 d_{yC}) \\ - \cos(\omega_2 d_{yB} - \omega_1 d_{xB}) + \cos(\omega_1 d_{xD} + \omega_2 d_{yD}) \\ + j[-\sin(\omega_2 d_{yA} + \omega_1 d_{xA}) - \sin(\omega_1 d_{xC} - \omega_2 d_{yC}) \\ + \sin(\omega_1 d_{xB} - \omega_2 d_{yB}) + \sin(\omega_1 d_{xD} + \omega_2 d_{yD})]. \quad (7)$$

If the probes are symmetrically placed, that is, $d_{xA} = d_{yA} = d_{xB} = d_{yB} = d_{xC} = d_{yC} = d_{xD} = d_{yD} = L/2$, the transfer function becomes

$$H(j\omega_1, j\omega_2) = -4 \cdot \sin \frac{L}{2} \omega_1 \cdot \sin \frac{L}{2} \omega_2. \quad (8)$$

From Eq. (6) we have the Fourier transform of $f(x, y)$ given by

$$F(j\omega_1, j\omega_2) = M(j\omega_1, j\omega_2) / H(j\omega_1, j\omega_2). \quad (9)$$

Therefore the measured profile can be obtained by the inverse Fourier transform of $F(j\omega_1, j\omega_2)$. The transfer function of this method is illustrated in Fig. 2.

From Eq. (9) we know that if the value of the transfer function is zero, the measured profile cannot be obtained. This means that at the zero points of the transfer function the information on the profile is lost totally. Accordingly, the zero points are one of the most important features for evaluating the measurement method. From Eq. (8) and Fig. 2 we know that the zero points of the transfer function

$$\frac{L}{2} \omega_1 = k_1 \pi, \quad \frac{L}{2} \omega_2 = k_2 \pi \quad (k_1, k_2 = 0, 1, 2, \dots).$$

2.2 Compound method

Figure 3 shows the construction of the measuring head of the compound method. It is composed of three displacement sensors placed at A, B and D, and an angular sensor in the x direction at A, with the

Fig. 2 Transfer function of 2-D four-point method

Fig. 3 Probe arrangement for the compound method

Fig. 4 Transfer function of the compound method

outputs of m_A , m_B , m_D and μ_{Ax} , respectively. The following differential calculation cancels the datum errors.

$$\begin{aligned} m(x, y) = & m_B - m_D - \frac{(d_{yB} - d_{yD})}{(d_{yB} + d_{yA})} (m_B - m_A) \\ & + (d_{xA} + d_{xD}) \mu_{Ax} = f_B - f_D - \frac{(d_{yB} - d_{yD})}{(d_{yB} + d_{yA})} (f_B - f_A) \\ & + (d_{xA} + d_{xD}) \frac{\partial f_A}{\partial x} \end{aligned} \quad (10)$$

In the case of symmetric arrangement, i.e. $d_{xA} = d_{yA} = d_{yB} = d_{xD} = L/2$ and $d_{yD} = L/4$, the transfer function is

$$\begin{aligned} H(j\omega_1, j\omega_2) = & \frac{3}{4} \cos \frac{L}{2} (\omega_1 - \omega_2) - \cos \frac{L}{2} \left(\omega_1 + \frac{\omega_2}{2} \right) \\ & + \frac{1}{4} \cos \frac{L}{2} (\omega_1 + \omega_2) + \omega_1 L \sin \frac{L}{2} (\omega_1 + \omega_2) \\ & + j \left[\frac{3}{4} \sin \frac{L}{2} (\omega_1 - \omega_2) - \sin \frac{L}{2} \left(\omega_1 + \frac{\omega_2}{2} \right) \right. \\ & \left. - \frac{1}{4} \sin \frac{L}{2} (\omega_1 + \omega_2) + \omega_1 L \cos \frac{L}{2} (\omega_1 + \omega_2) \right]. \end{aligned} \quad (11)$$

The transfer function is shown in Fig. 4. From Eq. (11) and Fig. 4, we know that the zero points of the transfer function are $(0, 0)$ and $(0, 8k\pi L)$ ($k=0, 1, 2, \dots$) the number of which is much less than in the four-point method. It is apparent that the form of the transfer function is similar to that in the 1-D mixed method in the ω_1 direction and in the 1-D asymmetric three-point method in the ω_2 direction because the present method is a combination of the 1-D mixed method and the 1-D three-point method.

