Alternate Optimization of Speed Control
Hump for Automobiles and
Automobile Suspension\footnote{Received 20th January, 1994.}
(Minimization of Each Single Objective Function)

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This paper deals with a study on the problem of minimizing each single objective function in two different design spaces. Whereas designers of automobile suspensions require that automobiles provide a comfortable ride during all automobile speed ranges, designers of humps require that humps induce an uncomfortable riding condition when driving at speeds greater than the hump design speed in order to indicate to the driver that he is speeding. Thus from the former standpoint, the maximum acceleration of the driver should be minimized at a speed greater than the hump design speed under adequate constraints for other maximum accelerations, and the optimum damping coefficients of the shock absorbers can be found. From the latter standpoint, the maximum acceleration of the driver should be minimized at the hump design speed under adequate constraints, and the optimum dimensions of the hump can be properly designed. The above two optimization processes are alternately repeated and the optimum dimensions of the hump are confirmed.

\textbf{Key Words:} Automobile, Transportation Engineering, Optimum Design, Hump, Automobile Suspension, Shock Absorber, Transient Response

1. Introduction

Traffic accidents, causing many injuries to citizens, are still a serious problem. To allow for more safety and reduce the number of traffic accidents, slightly protruding humps have been installed on roads. This is one means of reducing the speed of vehicles traveling faster than the designated limit. This device has been adopted extensively abroad, with its use in Japan recently becoming more widespread\textsuperscript{11-19}. It is, however, causing a variety of problems\textsuperscript{20-29}. In past reports\textsuperscript{30-34}, we have proposed methods to reduce excessive shock experienced when heavy vehicles pass over the humps at normal speeds. Furthermore, in the previous report\textsuperscript{35}, we have clarified, by the calculation using simulation, a problem that the speed control effect is reduced for vehicles with semi-active suspension\textsuperscript{36}, and proposed a method to establish optimum humps by optimizing the hump and semi-active suspension alternately, in order to resolve this problem.

This method adopts a suspension design which provides the best riding condition for a wide range of vehicle speeds, and alternately, from the standpoint of hump design, which induces a rough ride in the case of traveling too fast. First, suspension is optimized for the vehicle passing over a hump. The hump is then optimized under the condition of the optimum suspension obtained above, and this procedure is repeated.

As an objective function, we have selected the maximum acceleration of the driver at a speed higher than that specified by the hump design. This objective function is maximized for the optimization of the hump and minimized for the optimization of the suspension.

In this report, though we alternately take the standpoints of hump design and suspension design to optimize a circular arc hump and semi-active suspension as in the previous report, we have selected maximum acceleration of the driver at the speed specified by hump design as the objective function for the optimization of the humps, and maximum acceleration of the driver at a speed in excess of the speed
specified by the hump design as the objective function for the optimization of the suspension. We propose here a method to obtain an optimum hump by minimizing these two objective functions.

In the previous report, we optimized the hump and suspension on the basis of two points, i.e., the designed speed of the hump and a speed exceeding it. In this report, however, we optimized both by setting the constraint condition of maximum acceleration of the driver more precisely with reference to speed at three points.

In the previous study, depression of the maximum acceleration of the driver was prominent at one and a half times the designed speed for the humps, as a result of optimization of the suspension. This depression was almost compensated by maximizing the acceleration during the optimization of the hump. In this work, however, we aimed at reliable recovery of this depression by setting a lower limit for acceleration during optimization of the hump.

For the optimization of suspension, we sought to improve convergence characteristics of the alternating optimization by appropriately adopting the maximum acceleration of the driver, which was obtained by optimization of the hump during preceding steps, as the upper limit.

2. Optimum Design Procedure

The thick line in Fig. 1 shows the ideal characteristics of the hump[1]. They are different from those of the suspension (fine line) at the vehicle speed \( \hat{Z} \) higher than \( \hat{Z}_i \), which is the speed specified by the hump design. Figure 2 shows the relationship between \( \hat{Z} \) and \( |\dot{X}_o|_{\max} \), where \( |\dot{X}_o|_{\max} \) is the maximum acceleration of the driver when the vehicle passes over the hump. In this optimum design, the above-mentioned ideal characteristics are examined as the relationship between the maximum acceleration and the passing speed of the vehicle. The optimum hump and optimum suspension are studied to determine the ideal performance of these two items. In the present paper, various parameters are expressed in nondimensional form[1b].

