Effect of Boundary Conditions on the Stability of a Cantilever Pipe Discharging and Aspirating Fluid*

Hongwu CUI** and Junji TANI**

In this paper, the governing equation for a pipe conveying fluid is modified by including the influence of the following factors: (1) differences in pressure within and around the pipe; (2) boundary conditions of the fluid and the pipe. The solution of the new general basic equation depends on the boundary conditions. Application of the new equation to a cantilever pipe discharging fluid gives a result closer than the previous ones to the experimental value. The critical speed obtained from the new theory for the cantilever pipe aspirating fluid is higher than that obtained from the previous theory.

Key Words: Flutter, Vibration Coupled with Fluid Motion, Self-Excited, Vibration, Pipe Conveying Fluid, Basic Equation, Boundary Condition

1. Introduction

It is well known that a cantilevered pipe discharging fluid becomes unstable when the flow speeds exceed a critical value. The conclusion was proved both theoretically and experimentally¹⁾⁻⁶⁾. However, in the case of a cantilever pipe aspirating fluid, although the previous theoretical results indicate that the critical flow speed is close to zero, unstable phenomenon appears never to have been observed experimentally⁷⁾⁻⁹⁾. The authors had never observed the phenomenon in the present setup. This can be explained in that an important factor had been neglected: for a pipe aspirating fluid, the free end must be in the fluid, and thus there is a shear force acting on the pipe end when the free end vibrates laterally. The boundary condition of shear force will affect the stability.

In this paper, the governing equation of a cantile-

ver pipe conveying fluid is obtained by including the influence of two factors: (1) the differences in pressure within and around the pipe; (2) the boundary conditions of the fluid and the pipe. Considering the two factors, the theoretical and experimental results show that when the free end is in the fluid, a pipe aspirating fluid has a high critical flow speed (in the perfect case it may be infinity), and a pipe discharging fluid has a lower critical flow speed than discharging to air.

Nomenclature

EI : flexural rigidity of pipe
 E^* : coefficient of internal viscosity
 x : axial coordinate
 W : transverse displacement of pipe
 A : cross-sectional area of pipe
 L : length of pipe
 V : flow speed of fluid in pipe
 m_f : mass of fluid per unit length
 m_t : mass of pipe per unit length
 p : static pressure in internally flowing fluid
 p_0 : external pressure
 qS : shear force on the internal surface of the pipe

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** Institute of Fluid Science, Tohoku University, 2-1-1 Katahira, Aoba-ku, Sendai 980, Japan

F : transverse force per unit length between pipe wall and fluid
 g : acceleration of gravity
 K_n : nozzle parameter

2. Motion Equation and Boundary Condition of a Pipe Conveying Fluid

Before the analysis, the following assumptions were made.

- (i) the pipe is a Euler beam, and its vibration is small
- (ii) the flow speed distribution on a crosssection is uniform and does not depend on time
- (iii) for simplicity, the pipe vibrates in air except for the free end

The resulting motion equation will be derived later in detail.

2.1 Differences in pressure within and around the pipe

The static pressure distribution is shown in Fig. 1, from assumptions (i) and (ii), the difference in pressure between the extended and contracted sides is shown as the hatched part in Figs. 1 and 2, where

$$\Delta\theta = \frac{\partial W}{\partial x} \delta x. \quad (1)$$

For the fluid element in the lateral direction, the difference per unit length of pressures ΔP_f is given as

$$\begin{aligned} \Delta P_f &= \int_0^{2\pi} p \Delta\theta a^2 \sin^2 \varphi d\varphi \\ &= \pi a^2 p \frac{\partial^2 W}{\partial x^2} \delta x, \end{aligned} \quad (2)$$

(a) Fluid element

(b) Tube element

Fig. 1 Distribution of pressure

and for the pipe element, the difference per unit length of pressures ΔP_t is given as

$$\Delta P_t = \pi a^2 (p_0 - p) \frac{\partial^2 W}{\partial x^2} \delta x. \quad (3)$$

2.2 Force balance of element

Forces acting on an element are shown in Fig. 3.

Fig. 2 Area difference between extended and contracted sides

(a) Fluid element

(b) Tube element

Fig. 3 Forces and moments on elements

For the fluid element, resolution along and perpendicular to the tangent to the centerline of the deflected element yields the equations

$$A \frac{\partial p}{\partial x} + qS = 0 \quad (4)$$

$$F - m_f \frac{D^2 W}{Dt^2} - A p \frac{\partial^2 W}{\partial x^2} + \Delta P_f = 0, \quad (5)$$

where

$$\frac{D^2 W}{Dt^2} = \frac{\partial^2 W}{\partial t^2} + 2V \frac{\partial^2 W}{\partial x \partial t} + V^2 \frac{\partial^2 W}{\partial x^2}.$$

For the pipe element, the force balance equation can be obtained in the same way and is expressed in the following form.