Fig. 5 Configuration of the 2-D mixed method

Fig. 6 Transfer function of the 2-D mixed method

2.3 2-D mixed method

Figure 5 shows the configuration of this measuring head. It contains two displacement sensors placed at B and C with the outputs of m_B and m_C , and a 2-D angular sensor at the typical point P with the outputs of μ_{px} and μ_{py} .

The following differential calculation eliminates the datum errors.

$$\begin{aligned} m(x, y) = & m_B - m_C + d_C \cdot \mu_{px} - d_B \cdot \mu_{py} \\ = & f_B(x, y) - f_C(x, y) + d_C \cdot \frac{\partial f_P(x, y)}{\partial x} \\ & - d_B \cdot \frac{\partial f_P(x, y)}{\partial y} \end{aligned} \quad (12)$$

The transfer function of this method is

$$\begin{aligned} H(j\omega_1, j\omega_2) = & \cos(\omega_2 d_{yB} - \omega_1 d_{xB}) - \cos(\omega_1 d_{xC} - \omega_2 d_{yC}) \\ & + j[\sin(\omega_2 d_{yB} - \omega_1 d_{xB}) - \sin(\omega_1 d_{xC} - \omega_2 d_{yC}) \\ & + (\omega_1 d_C - \omega_2 d_B)]. \end{aligned} \quad (13)$$

In the case of symmetric arrangement, i.e. $d_{xB} = d_{yB} = d_{yC} = d_{xC} = L/2$, the transfer function becomes

$$|H(j\omega_1, j\omega_2)| = 4 \cdot \sin \frac{L}{2} (\omega_2 - \omega_1) + L(\omega_2 - \omega_1), \quad (14)$$

which is illustrated in Fig. 6. From Eq. (14) and Fig. 6 we see that $\omega_1 = \omega_2$ are the zero points of the transfer function. The number of zero points is less than in the four-point method but more than in the compound method in section 2.2. It is seen that the form of the transfer function is similar to that in the 1-D mixed

Fig. 7 Arrangement of the 2-D differential autocollimation method

method in both directions.

2.4 2-D differential autocollimation method

Figure 7 shows the structure of this measuring head. It consists of a 2-D angular sensor placed at A and two 1-D angular sensors at B in the y direction and C in the x direction with the outputs of μ_{Ax} , μ_{Ay} , μ_{Cx} and μ_{By} , respectively.

The datum errors of the scanning stage are removed by the following differential calculation:

$$m(x, y) = (\mu_{Ax} - \mu_{Cx}) + (\mu_{By} - \mu_{Ay}) \\ = \left(\frac{\partial f_A}{\partial x} - \frac{\partial f_C}{\partial x} \right) + \left(\frac{\partial f_B}{\partial y} - \frac{\partial f_A}{\partial y} \right). \quad (15)$$

The transfer function of this method is

$$H(j\omega_1, \omega_2) \\ = \omega_1 [\sin(\omega_2 d_{yA} + \omega_1 d_{xA}) + \sin(\omega_1 d_{xC} - \omega_2 d_{yC})] \\ + \omega_2 [\sin(\omega_1 d_{xB} - \omega_2 d_{yB}) - \sin(\omega_1 d_{xA} + \omega_2 d_{yA})] \\ + j \{ \omega_1 [\cos(\omega_2 d_{yA} + \omega_1 d_{xA}) - \cos(\omega_1 d_{xC} - \omega_2 d_{yC})] \\ + \omega_2 [\cos(\omega_2 d_{yB} - \omega_1 d_{xB}) - \cos(\omega_1 d_{xA} + \omega_2 d_{yA})] \}. \quad (16)$$

In the case of symmetric arrangement, the transfer function becomes

$$H(j\omega_1, j\omega_2) = (\omega_1 - \omega_2) \sin \frac{L}{2}(\omega_1 + \omega_2) \\ + (\omega_1 + \omega_2) \sin \frac{L}{2}(\omega_1 - \omega_2) + j(\omega_1 - \omega_2) \\ \times \left[\cos \frac{L}{2}(\omega_1 + \omega_2) - \cos \frac{L}{2}(\omega_1 - \omega_2) \right]. \quad (17)$$

The transfer function is shown in Fig. 8. From Eq. (17) and Fig. 8 we see that the zero points of the transfer function are $\omega_1 = \omega_2$, $(0, k_1 \pi L)$, and $(k_2 \pi L, 0)$ where k_1 and k_2 are integers. The number of zero points is less than in the four-point method and more than in the compound method and the 2-D mixed method. It is seen that the form of the transfer function is similar to that in the 1-D differential autocollimation method in both directions.