2.1 Optimization of hump

First, the vehicle is considered to pass over the hump at the speed less than or equal to \( \hat{Z}_i \). In this case, maximum acceleration of the driver \( |\dot{X}_{o,j}|_{\max} \) at speed \( \hat{Z}_j \) should be as small as possible, thus \( |\dot{X}_{o,j}|_{\max} \) selected as the objective function which we tried to minimize.

Next, when the vehicle passes over the hump at speed \( \hat{Z}_{j+1} \) which is faster than \( \hat{Z}_j \), the driver of the vehicle should be warned that he is exceeding the speed limit. Accordingly, to constrain the range, the lower limit \( |\dot{X}_{o,j+1}|_{\max, LH} \) is set as the maximum acceleration of the driver \( |\dot{X}_{o,j+1}|_{\max} \). This condition is expressed as

\[
|\dot{X}_{o,j+1}|_{\max, LH} - |\dot{X}_{o,j+1}|_{\max} \leq 0. \tag{1}
\]

When the velocity of the vehicle is \( \hat{Z}_{j+2} \) which is faster than \( \hat{Z}_{j+1} \), the maximum acceleration of the driver \( |\dot{X}_{o,j+2}|_{\max} \) is also constrained by the lower limit \( |\dot{X}_{o,j+2}|_{\max, LH} \). The upper limit \( |\dot{X}_{o,j+2}|_{\max, UB} \) is also set for vehicle safety. Namely,

\[
|\dot{X}_{o,j+2}|_{\max, LH} - |\dot{X}_{o,j+2}|_{\max} \leq 0 \tag{2}
\]

The dimension of the cross section of the hump \( Y = (Y_1, Y_2, \ldots, Y_N)^T \) is adopted as the design variables. We set the upper limit \( Y_{n,0} \) and lower limit \( Y_{n,1} \) for the \( Y \) values.

\[
\begin{align*}
Y_{n,0} - Y_{n,1} & \leq 0, \\
Y_{n,1} - Y_{n,0} & \leq 0, \\
(n & = 1, 2, \ldots, N) \tag{3}
\end{align*}
\]

Constraint conditions (1) - (3) are expressed as

![Fig. 1 Ideal characteristics of hump and suspension](image)

(0: Comfortable, 2: Slightly uncomfortable, 4: Uncomfortable, 6: Very uncomfortable)

![Fig. 2 Relationship between maximum acceleration of the driver and passing speed of the vehicle over hump (j = 1, 2, ..., j+1, j+2)](image)
\( g_d(Y) \leq 0 \quad (s = 1, 2, \ldots, S). \) (4)

By converting objective function \( |\dot{X}_{\alpha,j}|_{\max} \) using SUMT (sequential unconstrained minimization technique) and the unit step function of Heaviside \( H \), we obtain the objective function without any constraint conditions:

\[
P_K = |\dot{X}_{\alpha,j}|_{\max} + r_K \left[ \sum_{s=1}^{S} \left( g_d(Y) \right)^2 \right. \times \left. H(-g_d(Y)) \right] \quad (K = 1, 2, \ldots),
\] (5)

where, the second term on the right side is a penalty function with positive coefficient \( r_K \), in which \( r_{K+1} > r_K \).

2.2 Optimization of suspension

A vehicle having a semi-active suspension is assumed to pass over the hump. Minimization of the maximum acceleration of the driver \( |\dot{X}_{\alpha,j+1}|_{\max} \) at the speed \( Z_{j+1} \) is sought.

