Motion Transmissibility of In-Parallel Actuated Manipulators*

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When all input links except one are fixed in the mechanism of an in-parallel actuated manipulator with N degrees of freedom (dof), N kinds of mechanisms with a single dof can be obtained, and pressure angles corresponding to each mechanism can be defined. In order to transmit power efficiently from input links to the output link, all of the cosines of the pressure angles must be as large as possible. The present paper proposes a new transmission index (TI) for in-parallel actuated manipulators based on the power transmitted from input links to the output link, which is defined as the minimum value of cosines of the pressure angles. For planar and spatial in-parallel actuated manipulators, the effectiveness of TI is investigated considering static characteristics of mechanisms. Working spaces of in-parallel actuated manipulators obtained with consideration of motion transmissibility are also shown.

Key Words: Robotics, In-Parallel Actuated Manipulator, Mechanism, Motion Transmissibility, Static Force Analysis, Singular Point, Working Space

1. Introduction

In an in-parallel actuated manipulator, the output link is supported by multiple connecting-chains. The input motions and forces generated by actuators which are arranged on the base link are transmitted to the output link under the geometric constraints of its mechanism. Thus, in the mechanism design of in-parallel actuated manipulators suitable for high precision, high speed, and high rigidity which are the typically advantageous characteristics of in-parallel actuated manipulators compared with conventional serial manipulators, the transmissibility of motion and force from the input links to the output link should be considered. Though the transmissibility of motion has been discussed by many researchers⁽¹⁾⁻⁽⁶⁾ for mechanisms with a single degree of freedom (dof), the transmissibility of mechanisms with multiple loops

and multiple dof has not yet been discussed. Therefore, in-parallel actuated manipulators with high motion transmissibility which contributes to positioning accuracy, operation speed and load capacity have never been developed, though some possible structures of in-parallel actuated manipulators⁽⁷⁾⁻⁽⁹⁾, some methods of analyzing kinematic and dynamic characteristics^{(10),(11)} and singular configurations^{(8),(9),(12)-(15)} of some in-parallel actuated manipulators have been presented.

In the present paper, a new transmission index for in-parallel actuated manipulators is proposed, which is based on the power transmitted from the input links to the output link. The effectiveness of the index is investigated considering the joint forces caused by the forces and moments exerted on the output link. The working spaces of planar and spatial in-parallel actuated manipulators obtained with consideration of motion transmissibility are also shown. Here, the friction at the joints is neglected.

2. Transmission Index

For mechanisms which satisfy the following conditions, the motion transmissibility is analyzed.

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1. Degrees of freedom of planar and spatial mechanisms are 3 and 6, respectively.

2. The output link is supported by the same number of connecting-chains as the dof of the mechanism, and forces are only transmitted from each connecting-chain to the output link.

3. Each connecting-chain contains an input link. For example, the spatial in-parallel actuated manipulator developed by the authors⁽⁹⁾, the Stewart Platform⁽¹⁶⁾, HEXA⁽¹⁷⁾, and the planar in-parallel actuated manipulator considered by Sugimoto⁽¹⁰⁾ and Gosselin⁽¹³⁾ satisfy the above-mentioned conditions.

The connecting-chains which connect the output link with the base link are numbered 1, 2, ..., N , where N denotes the dof of the mechanism. The input displacement and input velocity are denoted by $\theta = [\theta_1, \theta_2, \dots, \theta_N]^T$, $\dot{\theta} = [\dot{\theta}_1, \dot{\theta}_2, \dots, \dot{\theta}_N]^T$, respectively, where θ_i denotes the input displacement of the i -th connecting-chain. The case is considered where all input links except for the i -th input are fixed and $\dot{\theta}_j$ ($j=1, 2, \dots, N, j \neq i$) = 0, as shown in Fig. 1. The velocity caused by $\dot{\theta}_i$ at the point P_k ($k=1, 2, \dots, N$) where the k -th connecting-chain is connected to the output link is denoted by $v_{k,i}$. The force transmitted from the k -th connecting-chain to the output link at P_k , whose direction is determined only by the geometric condition of the mechanism, is denoted by f_k . Here, the following equation holds:

$$f_j \cdot v_{j,i} = 0 \quad (j=1, 2, \dots, N, j \neq i) \quad (1)$$

Thus, the power P transmitted to the output link when all the input links are driven becomes

$$P = \sum_{i=1}^N f_i \cdot v_{i,i} = \sum_{i=1}^N |f_i| |v_{i,i}| \cos \alpha_i \quad (2)$$

Here, α_i is the angle formed by f_i and $v_{i,i}$, and it corresponds to the pressure angle of the mechanism with a single dof in which only the i -th input link is driven. In Eq. (2), f_i is determined by the external load exerted on the output link, $v_{i,i}$ is determined by the output velocity, and α_i is determined by the geometric condition of the mechanism. Thus, P is

maximized when $\cos \alpha_i$ is equal to one ($i=1, 2, \dots, N$). Then, in order to efficiently transmit power from each input link to the output link, the minimum value of $|\cos \alpha_i|$ is required to be as large as possible.