Fig. 8 Transfer function of the 2-D differential autocollimation method

3. Transfer Function and Spatial Frequency Resolution

As mentioned above, if there exists a zero point in the transfer function, the information on the profile of this frequency is lost and the profile cannot be obtained correctly. In this sense, the spatial resolution is limited by the frequency at the zero point of the transfer function. Therefore, it is important to reduce the zero point of the transfer function or even to seek some methods whose transfer functions have no zero point.

There are mainly two kind of zero points in the 2-D transfer functions discussed above. The first is the zero point directly related to the distance between the probes, such as $L\omega/2 = k\pi$ or $fL = k$ where $k = 0, 1, 2, \dots$ in the 2-D four-point method. These are similar to that in the 1-D differential methods. If the spatial resolution is defined as the frequency f_R where the first zero point occurs, $f_R = 1/L$. Thus the frequency resolution can be increased by reducing the distance between the probes. However in most situations, it is difficult to make the distance between the probes sufficiently small.

The other is the zero point introduced by the combination of each of the 1-D transfer functions in the x and y directions. Generally, this kind of zero point appears on a special curve determined by the relative position of the probes. As a special case, if a 2-D angular sensor is placed in the middle of the two displacement sensors shown in Fig. 5, the zero points appear as a continuous line $\omega_1 = \omega_2$ (see Fig. 6). This is because the probes are symmetrically placed and the two angular sensors in the x and y directions are set at the same point. In such a case, the zero points have no relation with the distance between the probes and the frequency resolution cannot be increased by adjusting the distance between the probes.

On the other hand, if two angular sensors in the x and y directions are placed at different points, the number of zero points will be considerably reduced.

Fig. 9 Probe arrangement of the asymmetric 2-D mixed method

Fig. 10 Transfer function of the asymmetric 2-D mixed method

Moreover, when the angular sensors A in the x direction and D in the y direction are placed on the same side of line BC, and the ratio of the probe distances BC to AD is not an integer (called as the asymmetric 2-D mixed method for simplicity), the zero point of the second kind disappears. Figure 9 shows the probe arrangement and Fig. 10 the transfer function of this kind. It is seen from Fig. 10 that the zero point of the transfer function is only (0, 0). This improves the spatial resolution of the method such that the limitation is only the sampling theorem.

4. Transfer Function and Longitudinal Resolution

The longitudinal resolution in the differential method is limited by the random error of measurement. That is, if the signal is less than the measurement error, it is buried in noise and cannot be determined. Therefore it is common to define the longitudinal resolution as the signal level where the signal-to-noise ratio (SNR) is 1. In the same way, we can define the resolution of the probes and the differential output. Accordingly, instead of analyzing the resolution, we consider the random error of measurement. It is essential to analyze the resolution of each probe and the differential output before discussing the resolution of the method. For simplicity of comparison we take only the random error of the probe into consider-

ation.

In the case of the four-point method, if the random errors of the probes are taken into account, the output of the probes are given as,

$$m_A = f_A + e_z + d_{yA} \cdot e_{px} + d_{xA} \cdot e_{py} + \varepsilon_A(x, y) \quad (18)$$

$$m_B = f_B + e_z - d_{yB} \cdot e_{px} + d_{xB} \cdot e_{py} + \varepsilon_B(x, y) \quad (19)$$

$$m_C = f_C + e_z + d_{yC} \cdot e_{px} - d_{xC} \cdot e_{py} + \varepsilon_C(x, y) \quad (20)$$

$$m_D = f_D + e_z - d_{yD} \cdot e_{px} - d_{xD} \cdot e_{py} + \varepsilon_D(x, y), \quad (21)$$

where $\varepsilon_A(x, y)$, $\varepsilon_B(x, y)$, $\varepsilon_C(x, y)$ and $\varepsilon_D(x, y)$ are random errors of probes A, B, C and D, respectively, with the expectations of ε_A , ε_B , ε_C , ε_D and variances of σ_A^2 , σ_B^2 , σ_C^2 and σ_D^2 . Here σ_A , σ_B , σ_C , σ_D and ε_A , ε_B , ε_C , ε_D can be obtained by calibrating the probes.

According to the definition of the resolution, σ_A , σ_B , σ_C , and σ_D can be considered as the resolutions of probes A, B, C and D. On the other hand, ε_A , ε_B , ε_C and ε_D can be considered as the zero point outputs of the probes A, B, C and D.

