Suppression of Chatter Vibration in Inner-Diameter Cutting*

Kelson Z.Y. YEN** and Wen C. HSUEH**

The accuracy of a workpiece produced using metal-cutting machine tools is decided by the deviation at the cutting point from the demanded working displacements between the workpiece and tool. Chatter vibration, which is the most important phenomenon among the dynamic characteristics of machine tools, is hard to suppress after machine tools have been constructed. Also, the elimination of chatter vibration is important in order to improve cutting accuracy and productivity of the cutting operation. In this study, the structural dynamics of machine tools are identified by a normal transfer function that determines the necessary design change from simulation by FEA. The predicted local effects of component modification computed by FEA to improve the dynamic performance by reducing chatter were found to coincide with those observed in a practical cutting operation. This study shows that modification of structural units to overcome chatter is an effective and cheap way of improving the cutting performance, and reveals how the stiffness changes with changes in the stability.

Key Words: Cutting, Chatter Vibration, Finite-Element Analysis, Stability

1. Introduction

Machine tools evaluated by the machining accuracy and productivity performances are the most elementary and essential machines in the manufacturing industry. One of the most significant factors preventing an increase in the output from machine tools is the occurrence of chatter. Chatter adversely affects both the surface finish and the dimensional accuracy of workpieces. Furthermore, violent chatter not only reduces tool life but also interrupts cutting operations. Fundamental chatter theory has been developed in studies to formulate and implement standards for testing and evaluating the cutting performance of machine tools which include the limitations imposed by chatter and by the effect of the cutting force on the accuracy of the machine surface^{(1),(2)}.

Structural dynamics analyses of variables at the initial design stage are important in improving and optimizing the design to avoid structural

resonance^{(3)-(7),(20)}. Sometimes, forced vibrations in machine tools are induced by component defects, unbalanced members, gear-tooth engagement shock or poor assembly. Frequency analysis and power spectrum analysis can be used to determine the oscillation sources. In most cases the amplitude of forced vibrations can be reduced by moving the oscillation source in such a manner that it does not come close to any natural frequency of the machine system.

Vibration can occur even if there is no oscillation source. If machine tools dissipate only a portion of the energy supplied during cutting operations, then the relative displacements between the tool and workpiece will grow to unacceptable levels. This kind of vibration is called chatter vibration and causes dynamic instability in the metal cutting process. The stability of chatter has been investigated by a number of researchers⁽⁸⁾⁻⁽¹⁰⁾. Most have concentrated on chatter theories and stability analyses and have enhanced the comprehension and prediction of chatter phenomena⁽²¹⁾. However, it is technically difficult to develop techniques to suppress chatter completely after the machine tool has been constructed. Some success in suppressing self-excited chatter by varying

* Received 1st August, 1994.

** CNC Laboratory, Victor-Taichung Machinery Works Co., Ltd., 266, Sec. 3, Taichung Kan Road, Taichung, Taiwan

the rotational speed of the spindle has been reported^{(11),(12)}. Most studies have suggested reducing the chip thickness in the turning operation to eliminate chatter. However, the reduced removal rates resulting from a reduction in chip thickness limited productivity, which was unacceptable. Sometimes, chatter occurs even when successive cuts do not overlap, so that varying the chip thickness is not effective.

We aim to achieve an improved cutting performance by modifying components to increase the stability of the machining system against chatter vibration. In this study, chatter phenomena in turning workpieces are described and effective methods of suppressing chatter are evaluated. We identify the chatter using a normal transfer function and determine the optimum design variables to improve the machining accuracy and surface finish by FEA.

2. Chatter Vibration in Turning Workpieces

The machine tool used is a Victor Vturn-26 CNC lathe. It is driven by a 20 hp motor, which provides spindle speeds of up to 6 000 rpm, and it is numerically controlled by a FANUC 0-T controller. The maximum travelling distance of the carriage is 610 mm and the maximum travel of the cross slide is 190 mm. A third axis is provided by the end turret, which can carry a cutting center. The maximum travel of the end turret is 520 mm. The cutting tools are carried by a turret assembly that supports up to 10 tools.