Therefore, a new transmission index TI for in-parallel actuated manipulators is defined by

$$TI = \min(|\cos \alpha_1|, |\cos \alpha_2|, \dots, |\cos \alpha_N|) \quad (3)$$

TI is a nondimensional quantity and varies between zero and one, and when TI is equal to zero, the mechanism is located at a singular point and it cannot work. Thus, TI is a useful index in mechanism design and trajectory planning.

3. Joint Forces

For some planar and spatial in-parallel actuated manipulators, joint forces caused by external forces and moments exerted on the output link are investigated, and the relationships between joint forces and pressure angles and between joint forces and TI are discussed.

3.1 Planar in-parallel actuated manipulator with 3 dof

A planar in-parallel actuated manipulator with three dof is shown in Fig. 2, and static force analyses are carried out. In the mechanism, $J_{i,j}$ ($i, j=1, 2, 3$) are revolute pairs, $J_{1,i}$, $J_{2,i}$ are input links, and their angular displacements are described by $\theta = [\theta_1, \theta_2, \theta_3]^T$. The position of the output link is described by the coordinates (X_P , Y_P) of a point P on the output link in the fixed coordinate system $O-XY$. The orientation angle of the output link is described by φ which is formed by the X axis and x_P axis of the coordinate system $P-x_P y_P$ fixed on the output link. Since the direction of the force transmitted from the i -th connecting-chain to the output link at the joint $J_{3,i}$ is parallel to $\vec{J_{2,i}J_{3,i}}$ and the instantaneous center of the output link which is related to θ_i is the point C_i , each pressure angle α_i can be determined as shown in Fig. 2. For a given position and orientation (X_P , Y_P , φ), eight solutions to the inverse displacement analysis

Fig. 1 Velocities and forces at the points where the output link is connected with each connecting-chain

Fig. 2 A planar in-parallel actuated manipulator with 3 dof

exist, as shown in Fig. 3. Each solution has different kinematic and mechanical characteristics. The problem of selecting the solutions is discussed in the following.

The following three cases of external force $\mathbf{F}$ and moment $\mathbf{M}$ ($[\mathbf{F}, \mathbf{M}] = [F_x, F_y, M]$) applied at the point P are considered.

- (1) $[\mathbf{F}, \mathbf{M}] = [1, 0, 0]$
- (2) $[\mathbf{F}, \mathbf{M}] = [0, 1, 0]$
- (3) $[\mathbf{F}, \mathbf{M}] = [0, 0, 1]$

Here, since the objective of this analysis is to know the values of joint forces relative to external forces and moment, F_x, F_y , and M are set unity. Then, positions of all the joints are calculated, and joint forces and input torques are calculated from equations of equilibrium on the forces and moments of all links. Joint forces $\mathbf{f}_i$ ($i=1, 2, 3$) at joint $J_{3,i}$ are represented by $\mathbf{f}_i = f_i \bar{\mathbf{f}}_i$ ($|\bar{\mathbf{f}}_i| = 1$). For case (1) and case (2) of external forces, the maximum of six values of f_i is represented by $f_{F, \max}$, and for case (3) of external moment, the maximum of three values of f_i is represented by $f_{M, \max}$. For mechanisms with several kinematic parameters, $f_{F, \max}, f_{M, \max}, \alpha_i$ and TI are calculated at various positions and orientations.

As an example, the results of analyses are

Fig. 3 Solutions to the inverse displacement analysis

Fig. 4 A planar in-parallel actuated manipulator with 3 dof used in the analyses

presented for a mechanism shown in Fig. 4. In the mechanism, the Px_P axis is parallel to $\bar{J}_{3,1}\bar{J}_{3,2}$. The positions and orientation angles of the output link are given as follows:

$$X_P = \pm 0.2l \quad (l=1, 2, \dots)$$

$$Y_P = \pm 0.2m \quad (m=1, 2, \dots)$$

$$\varphi = \pm 30^\circ n \quad (n=1, 2, \dots)$$

Here, l, m and n are changed within the limits for which solutions to the inverse displacement analysis can be obtained. In the following, the results for the solution (c) of Fig. 3 are shown.