Thus, the differential output becomes

$$\begin{aligned} m(x, y) &= m_A + m_D - m_B - m_C \\ &= f_A + f_D - f_B - f_C + \varepsilon(x, y), \end{aligned} \quad (22)$$

where $\varepsilon(x, y)$ is the error of the differential output with the expectation of $\varepsilon_1 = \varepsilon_A - \varepsilon_B - \varepsilon_C + \varepsilon_D$ and the variance of $\sigma_1^2 = \sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2$. For the same reason, we can consider ε_1 as the zero point error and σ_1 as the resolution of the differential output. The resolution of the differential output is lower than that of each probe, which is one of the disadvantages of the differential methods; i.e., the more the probes are used the lower the longitudinal resolution becomes. The zero point error of the differential output is a systematic one mainly caused by the adjustment error of the zero points of the probes and it does not influence the analyses of the resolution of the differential method. Therefore we assume it to be zero here.

The Fourier transform of Eq.(22) gives

$$\begin{aligned} M(j\omega_1, j\omega_2) &= F(j\omega_1, j\omega_2) \cdot H(j\omega_1, j\omega_2) \\ &+ E(j\omega_1, j\omega_2), \end{aligned} \quad (23)$$

where $E(j\omega_1, j\omega_2)$ is the Fourier transform of the error of the differential output. If the error is assumed to be white Gaussian noise, it will be homogeneously distributed in the frequency domain with an amplitude of E_m which is a random variable with the expectation of ε_1 and the variance of σ_1 . It is reasonable to assign a maximum value of $E_{\max} = 3\sigma_1$ to E_m .

When SNR=1 we obtain the resolution of the differential method as

$$\begin{aligned} R(j\omega_1, j\omega_2) &= |E(j\omega_1, j\omega_2)/H(j\omega_1, j\omega_2)| \\ &\leq \frac{3\sigma_1}{|H(j\omega_1, j\omega_2)|}. \end{aligned} \quad (24)$$

This means that the resolution of the method can be described as the variance of the error of the

Fig. 11 Resolution of the 2-D differential autocollimation method

Fig. 12 Resolution of the asymmetric 2-D mixed method

differential output divided by the transfer function of the method. In the same way, we can obtain the resolution of the other methods as described by Eq. (24), except for the difference in the variance of the error of the differential output.

The variance of the combined method discussed in section 2.2 is

$$\sigma_z^2 = \sigma_B^2 + \sigma_D^2 + \left(\frac{d_{yB} - d_{yD}}{d_{yB} + d_{yA}} \right)^2 (\sigma_B^2 + \sigma_A^2) + (d_{xA} + d_{xD})^2 \sigma_{Ax}^2 \quad (25)$$

The variance of the 2-D mixed method is

$$\sigma_3^2 = \sigma_B^2 + \sigma_C^2 + d_C^2 \cdot \sigma_{px}^2 + d_B^2 \cdot \sigma_{py}^2 \quad (26)$$

The variance of the 2-D differential autocollimation method is

$$\sigma_4^2 = \sigma_{Ax}^2 + \sigma_{Cx}^2 + \sigma_{By}^2 + \sigma_{Ay}^2 \quad (27)$$

Figure 11 shows the resolution of the 2-D differential autocollimation method with a symmetric probe arrangement uniformed by $3\sigma_4$ and Fig.12 shows the resolution of the 2-D mixed method with an asymmetric probe arrangement uniformed by $3\sigma_3$.

As can be seen from Eq.(24), and Figs. 11 and 12, the resolution of the differential method is closely related to the transfer function of the method and the resolution of each probe. It is easy to see that the resolution becomes very low near the zero point and the worst at the zero point of the transfer function where no profile can be obtained. Furthermore, the resolution varies with the arrangement of the probes when the displacement sensor and angular sensor are used at the same time, and it is not difficult to find an

optimal arrangement where the resolution becomes the highest. Consequently, the asymmetric 2-D mixed method is superior to the others with respect to this point.

5. Conclusions

The basic characteristics of some proposed methods for 2-D profile measurement are considered in detail. The conclusions are as follows

1) Four probes (displacement sensor or angular sensor) are needed to construct a 2-D software datum by which the datum errors of the scanning stage can be removed. Among the possible differential methods, four typical arrangements are presented and discussed.

2) The spatial frequency resolution is limited by the zero point of the transfer function of the method and the number of zero points of the transfer function can be reduced by asymmetric arrangement of the probes.

3) The longitudinal resolution is determined from the resolution of each probe and the zero point of the transfer function. It also varies with the arrangement of the probes if the displacement sensor and angular sensor are used at the same time.

4) With respect to spatial and longitudinal resolutions the asymmetric 2-D mixed method is superior to the others.

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