$$\frac{\partial T}{\partial x} + qS - Q \frac{\partial^2 W}{\partial x^2} = 0 \quad (6)$$

$$\frac{\partial Q}{\partial x} + T \frac{\partial^2 W}{\partial x^2} - F + \Delta P_t - m_t \frac{\partial^2 W}{\partial t^2} = 0 \quad (7)$$

2.3 Motion equation

In Fig. 3(b) moment balance is given by the following equation.

$$Q = - \frac{\partial M}{\partial x} = - \frac{\partial}{\partial x} \left(EI \frac{\partial^2 W}{\partial x^2} \right) \quad (8)$$

Equations (5), (7) and (8) now yield

$$EI \frac{\partial^4 W}{\partial x^4} - [A(p_0 - p) + T] \frac{\partial^2 W}{\partial x^2} + m_f V^2 \frac{\partial^2 W}{\partial x^2} + 2m_f V \frac{\partial^2 W}{\partial x \partial t} + (m_t + m_f) \frac{\partial^2 W}{\partial t^2} = 0, \quad (9)$$

and q may be eliminated between Eqs. (4) and (6) to give

$$\frac{\partial(Ap - T)}{\partial x} = 0. \quad (10)$$

Considering gravity in the vertical direction, the motion equation can be given as

$$EI \frac{\partial^4 W}{\partial x^4} - [A(p_0 - p) + T] \frac{\partial^2 W}{\partial x^2} + m_f V^2 \frac{\partial^2 W}{\partial x^2} + 2m_f V \frac{\partial^2 W}{\partial x \partial t} + (m_t + m_f) \frac{\partial^2 W}{\partial t^2} + (m_t + m_f)g \frac{\partial W}{\partial x} = 0, \quad (11)$$

and Eq. (10) becomes

$$A(p_0 - p) + T = (L - x)(m_t + m_f)g + C. \quad (12)$$

The motion equation, Eq. (11) includes the terms of Coriolis force and centrifugal force.

2.4 Boundary conditions

When the free end of a cantilever pipe is not immersed in the fluid, the boundary conditions can be written as follows.

$$\left. \begin{aligned} W|_{x=0} = 0 \quad \frac{\partial W}{\partial x}|_{x=0} = 0 \\ \frac{\partial^2 W}{\partial x^2}|_{x=L} = 0 \quad \frac{\partial^3 W}{\partial x^3}|_{x=L} = 0 \end{aligned} \right\} \quad (13)$$

Therefore, if the free end is immersed in the fluid, the shear force condition will not be zero, i.e.,

$$EI \frac{\partial^3 W}{\partial x^3}|_{x=L} \neq 0, \quad (14)$$

Here we use the momentum of the ideal fluid to

derive the shear force boundary conditions of the pipe and fluid. The theorem has the form

$$\int_S \rho \mathbf{v}_n \cdot \mathbf{v} ds = - \int_S p \cdot \mathbf{n} ds, \quad (15)$$

where S is the fluid surface boundary and $\mathbf{n}$ is tangent to the direction of S .

It is assumed that flow speed along the x direction is positive. Utilizing the momentum of the ideal fluid, the shear force boundary condition at the end of the pipe is

$$EI \frac{\partial^3 W}{\partial x^3}|_{x=L} = - \rho A V \frac{\partial W}{\partial t}|_{x=L}, \quad (16)$$

and the fluid boundary condition is found as follows

$$(p_0 - p)A + T = (L - x)(m_f + m_t)g - \rho A V |V|. \quad (17)$$

3. Applications

3.1 Experimental setup

The experimental setup is shown in Fig. 4; the pipe conveying fluid is cantilevered vertically and the free end of pipe is immersed in fluid. The display unit and flow meter are used to measure the flow speed, and FFT analyzer and laser sensors to measure the

Table 1 Value of parameters

Length of Pipe	$L=1$ m
Outer Diameter of Pipe	$d_o=9.4$ mm
Inner Diameter of Pipe	$d_i=8.2$ mm
Young's Modulus	$E=1.3 \times 10^8$ N/m ²
Coefficient of Internal Viscosity	$E^*=6.0 \times 10^5$ N·s/m ²
Mass per Unit Length of Pipe	$m_t=1.6 \times 10^{-2}$ kg/m
Mass per Unit Length of Fluid	$m_f=5.3 \times 10^{-2}$ kg/m
Nozzle Parameter	$K_n=1.9$

Fig. 4 Setup

Table 2 Critical flow speed

Free End	Dimensionless Flow Speed	
	Theory	Experiment
Out of Water	4.66	4.35
In Water	2.24	3.36

displacement of the pipe. We use this setup to examine the stability of one discharging water to fluid and a pipe aspirating water.

The three dimensionless parameters is defined as follows

$$\beta = \frac{m_f}{m_t + m_f} = 0.768$$

$$\mu = \frac{E^*}{EL^2} \sqrt{\frac{EI}{m_t + m_f}} = 0.002544$$

$$v = VL \sqrt{\frac{m_f}{EI}}$$

3.2 Pipe discharging fluid to fluid (free end in water)

The stability of a cantilever pipe discharging water to air has been discussed by many researchers¹⁾⁻⁶⁾. Those papers showed that theoretical and experimental results are in agreement. Here, to examine the effect of boundary conditions induced by water, we analyze the stability of a pipe discharging fluid to fluid.