In the optimization of the suspension, maximum acceleration of the driver at speeds \( Z_j \) and \( Z_{j+2} \), obtained by optimization of the hump in the preceding step, is set as upper limits \( |\dot{X}_{\alpha,j}|_{\max,US} \) and \( |\dot{X}_{\alpha,j+2}|_{\max,US} \), respectively. Using three points, we reduce the maximum acceleration of the driver to below that in hump optimization, utilizing the above situation and minimization of \( |\dot{X}_{\alpha,j}|_{\max} \). The above-mentioned constraint conditions are expressed by

\[
\begin{align*}
|\dot{X}_{\alpha,j}|_{\max} - |\dot{X}_{\alpha,j}|_{\max,US} & \leq 0, \\
|\dot{X}_{\alpha,j+2}|_{\max} - |\dot{X}_{\alpha,j+2}|_{\max,US} & \leq 0.
\end{align*}
\] (6)

As a design variable, the damping coefficient of shock absorber \( C = (C_1, C_2, C_3, C_4)^T \) is adopted. By expressing the damping force of front and rear shock absorbers as \( F_{ad} \) and \( F_{ar} \), respectively and relative velocity of front and rear shock absorbers as \( \dot{X}_{ad} \) and \( \dot{X}_{ar} \), respectively, we obtain

\[
\begin{align*}
F_{ad} &= C_1 \dot{X}_{ad}(X_{ad} \geq 0) \\
&= C_2 \dot{X}_{ad}(X_{ad} > 0) \\
F_{ar} &= C_3 \dot{X}_{ar}(X_{ar} \geq 0) \\
&= C_4 \dot{X}_{ar}(X_{ar} > 0)
\end{align*}
\] (7)

The constraint condition for \( C \) is

\[
C_i - C_0 \leq 0, \quad C_i - C_0 \leq 0, \quad (i = 1, 2, 3, 4), \quad C_2 - C_1 \leq 0, \quad C_4 - C_3 \leq 0, \quad C_3 - C_1 \leq 0, \quad C_4 - C_2 \leq 0
\] (8)

where \( C_0 \) and \( C_i \) are the upper and lower limits of \( C_i \), respectively.

By expressing constraint condition formulas (6) and (8) as

\[ g_d(C) \leq 0 \quad (s = 1, 2, \ldots, S) \] (9)

and converting \( |\dot{X}_{\alpha,j+1}|_{\max} \) by SUMT, an objective function without the constraint condition is obtained.

\[
P_K = |\dot{X}_{\alpha,j+1}|_{\max} + r_K \left[ \sum_{s=1}^{S} \left( g_d(C) \right)^2 \right. \times \left. H(-g_d(C)) \right] \quad (K = 1, 2, \ldots)
\] (10)

Fig. 3 Macro-flow chart for alternate optimization

2.3 Alternate optimization of the hump and suspension

A macro-flow chart of alternate optimization is shown in Fig. 3. \( C_{\text{opt}} \) and \( Y_{\text{opt}} \) are the nth optimum solution of design variables \( C \) and \( Y \). \( Y_{\text{opt,0}} \) is the dimension of the hump in the existing design, \( (|\dot{X}_{\alpha,j+1}|_{\max})_{\text{opt}} \) the value of \( |\dot{X}_{\alpha,j+1}|_{\max} \) when the vehicle passes over the \( Y_{\text{opt,0}} \)-hump, \( (|\dot{X}_{\alpha,j+1}|_{\max})_{\text{opt}} \) the value of \( |\dot{X}_{\alpha,j+1}|_{\max} \) when the existing hump and shock absorber of the existing semi-active suspension are combined, and \( \varepsilon_a \) and \( \varepsilon_s \) are positive numbers used to determine the convergence.