The apparatus incorporates a PCB 353 A, 97.7 mV/g accelerometer and a HP 35655 A dynamic signal analyzer (DSA) to detect vibrational signals. An inner-diameter cutting tool was fixed on the tool holder of the lathe and fed toward a workpiece which was mounted on a three-jaw chuck and rotated. Channel 1 of the accelerometer used by the HP DSA was attached to the tool holder which was perpendicular to the feed direction, and channel 2 was mounted in the end of the spindle. Thus, the frequency and amplitude of chatter vibration were measured. Inner-diameter cutting was carried out with various cutting parameters.

Figure 1 shows a workpiece subject to chatter vibration during inner diameter cutting. Chatter results in regenerative waviness in the surface finish produced by inner-diameter cutting. The cutting parameters were a feed rate of 2.309 mm/rev, and the spindle speeds of 855 and 1 120 rpm. The tool material was high-speed tool steel and the workpiece material was stainless steel. The figure shows that chatter started at the beginning of the cutting operation. Therefore, the finished workpiece has a poor surface finish and distinguishable waviness. In turning, chat-

ter caused by the undulating surface formed during the previous revolution is undesirable because of its detrimental effects on machining accuracy and surface roughness. Theoretically, the chatter frequency f (Hz) is determined by the cutting speed, ω (rpm), the radius of the workpiece, R (mm), and the spacing of the chatter marks, Δw (mm) :

$$f = \left(\frac{2\pi\omega}{60\Delta w} \right) R. \quad (1)$$

For $\omega = 1\ 120$ rpm and 855 rpm, the spacings of chatter marks are found to vary from 0.87 mm to 0.66 mm. Thus the chatter frequencies, which are 2 156 Hz and 2 170 Hz, respectively, can be calculated using Eq. (1).

Figures 2(a) and (b) show the power spectra of chatter under the above cutting conditions. Recording to Fig. 2(a), chatter vibration occurred at a frequency of about 2 192 Hz and the amplitude of vibration at this frequency was 427.81 *mg* in channel 1. The acceleration unit used by the HP DSA is *mg* (e.g. $1 \times 10^{-3} g$) and one *g* is 9.8 m/sec². The chatter frequency was the same in Fig. 2(b), and this vibration cannot be successfully suppressed by varying the rotational speed during inner-diameter cutting. The only effect of varying the speed of the spindle is to change the spacing of the regenerative waviness in the surface finish.

Selection of optimum cutting conditions is the first consideration in suppressing chatter. The cutting speed has been changed in order to suppress chatter, but this was not effective. Decrease of the feed rate and the angle of the cutting edge, which cannot be used in this process, are sometimes quite effective. While the cutting operation continues on the wavy surface as the workpiece turns, a dynamic excitation of the machine at its resonance frequency is induced.

Fig. 1 Chatter vibration causes poor surface finish and distinguishable waviness in the workpiece

(a) 1120 rpm

(b) 855 rpm

Fig. 2 Measured results of frequency response of accelerometer signals during cutting operation. Chatter vibration occurs at a frequency of around 2 200 Hz

However, the instability of the machining process is not governed by the cutting parameters.

3. Suppression of Chatter Vibration

Chatter vibration due to structural resonance normally has comparatively low frequency^{(3),(13)}. In the present case, chatter occurred at very high frequency due to the low rigidity of local units. There are several ways to stabilize the cutting process against chatter.

- Increase the static stiffness.
- Increase the damping.
- Shift the resonant frequency.

3.1 Increasing the static stiffness

Insufficient static stiffness of machine tools is predominantly responsible for poor machining accuracy. Increasing the static stiffness of the machine tool, which is difficult after it has been constructed, is essential to stabilize the machining system. However, this causes undesirable increases in the weight and production costs of the machine tools. Also, the cost increase makes the machine tool builder less competitive. The maximum stiffness should be achieved with the minimum amount of material. Therefore, we did not adopt this strategy to enhance the cutting performance of machine tools.