First, relationships between α_i and f_i are investigated. As an example of the results, $|\cos \alpha_1|$ versus $f_{1, F, \max}$ and $|\cos \alpha_1|$ versus $f_{1, M, \max}$ are shown in Fig. 5. Here, $f_{1, F, \max}$ is the larger value of the two f_1 for case (1) and case (2), and $f_{1, M, \max}$ is f_1 itself for case (3). As shown in the figure, there are many points where $f_{1, F, \max}$ or $f_{1, M, \max}$ is large even in the region where $|\cos \alpha_1|$ is large. For example, at the position ($X_P = -0.4, Y_P = 1.4, \varphi = -30^\circ$) which is indicated by $\bullet$ in the figure, $|\cos \alpha_1| = 0.8806, |\cos \alpha_2| = 0.1226, |\cos \alpha_3| = 0.0731, f_{1, F, \max} = 6.281, f_{1, M, \max} = 14.155$. At this position, the relation between the input and output velocities, and the equation of equilibrium on the forces and a moment of the output link become the following:

$$\begin{bmatrix} \dot{X}_P \\ \dot{Y}_P \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} -0.479 & -0.963 & 0.271 \\ 3.721 & 4.180 & -6.021 \\ 8.387 & 7.766 & -11.874 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} -0.809 & 6.281 & 14.155 \\ 1.195 & -5.187 & -9.637 \\ 0.354 & -7.851 & -15.484 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ M \end{bmatrix} \quad (5)$$

From Eq. (4), it is known that when only θ_3 , the cosine of pressure angle of which is the smallest, is driven, the point P moves almost along the Y axis. On the other hand, from Eq. (5) it is known that all of f_i divided by $|F_y|$ are large. Then, the case is considered where an external force is applied along the Y axis and θ_3 is driven. In this case, since $|\cos \alpha_3|$ is small, f_3 must be large, and the component of $\mathbf{f}_3$ which contributes to effective work is very small. Since the residual component of $\mathbf{f}_3$ which does not contribute to effective work acts on the output link as an internal force, f_1 and f_2 also must be large in order to maintain

Fig. 5 Relationships between $|\cos \alpha_1|$ and $f_{1, F, \max}, f_{1, M, \max}$

the equilibrium on forces and a moment of the output link. Therefore, it is concluded that $|\cos \alpha_i|$ makes $f_{1,F,\max}$ large at this position.

Relationships between TI and $f_{F,\max}$, $f_{M,\max}$ are shown in Fig. 6. From the figure, it is seen that the joint forces caused by the external forces F_X , F_Y and the moment M become smaller as TI increases to 1. For different solutions and kinematic parameters from those cited here, similar results are obtained to those shown in Fig. 6. Therefore, it is found that TI can represent precisely the static characteristics of the in-parallel actuated manipulator.

3.2 Spatial in-parallel actuated manipulator with 6 dof

A Stewart Platform⁽¹⁶⁾ shown in Fig. 7, which is one of the spatial in-parallel actuated manipulators with six dof, is analyzed here. In it, the base and output links are connected by six connecting-chains, each with two spherical joints and a prismatic joint. Each prismatic joint is actuated. The position of the output link is described by the coordinates (X_P, Y_P, Z_P) of the point P on the output link in the fixed coordinate system O-XYZ, and the orientation of the output link is described by roll, pitch and yaw angles $(\varphi_R, \varphi_P, \varphi_Y)$ of the coordinate system P- $x_P y_P z_P$ fixed on the output link to O-XYZ. Since the direction of the force transmitted from each connecting-chain to the output link at the spherical joint $J_{2,i}$ is parallel to $\vec{J_{1,i}J_{2,i}}$ ($i=1, 2, \dots, 6$), the pressure angle α_i is determined by the angle between $\vec{J_{1,i}J_{2,i}}$ and the direction of motion of $J_{2,i}$ when the i -th input link is driven alone.