Combining Eqs. (12) and (17) yields

$$(p_0 - p)A + T = (L - x)(m_f + m_t)g - \rho A V^2. \quad (18)$$

Considering the influence of internal viscosity, and combining Eqs. (11) and (18), the motion equation of a pipe discharging fluid to fluid may be obtained as

$$\begin{aligned} EI \frac{\partial^2 W}{\partial x^4} + [2m_f V^2 - (L - x)(m_f + m_t)g] \frac{\partial^2 W}{\partial x^2} \\ + 2m_f V \frac{\partial^2 W}{\partial x \partial t} + E^* I \frac{\partial^5 W}{\partial x^4 \partial t} + (m_f + m_t) \frac{\partial^2 W}{\partial t^2} \\ + (m_f + m_t)g \frac{\partial W}{\partial x} = 0. \end{aligned} \quad (19)$$

From Eq. (16), the boundary conditions are

$$\left. \begin{aligned} W|_{x=0} = 0 \quad \frac{\partial W}{\partial x} \Big|_{x=0} = 0 \\ \frac{\partial^2 W}{\partial x^2} \Big|_{x=L} = 0 \quad EI \frac{\partial^3 W}{\partial x^3} \Big|_{x=L} = -\rho A V \frac{\partial W}{\partial t} \Big|_{x=L} \end{aligned} \right\}. \quad (20)$$

In Eq. (19), the centrifugal force turns into two times, and in Eq. (20), there is a shear force acting on the free end of the pipe.

The theoretical and experimental results are shown in Table 2. These results imply that the change in stability is induced by the fluid and shear force boundary conditions.

3.3 Pipe aspirating fluid

The previous research results regarding cantilevered pipes aspirating fluid indicated that in the absence of dissipative forces, when the fluid is

Table 3 Comparison of results

Dimensionless flow speed	
Previous Theory	0.01
Present Theory	> 1000
Experimental Result	> 6.54

conveyed from the free end towards the clamped end, stability is lost instantaneously. Those studies were only theoretical, and no physical experiments have verified this phenomenon. Later, we will use the above results to discuss and verify the observed stability both theoretically and experimentally.

According to the analysis in Section 3, combining Eqs. (12) and (17) yields

$$(p_0 - p)A + T = (L - x)(m_f + m_t)g + \rho A V^2; \quad (21)$$

the difference between Eqs. (18) and (21) is due to the direction of flow speed.

Substituting Eq. (21) in to Eq. (11), the motion equation of a pipe aspirating fluid can be obtained as

$$\begin{aligned} EI \frac{\partial^4 W}{\partial x^4} - (L - x)(m_f + m_t)g \frac{\partial^2 W}{\partial x^2} \\ - 2m_f V \frac{\partial^2 W}{\partial x \partial t} + E^* I \frac{\partial^5 W}{\partial x^4 \partial t} + (m_f + m_t) \frac{\partial^2 W}{\partial t^2} \\ + (m_f + m_t)g \frac{\partial W}{\partial x} = 0, \end{aligned} \quad (22)$$

and from Eq. (16), the boundary conditions are as follows.

$$\left. \begin{aligned} W|_{x=0} = 0 \quad \frac{\partial W}{\partial x} \Big|_{x=0} = 0 \\ \frac{\partial^2 W}{\partial x^2} \Big|_{x=L} = 0 \quad EI \frac{\partial^3 W}{\partial x^3} \Big|_{x=L} = \rho A V \frac{\partial W}{\partial t} \Big|_{x=L} \end{aligned} \right\} \quad (23)$$

Comparing Eqs. (22) and (23) with the previous results, it is clear that in Eq. (22) there is no centrifugal force term, and in Eq. (23) there is a shear force acting on the free end of the pipe. The difference in motion equations and boundary conditions may affect theoretical results of stability differently. Using the present theory, calculations showed that the critical dimensionless flow speed is larger than 1 000 and much larger than the previous theoretical result of only 0.01. This value is large enough to explain the result of the physical experiments. (The true value may be infinity, but for larger values, the numerical method loses reliability.)

In the present experiment, at high dimensionless flow speed, our system did not lose stability. Because of the limited capacity of the pump, we could not examine cases with dimensionless flow speeds greater than 6.54. However, this flow speed is more than 600 times larger than the previous theoretical result, and the critical value may be even larger according to the present theoretical result.

The theoretical and experimental results are shown in Table 3.

5. Conclusions

The dynamic system investigated was a cantilever pipe conveying fluid, with the free end of the pipe in fluid. The fluid and shear force boundary conditions differ from those of the case in which the free end is in air (for a pipe aspirating fluid, the free end must be in the fluid).

Theoretical and experimental results showed that the critical flow speed of a pipe discharging fluid to fluid (free end in fluid) is lower than that of one discharging to air; and in the case of a pipe aspirating fluid, the pipe system will not lose stability, i.e., no flutter occurs.

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