Vehicle Cornering Characteristics in Acceleration and Braking through Attitude Control of Front and Rear Tires*

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A new active control method for the attitude angles of wheels is proposed to improve the maneuverability and stability of automobiles. From theoretical analysis and computer simulation, it is clarified that this control yields satisfactory vehicle cornering performances during steering, acceleration and braking. The dynamics of lateral movement and yawing of the vehicle as a result of tire attitude control are investigated using a quasi steady-state analysis model for the case of tractive or braking force. An extended stability factor related to the characteristics of toe and camber angles of wheels is defined, which is applicable to circular turnings with lateral and longitudinal accelerations. Effects of control are analyzed using this factor. Furthermore, a modified control method is proposed, which makes it possible not only to improve vehicle dynamics but also to decrease actuating values of attitude angles in actual driving.

Key Words: Automobile, Vehicle Dynamics, Motion Control, Maneuverability, Stability, Frequency Response, Acceleration and Braking, Simulation, Attitude Control

1. Introduction

It is expected that an active control system for the attitude angles of front and rear wheels will enhance vehicle dynamics in wide maneuvering ranges.

We have already proposed a combined method for controlling not only steer angles of tires but also camber angles, and have shown that a system equipped with this control method exhibits improved steering response characteristics under running conditions without driving or braking force over a system equipped with four-wheel-steering depending on steer angle control only. Improvement of cornering performance with consideration of braking is important for automobile safety. Therefore it is necessary to analyze vehicle dynamics and cornering characteristics with consideration of tractive and braking forces.

This paper describes the results of theoretical analysis and computer simulation of the effects of attitude control of tires during circular turnings with lateral and longitudinal accelerations. Under the assumption of driving with relatively small longitudinal acceleration, simplified nonlinear equations of vehicular motion under tractive and braking forces are derived using a quasi-steady-state analysis model, and the effects of control are analyzed by using these equations. Consequently, it is clarified that the attitude control system of tires can provide better maneuverability and stability of automobiles with and without longitudinal acceleration than a four-wheel-steering control system, particularly in case of cornering with large lateral and longitudinal accelerations. A modified control method is also proposed, in which variables applied to attitude angles are controlled according to the lateral and longitudinal accelerations. It is shown that the modified control law enables not only the improvement of vehicle dynamics but also a decrease actuating values of attitude angles in actual driving.
2. Description of Vehicle Model

2.1 Nomenclature

The notations used in the vehicle model, as well as main vehicle specifications and tire characteristics used in computer simulation are listed below.

\( M, m_s \) : vehicle mass, sprung mass \{1500, 1300 kg\}

\( I \) : yaw moment of inertia \(240 \text{ kgm}^2\)

\( l, t \) : wheelbase, wheel tread \{2.62, 1.45, 1.45 m\}

\( a, b \) : distance from front/rear axle to center of gravity \{1.18, 1.44 m\}

\( h, h_t \) : vehicle height at center of gravity, height of roll center

\( h_0 \) : arm length of roll moment

\( K_s \) : total roll stiffness

\( K_v \) : roll stiffness \{38, 32 \text{ kN/rad}\}

\( N \) : steering overall gear ratio \{15.4\}

\( a_c \) : driving or braking torque split to the front wheels

\( K_{ct} \) : cornering power of both wheels \{51.6, 96.9 \text{ kN/rad}\}

\( K_{ct} \) : camber stiffness of both wheels \{6.7, 12.6 \text{ kN/rad}\}

\( \theta \) : steering wheel angle

\( \delta_0 \) : front wheel steer angle on maneuvering steering wheel \( \delta_0 = \theta/N \)

\( \delta_{ct} \) : active steer angle of front wheels

\( \delta_{ct} = \delta_0 - \delta_0 \)

\( \delta_1, \gamma \) : wheel steer angle (toe angle), wheel camber angle

\( r, \Delta \phi \) : yaw rate, yaw rate gain in steady state

\( \alpha, \alpha_0 \) : longitudinal acceleration, lateral acceleration at \( g \)

\( \beta, \beta_0 \) : body slip angle at \( g \), tire slip angle

\( W, w_s, w_l \) : normal reaction on wheel, load shift in right and left wheels, load shift in front and rear wheels

\( F_{st}, F_v \) : tire force in vehicle longitudinal direction, tire force in vehicle lateral direction

\( F_{st}, F_v \) : longitudinal tire force, lateral tire force

\( F_{c}, F_{o} \) : tire cornering force, tire camber thrust

\( R_i, r_i \) : control parameters of steer angles

\( k_i, d_i \) : control parameters of camber angles

\( K \) : extended stability factor

\( R, v \) : radius of curvature, vehicle velocity

\( g, \mu \) : acceleration due to gravity, road-tire friction coefficient \{9.8 \text{ m/s}^2, 1.0\}