Fig. 6 Relationships between TI and $f_{F,\max}$, $f_{M,\max}$

Fig. 7 A Stewart Platform

As an example, Mechanisms A, B and C whose kinematic parameters are shown in Fig. 8 are analyzed. For each mechanism, four sets of orientation angles are given as follows:

$$\begin{aligned} \varphi_R=0^\circ, \quad \varphi_P=0^\circ, \quad \varphi_Y=0^\circ \\ \varphi_R=30^\circ, \quad \varphi_P=0^\circ, \quad \varphi_Y=0^\circ \\ \varphi_R=0^\circ, \quad \varphi_P=30^\circ, \quad \varphi_Y=0^\circ \\ \varphi_R=0^\circ, \quad \varphi_P=0^\circ, \quad \varphi_Y=30^\circ \end{aligned}$$

For each orientation, the positions (X_P, Y_P, Z_P) are given as follows:

$$\begin{aligned} X_P &= 0.1l \quad (l=1, 2, \dots) \\ Y_P &= 0.1m \quad (m=1, 2, \dots) \\ Z_P &= 0.1n \quad (n=1, 2, \dots) \end{aligned}$$

Then, TI and joint forces f_i at $J_{2,i}$, caused by external forces $[\mathbf{F}, \mathbf{M}] = [F_X, F_Y, F_Z, M_X, M_Y, M_Z]$, which are equal to the driving forces in the i -th input link, are calculated at each position. The next six cases are considered here.

- (1) $[\mathbf{F}, \mathbf{M}] = [1, 0, 0, 0, 0, 0]$
- (2) $[\mathbf{F}, \mathbf{M}] = [0, 1, 0, 0, 0, 0]$
- (3) $[\mathbf{F}, \mathbf{M}] = [0, 0, 1, 0, 0, 0]$
- (4) $[\mathbf{F}, \mathbf{M}] = [0, 0, 0, 1, 0, 0]$
- (5) $[\mathbf{F}, \mathbf{M}] = [0, 0, 0, 0, 1, 0]$
- (6) $[\mathbf{F}, \mathbf{M}] = [0, 0, 0, 0, 0, 1]$

Here, the minimum value of $\vec{J_{1,i}J_{2,i}}$ is denoted by l_B and the stroke Δl of each input displacement is assumed to be limited within 40 percent of l_B . Here l_B is fixed to be 2. At each position, the maximum value among eighteen of $|f_i|$ for the cases (1)~(3) of the external forces and cases (4)~(6) of the external moments are denoted by $f_{F,\max}$ and $f_{M,\max}$, respectively.

Relationships between TI and $f_{F,\max}$ and $f_{M,\max}$ are shown in Fig. 9. From the figure, it is seen that joint forces are small if TI is large and near one, which means that TI can accurately indicate the motion transmissibility of the in-parallel actuated

	Mech.A	Mech.B	Mech.C
$\beta_{2,1}$	30°	330°	0°
$\beta_{2,2}$	150°	210°	180°
$\beta_{2,3}$	150°	90°	120°
$\beta_{2,4}$	270°	330°	300°
$\beta_{2,5}$	270°	210°	240°
$\beta_{2,6}$	30°	90°	60°
R	2	r	1

Fig. 8 Kinematic constants

Fig. 9 Relationships between TI and $f_{F,\max}$, $f_{M,\max}$

Fig. 10 TI map in the working space for the solution (a) of Fig. 3 ($\varphi=30^\circ$)

manipulators. In addition, it is seen from the figure for Mech. C that many positions of the output link where TI is small, $f_{F,\max}$ is small and $f_{M,\max}$ is large exist. At such positions, a small value of TI means that at least one of the joint forces caused by the external moments is large. Here, motion transmissibility cannot be evaluated from the value of the ratio of an external moment and a joint force, because the ratio has the dimension of length. On the other hand, since TI is a nondimensional value, it is a very useful index in the mechanism design.

4. Working Space

4.1 Planar in-parallel actuated manipulator with 3 dof

With consideration of motion transmissibility, working spaces of the planar in-parallel actuated manipulator with 3 dof shown in Fig. 4 are investigated. The working space, where the point P is reachable, is obtained as the common region of the interiors of three circles when the orientation angle φ is kept constant⁽¹³⁾. However, since there are singular points and regions in the working space where motion transmissibility is poor and the manipulator cannot operate normally, the regions which can be used in actual work are fairly limited.