3.2 Increasing the damping

Heavy damping is desirable not only from the viewpoint of suppressing chatter but also for the purpose of absorbing free or forced vibration⁽¹⁴⁾. Increased damping can be achieved by incorporating vibration absorbers or by introducing friction surfaces in the machine tool design^{(15),(16)}. There are a variety of vibration absorbers which increase damping.

- Dynamic damper⁽¹⁷⁾
- Lanchester damper⁽¹⁸⁾
- Impact damper⁽¹⁹⁾
- Hydraulic damper
- Squeeze damper
- Self-optimizing damper
- Active damper

We chose the “impact damper” to increase the damping and stability of the cutting operation against chatter vibration. This damper is extremely simple in construction and suitable when the vibration primarily affects the tool. The damper consists of a screw which contains a small mass and is mounted on the tool or tool holder. By the interaction of the mass and the spring, the damper alters the natural frequency of the tool and the dynamic characteristics of the system.

3.3 Shifting the resonant frequency

An actual system has an infinite number of degrees of freedom and the equation of motion in a linear vibration system is

$$[M]\{X''\} + [C]\{X'\} + [K]\{X\} = \{F\}. \quad (2)$$

We want to shift the resonant frequency which is almost unaffected by damping. Thus the system can be considered to have no damping and Eq.(2) becomes

$$[M]\{X''\} + [K]\{X\} = \{F\}e^{i\omega t}, \quad (3)$$

where $[M]$ and $[K]$ are the mass and stiffness matrices of a component when the system is subject to a harmonic excitation. At the m -th natural frequency ω_m , Eq.(3) is expressed in terms of the displacement eigenvector $\{X_m\}$:

$$[K]\{X_m\} = \omega_m^2 [M]\{X_m\}. \quad (4)$$

The redesign of components corresponds to the modification of the stiffness matrix $[K]$ and the mass matrix $[M]$. It is assumed that the stiffness matrices for subsystems s and r of the component are $[K_s]$ and $[K_r]$, and the mass matrices for subsystems s and r are $[M_s]$ and $[M_r]$. When a component is redesigned, the stiffness and mass matrices are changed as follows:

$$[K_s'] = [K_s] + \alpha[K_s] \quad (5)$$

$$[M_r'] = [M_r] + \beta[M_r]. \quad (6)$$

Therefore, the equation governing the resonant frequency shift due to design modification is

$$\frac{d\omega_m^2}{\omega_m^2} = \frac{\alpha\{X_m\}_s^T[K_s]\{X_m\}_s - \beta\omega_m^2\{X_m\}_r^T[M_r]\{X_m\}_r}{\omega_m^2\{X_m\}^T[M]\{X_m\}}. \quad (7)$$

The above relationships have been extensively analyzed by Yoshimura⁽⁷⁾. When α has a positive value, that is, the component has increased rigidity, the natural frequency ω_m is increased. On the other hand, when β has a positive value, that is, the component has increased mass, the natural frequency ω_m decreases because $d\omega_m^2/\omega_m^2$ is negative⁽⁷⁾. From Eq. (7), the natural frequency increase is greater when the component is redesigned so that its rigidity increases or its mass decreases. Thus we can shift the natural frequency in order to suppress resonant vibration by redesigning the component.

4. Finite Element Model of a CNC Lathe

All the finite-element calculation described below were carried out with MSC/NASTRAN version 67.5, running on the HP 9000/735 workstation available at the CNC Laboratory of VTMW. MSC/NASTRAN is a general purpose FEA code especially suited for structural dynamic analyses.