\( s, t \) : Laplace transform operator, time

\( o, xy, o, XY \) : coordinates fixed to the vehicle body, coordinates fixed to the road

Subscripts

\( i \) : 1 front wheels, 2 rear wheels

\( j \) : 1 right front wheel, 2-right rear wheel, 3-left front wheel, 4-left rear wheel

\( 0 \) : initial state variable in steady circular turning

2.2 Equations of Motion

For the analysis of vehicle dynamics with tractive and braking forces, the following equations of motion are obtained by considering a two-dimensional plane model shown in Fig. 1\(^{11}\).

\[
M(\dot{x} - \dot{y}) = \sum F_{st}
\]

\[
M(\dot{y} + \dot{x}) = \sum F_v
\]

\[
I\ddot{\theta} = a(F_{stx} + F_{sty}) - b(F_{tvx} + F_{tvy})
\]

\[
+ (F_{tvx} - F_{stx})/2 + (F_{tvy} - F_{sty})/2
\]

Assuming that the attitude angles of right and left wheels are equal in order to analyze vehicle motions easily, the tire slip angles of each wheel are given as follows.

\[
\delta_0 = \tan^{-1}((\dot{y} + g \dot{r})/\dot{x})
\]

\[
\delta_2 = \tan^{-1}((\dot{y} - g \dot{r})/\dot{x})
\]

Then the vehicle velocity and the body slip angle are described by

\[
v = \sqrt{\dot{x}^2 + \dot{y}^2}
\]

\[
\tan \beta = \dot{y}/\dot{x}
\]

Considering the load shift caused by the longitudinal acceleration \( \alpha = \ddot{x} - \dot{y} \dot{r} \) and the lateral acceleration \( \alpha = \ddot{y} + \dot{x} \dot{r} \), the normal load on each wheel is expressed by

\[
W_i = hMg/(2l) + \nu_{w1} - \nu_{w2}
\]

\[
W_2 = aMg/(2l) + \nu_{w1} + \nu_{w2}
\]

\[
W_3 = bMg/(2l) - \nu_{w1} - \nu_{w2}
\]

\[
W_4 = aMg/(2l) - \nu_{w1} + \nu_{w2}
\]

and

\[
\nu_{w1} = d_{w1} \alpha
\]

\[
\nu_{w2} = bM\delta_t/(2l)
\]

where \( d_{w1}, d_{w2} \) are coefficients which are determined according to vehicle specifications and suspension properties.

As shown in Fig. 2, the relationship among the tire forces is defined by

\[
\begin{bmatrix}
F_{stx} \\
F_{sty}
\end{bmatrix} =
\begin{bmatrix}
\cos \delta_1 & -\sin \delta_1 \\
\sin \delta_1 & \cos \delta_1
\end{bmatrix}
\begin{bmatrix}
F_{tx} \\
F_{ty}
\end{bmatrix}
\]

Fig. 1 Theoretical analysis model
Assuming that the lateral tire force $F_{ty}$, which is expressed as a function of $\beta_t$, $\gamma$, $W_t$, and the longitudinal tire force $F_{tx}$, is calculated by simply adding the cornering force $F_{cy}$ to the camber thrust $F_{cy}$:

$$F_{cy} = F_{cy}^0$$

$$F_{cy} = F_{cy}^0 + F_{cy}(\beta_t, W_t, F_{tx})$$

(11)

Here, assuming that the relationship between the longitudinal tire forces and the lateral tire force is expressed in the form of an elliptic equation, the tire cornering characteristics are given by the following equations based on the Fiala equations$^{[9]}$:

$$F_{cy} = f(\psi)\sqrt{(\mu W_t)^2 - F_{tx}^2}$$

(12)

where

$$f(\psi) = \begin{cases} \psi^2 & (|\psi| > 3) \\ 3\psi^2 + 27 & (|\psi| \leq 3) \end{cases}$$

(13)

$$\psi = K_n \tan \beta_t / (\mu W_t)$$

In the same way, we consider that $F_{cy}$ is given by the equations substituting $K_{in}$, $\gamma$, for $K_n$, $\beta_t$ in Eqs. (12) and (13), respectively. The effects of tire slip ratio are ignored in order to simplify the theoretical analysis.

Thus, upon determining the tire attitude angles $\delta_t$ and $\gamma$, the motions of the vehicle applied to the tire attitude control are calculated from Eqs. (1) - (13) and the system performance is estimated.