3. Example of Design

When the road condition is anticipated for the \( L_v=1 \) vehicle in Table 1 (Fig. 4), it is assumed that damping force is adjustable to match this condition. For the case in which this vehicle passes over a circular arc hump (Fig. 5), alternate optimization was carried out using three vehicle speeds, i.e., \( Z_1 = 1, Z_2 = 1.5, Z_3 = 2 \), where \( Z_i \) is the designed speed of the hump. The constraint conditions were \( C_i = 0, \) and \( C_0 \) for the optimization of the suspension, and \( Y_{\text{opt,1}} = 1, Y_{\text{opt,2}} = 5, Y_{\text{opt,3}} = 0.01, Y_{\text{opt,4}} = 0.04, |\dot{X}_{\alpha,j+1}|_{\max,US} = 0.35, |\dot{X}_{\alpha,j+1}|_{\max,US} = 0.4, |\dot{X}_{\alpha,j+1}|_{\max,US} = 0.5 \) for the optimization of the hump.

The following equations of motion were adopted for the simulation calculation.

\[
\begin{align*}
M_{\alpha} \ddot{X}_{\alpha} &= -K_d(X_{\alpha} - (X_{\alpha} + L_0 \theta_0)) \\
&\quad - C_d(X_{\alpha} - (X_{\alpha} + L_0 \theta_0)) \\
M_{\alpha} \ddot{X}_{\alpha} &= -K_d((X_{\alpha} + L_0 \theta_0) - X_{\alpha})
\end{align*}
\]
Table 1 Specifications of dynamic model for automobiles

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<th>L</th>
<th>M₀</th>
<th>M₁</th>
<th>M₂</th>
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Fig. 6 Effect of alternate optimization of the suspension and hump regarding vehicle speed control effect

\[ X_r = \sqrt{\left(-\left(\frac{Z - L_w}{R} + Y_1/2\right)^2 + R^2\right)} - \left(Y_1/8Y_2 + Y_2/2\right) \]

\[ R = Y_1/8Y_2 + Y_2/2, \quad Z = \dot{Z}T \]  \hspace{1cm} (11)

When the tires are not in contact with the hump or the road:

\[ X_u = X_r - L_U, \quad X_o = X_r - L_o, \]  \hspace{1cm} (12)

where \( M \) is the mass, \( K \) the spring constant, \( C \) the damping coefficient, \( F \) the damping force of the shock absorber or resisting force of the suspension spring, \( I_0 \) the inertia moment of sprung mass around the center of gravity (c.g.), \( L_r, L_o \) the distances from c.g. of the sprung mass to front and rear axles, respectively, \( L_0 \) the distance from c.g. of the sprung mass to c.g. of the driver, \( L_w \) the wheel base, \( X \) the vertical displacement, \( \theta_b \) the angular displacement of the sprung mass during pitching, \( Z \) the forward displacement of c.g. of the sprung mass, \( T \) the time, and \( \dot{Z} \) the passing speed of the vehicle over the hump. Suffix \( o \) designates the driver, \( b \) the sprung mass, \( s \) the seats, \( f, r \) the front and rear unsprung masses, respectively, \( sf \) and \( sr \) are the front and rear suspension springs, respectively, \( df \) and \( dr \) are the front and rear shock absorbers, respectively, \( tf \) and \( tr \) are the front and rear tires, respectively, \( X_u \) and \( X_o \) the tire displacements on flat roads assuming these values are zero for vehicles at rest, \( L_U \) and \( L_o \) the deflections of the tires for vehicles at rest, and \( X_{sf}, X_{sr}, X_{sf}, X_{sr}, X_{sb}, \) and \( X_{sb} \) the deflections of springs \( K_{sf}, K_{sr}, K_{sb}, \) and \( K_{sb} \), respectively.

Figure 6 shows the results of revised simulation calculation using the hump dimensions and damping forces of the shock absorbers obtained by the optimization. The solid line shows the results when the vehicle with semi-active suspension, the shock absorbers of which are adjusted to the existing damping force characteristic "soft" passes over the existing circular arc hump.