Figure 3 shows a schematic diagram of a CNC lathe and the detail meshes of a turret for modal and frequency response analyses. The complete structure of the lathe was represented by 8793 QUAD 4, 1209 TRIA 3, and 573 HEXA elements and a total of 58 162 degree of freedom. The lathe is constructed from a Meehanite plate and is mounted on the floor by means of 12 rubber pads. The boundary conditions at the 12 rubber pads were simulated by the free-free condition in this case. First, the computed results were compared with the experimental modal testing results in order to verify the finite-element analysis mesh model and made it appropriate for evaluating design changes based on frequency response and stability analyses. The measured natural frequencies and the percentage errors in the calculated values are shown in Table 1. The four lowest resonance frequencies are 640 Hz, 1 142 Hz, 1 394 Hz and 1 621 Hz, which are comparable to the experimental results of 635.28 Hz, 1 190

Table 1 Comparison of the natural frequencies from finite-element analysis and experimental observation

Mode	FEA (Hz)	Experiment (Hz)	Error (%)
1	640	635.28	0.74
2	1142	1190	-4.0
3	1394	1390	0.28
4	1621	1460	11.03

(a) Model of machine tool structure for finite-element method calculations

(b) Meshes of turret for frequency response and stability analyses. The diameter, D , and thickness, b , of the turret are 420 mm and 85 mm, respectively

Fig. 3

Hz, 1 390 Hz and 1 460 Hz, respectively. The calculated and experimental mode shapes for the turret are shown in Fig. 4. From the mode shapes, the relative displacements of different parts of the structure can be examined. The calculated and experimental natural frequencies and mode shapes agree well. Therefore, the finite-element model in Fig. 3 can be considered a realistic simulation of the dynamic characteristics.

5. Stability Analysis of the Turret

An understanding of the relative flexibility at the cutting point is essential for chatter analysis. The cutting force, which is applied at the cutting point, has three components along the axes of a Cartesian

Fig. 4 Comparison of calculated and experimental mode shapes

coordinate system. A force applied in any given direction produces a displacement not only along the same axis but also along the other two axes. Therefore, the flexibility of the system at any cutting point can be written as the tensor form

$$x_i = g_{ij} f_j \quad (8)$$

where x_i is the Fourier transform of the relative displacement of the tool and the workpiece and f_j is the Fourier transform of the force. Both x_i and f_j , which are functions of frequency, lie along arbitrary orthogonal directions, ζ , η and ξ . g_{ij} is the flexibility matrix which consists of nine transfer function elements when i and j range over 1, 2, 3. g_{ij} corresponds to the response of the system in the i -th direction to a force applied in the j -th direction when the other two force components are zero. It is shown in this study that the state of displacement, force, and transfer function at any point is completely characterized by a subscript, called the components of tensor. On expanding Eq. (8), we have

Fig. 5 Frequency response at the tip of tool in a conventional tool sleeve

$$\begin{aligned} x_1 &= g_{11}f_1 + g_{12}f_2 + g_{13}f_3, \\ x_2 &= g_{21}f_1 + g_{22}f_2 + g_{23}f_3, \\ x_3 &= g_{31}f_1 + g_{32}f_2 + g_{33}f_3. \end{aligned} \quad (9)$$

Maxwell's reciprocity theorem for linear systems such as those normally encountered on machine tools is symmetric ($g_{ij} = g_{ji}$). In other words, the displacement in the direction i due to a unit load in another direction j is equal to the displacement along j due to a unit load along i , provided that the displacements and forces "correspond", i.e., that they are measured in the same direction. Consider a unit vector $\mathbf{n}^c$ normal to the cutting surface, oriented away from the cutting point. The traction transfer function G_i^c at the contact point for any orthogonal direction i can be expressed as

$$G_i^c = g_{ji} n_j^c. \quad (10)$$

Equation (10) shows that a transfer function for the cutting process at a contact point can be associated with a traction transfer function for an interface having any direction in space. Thus we can express the normal transfer function, G_n^c , acting at the cutting point by the dot product of the vectors $\mathbf{n}_i^c$ and $\mathbf{G}_i^c$:

$$G_n^c = G_i^c n_i^c = g_{ji} n_j^c n_i^c = g_{ij} n_i^c n_j^c. \quad (11)$$

Of the nine elements of the transfer matrix that were calculated in the frequency response analysis, only three are needed for stability analysis of the machining system during orthogonal turning. This is because the orientation of the structure's coordinate system was selected in such a way that the X and Y axes coincide with the cutting direction and the thrust force, respectively. Thus only the transfer elements g_{xx} , g_{yy} and g_{xy} are necessary for stability analysis. Figure 5 shows the frequency response of the receptance g_{ij} , which is the i -directional displacement due to excitation in the j direction, computed at the tip of

the tool using the normal transfer G_n^c function for the contact surface. The unit vectors n_x^c , n_y^c and n_z^c normal to the cutting surface are $\cos(5)$, $\cos(85)$ and $\cos(90)$, respectively. Using Eq.(11), Fig. 5 shows that the maximum response of G_n^c occurs at a frequency of 2 639 Hz. In the cutting test, chatter vibration occurred at a frequency of around 2 192 Hz due to the resonance at 2 639 Hz. The difference between the calculated and experimental values is about 20 percent which is reasonable for the high frequency range.

Actually, g_{ij} is a frequency response function of dynamic modes. Theoretically, there are an infinite number of vibration modes that contribute to the displacement between the tool and the workpiece in all directions. However, few of these modes affect the vibration magnitude. Nevertheless, the maximum response of the structure cannot be predicted using the individual components of g_{ij} . The normal transfer function, G_n^c , acting at the cutting point must be used to analyze chatter vibration in the machine tool.

The stability limit depends on the maximum real negative part of the relative displacement receptance of the tool and the workpiece in the direction normal to the cutting surface caused by the cutting force at the contact point. Therefore the vector locus and frequency response are used to evaluate the dynamic characteristics of the system. Figure 6 shows the calculated vector locus of the receptance in the direction normal to the cutting surface. This is used to estimate the stability of the system using the Nyquist criterion on the complex plane. These curves represent the receptance and phase difference at a polar coordinate calculated using the finite-element model shown in Fig. 3. As seen from Fig. 6, the maximum real negative receptance occurs at around a frequency

Fig. 6 Stability analysis of tool in the cutting direction on the complex plane

of 2 639 Hz. From the frequency response and stability analyses, it can be concluded that the CNC lathe is unstable at the resonant frequency of 2 639 Hz. These analytical results correspond well to the results obtained in the cutting tests. Our design modification objective is to maximize the stability of the machining system against chatter vibration in order to improve the quality of the surface of the finished workpiece.

6. Design Improvement and Experimental Results

The diameter, D , and thickness, b , of the turret in the original design are 420 mm and 85 mm, respectively. The first modification is to change the thickness of the turret in the range 75 mm to 115 mm, to give revised designs 1, 2 and 3. Figure 7 shows the calculated frequency responses of the receptance G_n^c at the cutting point for various turret thicknesses. The dynamic response of revised design 3 in the last

Fig. 7 Frequency responses for various turret thicknesses. b is the turret thickness

Fig. 8 Influence of turret thickness on dynamics stiffness at the last resonance frequency

resonant mode is 3.4 times better than that of revised design 1. Figure 8 shows the relationship between the thickness of the turret and the dynamic stiffness in the last resonant mode and shows that the optimum turret thickness is about 95 mm. The thickness is less significant and the dynamic characteristics of the system cannot be enhanced sufficiently when the turret thickness approaches the optimum value.