The TI map in the working space is shown in Fig. 10 in the case where the orientation angle φ is kept at 30° and the solution (a) of Fig. 3 is taken. From the figure, it is seen that TI varies throughout the working space. Moreover, with consideration of motion transmissibility, the regions of $TI \geq 0.5$ for each solu-

Fig. 11 Working spaces with consideration of motion transmissibility (Hatched region: $TI \geq 0.5$)

tion to the inverse displacement analysis are calculated and shown as hatched regions in Fig. 11. For the solutions (c), (e), (f), (g) of Fig. 3, the regions of $TI \geq 0.5$ can be obtained by rotating the region of (b) or (d) about the origin. For example, the region of solution (c) can be obtained by rotating the region of (b) clockwise by 120° . The values in parentheses in the figure indicate the ratio of the areas of the hatched regions and the region of the working space (6.7, as shown in Fig. 10). From Fig. 11, it is seen that the areas of the working spaces obtained with consideration of motion transmissibility are considerably smaller than those of the corresponding working spaces obtained without considering the motion transmissibility. The shapes and areas of the working spaces for each solution obtained with consideration of motion transmissibility are quite different from one another. Therefore, in motion planning, an appropriate solution must be chosen with consideration of motion transmissibility.

When the solution to the inverse displacement analysis is selected, it is necessary to know whether or not it is possible to change from the current configuration of the mechanism to another desirable configuration only by driving input links. Here, the configuration includes the position and orientation of the output link and the solution to the inverse displacement analysis. The mechanism can transit between two solutions in the positions where the solutions coincide. For example, between the solutions (a) and (b), the mechanism can transit at points on the arc AB in Fig. 11(a), where $J_{1,3}$, $J_{2,3}$ and $J_{3,3}$ are colinear and $TI \geq 0.5$. Therefore, if the solution is changed from (a) to (b) or from (b) to (a) through any points on the arc AB, the mechanism can

Fig. 12 Relationships between l_B and $\overline{S_G}$, $\overline{S_T}$

Fig. 13 Working space of the Stewart Platform

reach any points in the sum of the hatched regions (a) and (b) in Fig. 11.

4.2 Spatial in-parallel actuated manipulator with 6 dof

For Mech. A shown in Fig. 8, working spaces are investigated under the condition of $l_B \leq \sqrt{J_{1,i} J_{2,i}} \leq 1.4 l_B$ by changing l_B . When φ_R , φ_P and φ_Y are kept at zero, the cross-sectional area S in the X - Z plane of $Y=0$ is calculated. The area which the point P on the output link can reach is described by S_G , and the area of $TI \geq 0.5$ is described by S_T , and they are divided by the square of the stroke $(\Delta l)^2 = (0.4 l_B)^2$ and the non-dimensional values $\overline{S_G} = S_G / (\Delta l)^2$ and $\overline{S_T} = S_T / (\Delta l)^2$ are denoted by $\overline{S_G}$ and $\overline{S_T}$, respectively. The values $\overline{S_G}$ and $\overline{S_T}$ indicate how the stroke Δl contributes to the area of the working space. Relationships between l_B and $\overline{S_G}$, $\overline{S_T}$ are shown in Fig. 12. From the figure, it is known that $\overline{S_T}$ becomes maximum at about $l_B = 4.75$, and that at all positions inside the working space the condition of motion transmissibility ($TI \geq 0.5$) is satisfied when l_B is selected to be between 2 and 4.5. In addition, when $l_B = 1.5$ and $l_B = 5$, it is known that $\overline{S_T}$ is considerably smaller than $\overline{S_G}$. This is because motion transmissibility is poor at the lower part of the working space when $l_B = 1.5$, and at the upper part when $l_B = 5$,

as shown in Fig. 13.

5. Conclusions

In the present paper, motion transmissibility of in-parallel actuated manipulators has been investigated. The results are summarized as follows.

(1) Based on the power transmitted from the input links to the output link of mechanisms having multiple closed loops and multiple degrees of freedom, a new transmission index (TI) for in-parallel actuated manipulators has been proposed.

(2) Static force analyses have been carried out, and relationships between TI and joint forces caused by external loads have been investigated. TI has been confirmed to accurately evaluate the motion transmissibility of in-parallel actuated manipulators.

(3) Working spaces of in-parallel actuated manipulators obtained with consideration of motion transmissibility have been investigated. It has been shown that the regions where the motion transmissibility is good are considerably smaller than the working spaces obtained without considering the motion transmissibility. Also, for mechanisms with multiple solutions to the inverse displacement analysis, the areas and the shapes of the working spaces of each solution with consideration of motion transmissibility are quite different from each other.

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