Fig. 4 Dynamic model of suspension system for automobiles

Fig. 5 Cross-sectional profile of the circular arc hump
Dotted line $C_{\text{opt},1}$ shows the results when the maximum acceleration of the driver at $\dot{Z} = 1.5$ is minimized by adopting the upper limits $|\dot{X}_{a,1}|_{\text{max},0.5}$ and $|\dot{X}_{a,2}|_{\text{max},0.5}$ for maximum acceleration of the driver at $\dot{Z} = 1$ and $\dot{Z} = 2$, respectively. This is the condition under which damping force of the shock absorber is optimized for the existing circular arc hump. The vehicle speed control effect of the existing hump is generally reduced, as seen in the lowering of the maximum acceleration of the driver over a wide range, compared with the previously mentioned soft characteristics. In particular, 33.5% reduction of maximum acceleration was achieved at $\dot{Z} = 1.5$, where minimization was targeted.

$Y_{\text{opt},1}$ hump was developed to minimize the maximum acceleration of the driver at $\dot{Z} = 1$ for this optimized $C_{\text{opt},1}$ suspension. In this case, maximum acceleration of the driver was higher at $\dot{Z} = 1.5$ compared with the above two cases, but it is lower around $\dot{Z} = 1$. Therefore, vehicles will have a tendency to pass the hump at a speed of around $\dot{Z} = 1$.

Broken line $C_{\text{opt},3}$ shows results of the optimization for this $Y_{\text{opt},1}$ hump. The curve ($C_{\text{opt},3}$) is almost identical to the $Y_{\text{opt},1}$ curve and satisfies decision conditions of convergence for maximum acceleration of the driver at $\dot{Z} = 1.5$.

From these results, we can confirm that the optimum hump is $Y_{\text{opt},1} = (1.128, 0.0305 56)^T$ and optimum suspension is $C_{\text{opt},3} = (1.105, 0.9076, 1.104, 0.799 7)^T$. The dimension of the existing hump expressed in nondimensional form with a wheelbase of 2.52 m are $Y = (1.452, 0.039 68)^T$.

Figure 7 shows a comparison of the speed control effect of the existing and optimized humps, using the four types of vehicles shown in Table 1. Except for large vehicles of $L_w = 1.7$, the optimized hump shows a vehicle speed control effect with less deviation among the vehicle types. For this optimized hump, maximum acceleration of the driver at the design speed $\dot{Z} = 1$ is almost identical for all vehicles. For the vehicles with $L_w = 1.15$, maximum acceleration of the driver decreases from around $\dot{Z} = 1.3$, exhibiting a margin for the improvement of the hump.

Convergence of alternate optimization was greatly improved using the new procedure; seven optimization processes were necessary for the method explained in the previous report, while the procedure adopted in this report required only three.

4. Conclusion

Cross-sectional dimensions of humps and the damping coefficient of shock absorbers were optimized alternately for vehicles with semi-active suspension traveling over circular arc humps, by alternating the standpoint between hump design and suspension design. Results obtained are summarized as follows.

(1) For an optimized hump, maximum acceleration of the driver at the designed speed of the hump was selected as the objective function. For the optimization of shock absorbers, maximum acceleration of
the driver at 1.5 times the designed speed of the hump was selected as the objective function. These two objective functions were alternately minimized. Optimum dimensions of the hump were determined by repetition of the above procedure.

(2) During the optimization of shock absorbers, maximum acceleration of the driver at the hump designed speed and at twice that speed, which were obtained in the preceding step, were set as the upper limit. This resulted in the improvement of convergence for alternate optimization.

(3) For vehicles with optimum shock absorbers, the speed control effect of the existing hump decreased, but this weak point was compensated by optimization of the hump. These situations were clarified by a simulation calculation.

(4) Speed control effects on three types of light vehicles with different wheelbases and one type of heavy vehicle were compared for existing and optimized humps. The optimized hump exhibited less deviation among different vehicle types.

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References

(7) Stephens, B. W., Road Humps for the Control of Vehicular Speeds and Traffic Flow, Public Road, Vol. 50, No. 3 (1986), p. 82.
(17) Watts, G. R., Road Humps for the Control of Vehicle Speed, TRRL Rep. LR-597 (1973), Transport and Road Research Laboratory.
(20) Baguley, C., Speed Control Humps—Further Public Road Trials, TRRL Rep., LR-1017 (1981), Transport and Road Research Laboratory.