Based on Eqs. (5), (6) and (7), the component structure can be modified in order to redistribute the stiffness and mass in this system. In revised design 4, we kept the same turret thickness and enlarged the sleeve length from 85 mm to 100 mm which enhanced the rigidity of component and increased very little mass. The frequency responses of the system after modification, revised design 4, and the conventional system are compared in Fig. 9 which clearly shows that the modification of components in this system dramatically reduces chatter vibration. Figure 10

Fig. 9 Comparison of the frequency response of redesigned sleeve to that of conventional design. The turret thickness, b , is the same for both designs

Fig. 10 (a) Conventional design of sleeve and tool holder, and (b) redesigned sleeve and tool holder. Sleeve length varies from 85 mm to 100 mm, and tool holder consists of two impact dampers by the screws

shows the conventional design and revised design 4 of the sleeve and the tool holder. Finally, the receptances of revised and conventional designs are compared and the results are shown in Table 2. The frequency response analyses show that the modification of components is much more effective than increasing the turret thickness in improving the dynamic characteristics of the system against chatter vibration.

Various methods for improving the dynamic stability against the chatter vibration have been investigated. Revised design 4 was used to confirm the analytical predictions experimentally. The cutting conditions and cutting tool were identical to those used to make the workpiece shown in Fig. 1. The workpiece fabricated using the redesigned system is shown in Fig. 11. The quality of the workpiece surface is much better than that shown in Fig. 1.

Figure 12 shows the measured frequency responses during the cutting operation after modification of the set components. The amplitude of vibration is about 43.676 6 mg at a frequency of around 2 284 Hz. The amplitude of the spectrum is about 10 times smaller than that shown in Fig. 2 and the effect of the design modification predicted in

Table 2 Change of design parameters to reduce the receptance

	Revised design 1	Convention design	Revised design 2	Revised design 3	Revised design 4
Turret thickness, b	75 mm	85 mm	95 mm	115 mm	85 mm
Sleeve length, L	85 mm	85 mm	85 mm	85 mm	100 mm
Last resonance frequency	2405 Hz	2637 Hz	2773 Hz	3017 Hz	2848 Hz
receptance	2.0544 μ /kgf	0.8533 μ /kgf	0.653 μ /kgf	0.60 μ /kgf	0.089 μ /kgf
ratio to conventional design	2.4	1	0.765	0.7	0.1

Fig. 11 Finished surface of workpiece after modification of sleeve and tool holder

Fig. 12 Power spectrum after modification of sleeve and tool holder

Table 2 is verified by the experimental results. A comparison of the workpiece shown in Fig. 1 and Fig. 12, shows that the energy concentration at the chatter frequency 2 192 Hz is significantly reduced by the design modification under identical surface speed and feed rate conditions.

7. Conclusions

In this study, the dynamic characteristics of machine tool components were investigated by modal testing and finite-element analysis. The excitation mechanism causes particularly large displacements at the resonance frequencies. The natural frequencies, frequency responses and unstable frequency range of chatter vibration were calculated and measured. Techniques for improving the stability of the system against chatter vibration were discussed. A dramatic reduction in the surface roughness of the finished workpiece was observed as a result of reduced chatter due to the modification of components. We conclude that:

- The relationship between the dynamic stiffness of a specific mode and the turret thickness was clarified. Thus, the optimum thickness can be determined.
- The effect of component modification, calculated by finite-element analysis, on the dynamic performance of the system was found to be in excellent agreement with results obtained in practical cutting operations.
- The stability of the turret is greatly enhanced by slightly redistributing its stiffness and mass. Thus revising the design of some components in order to overcome chatter is a cheap and effective method of improving the cutting performance after the machine tool has been constructed.
- We performed structural dynamic analysis for a real case of chatter vibration in turning workpieces. The finite-element method is useful for evaluation and analysis of the performance of machine tools.

Acknowledgments

This work was carried out as part of the research

program of the CNC Laboratory of Victor-Taichung Machinery Works, Co., Ltd. The author appreciates the financial support of VTMW in conducting this research. The author also wishes to thank Dr. S.M. Yang, a Professor at National Cheng Kung University, for his suggestions and comments and T.C. Hsui for his invaluable assistance in calculations.

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