These nonlinear equations are difficult to solve exactly. Therefore, we consider the vehicle dynamics in the quasi-steady state assuming that the elapsed time is extremely short in order to neglect the fluctuation of velocity of a vehicle due to longitudinal acceleration or deceleration. From the foregoing nonlinear equations, the quasi-steady state analysis aims to obtain the equations of motion with a form similar to those in the case of treating longitudinal acceleration $a_t$. This enables the quantitative analysis of the influences of various design parameters on the steering response and the vehicle stability during cornering with tractive and braking forces.

In this work, we pay attention to the vehicle response and stability when a driver executes nominal steering maneuver after holding the steering wheel and making a circular turn with constant longitudinal acceleration or deceleration (called initial state). When $a_t$ is taken to be zero in the quasi-steady state, it becomes unnecessary to consider Eq. (1). The following approximate relations can be obtained, because both $\theta$ and $R$ are constant in the initial state.

$$\begin{cases} \dot{y}_t = 0, \dot{x}_t = v_t \\ a_{\theta} = v_t / R_0 \end{cases}$$

(14)

Furthermore, the driving/braking torque distribution between front and rear wheels $a_t$ is important for vehicle behaviors in the case of longitudinal acceleration or deceleration, and we assume that $a_t$ is constant in order to simplify the analysis model and to facilitate understanding of vehicle motion due to tire attitude control:

$$F_{x1} + F_{x2} = Ma_{\text{total}}$$

$$F_{x2} + F_{x4} = Ma_{\text{total}}(1 - a_t)$$

(15)

By determining the torque distribution between right and left wheels and the tire attitude angles $\delta_t$ and $\gamma$, Eqs. (2) - (15) are transformed into nonlinear coupled equations for the front wheel steer angle on maneuvering the steering wheel $\delta_t$ and the body slip angle $\beta_t$. As a result of solving the above equations using an iteration method, two approximate solutions are obtained. In addition, upon obtaining both $\delta_t$ and $\beta_t$, all values of the state variables in the initial state are fixed by the equations as mentioned previously.

Next, we attempt to linearize Eqs. (2) - (15) to examine the vehicle steering response and stability when the driver performs a transient steering maneuver whose amount is small after the initial state$^{[10]}$. Assuming that each increment of the state variable is much less than the value of the state variable in the initial state, we consider increments of the terms of the first order and below after application of the Taylor expansion. The increments are indicated by the sign $\Delta$. When Laplace transformation is applied to the linearized equations of motion, and is expressed in the form of a matrix, the following equation is obtained.

$$\begin{bmatrix} M_{tx0} + \xi \dot{v}_t \\ M_{tx0} + \xi \ddot{v}_t \end{bmatrix} = \begin{bmatrix} A_{12} & A_{22} \\ aA_{12} & -bA_{22} \end{bmatrix} \begin{bmatrix} \Delta \delta_t(s) \\ \Delta \beta_t(s) \end{bmatrix}$$

(16)

where the coefficients of terms of $s$ and the constant terms depend on vehicle specifications, the characteristics of tire and tractive/braking force, and the values of variables in the initial state. The state variables after Laplace transformation are represented by the same notations as those before for
convenience.

Substituting the linearized increments of the tire attitude angles into Eq. (16), the transfer functions of \( \Delta r \), \( \Delta \beta \), and \( \Delta \theta \) for \( \Delta \phi \) are derived in the form of rational functions, and this enables easy analysis of the frequency response characteristics of the vehicle motion.

3. Analysis of Vehicle Dynamics

The maneuverability and stability of a vehicle through the tire attitude control (Active Control of Tires-1) are compared with those of four-wheel-steering (4 WS) and 2-wheel-steering vehicles by computer simulation. The control laws of both the tire attitude control and 4 WS use the proposed control functions that are determined so that the yaw rate for steering maneuver can be the objective first-order delay characteristic and the dynamic body slip angle which is equal to zero (yawing center) can keep the objective position at all times. The following equations represent a control law of ACT-1:

\[
\begin{align*}
\dot{\delta}_1 &= R_1 \delta_3 + r_1 r \\
\dot{\delta}_2 &= R_2 \delta_3 + r_2 r \\
\gamma_1 &= -k_1 \beta_1 \\
\gamma_2 &= -k_2 \beta_1 
\end{align*}
\] (17)

where \( R_1 \) and \( r_1 \) are control gain parameters calculated using vehicle specifications and objective vehicle characteristics. \( k_i \) is the adjustable control gain parameter. For ease of comparison, we set the objective yaw rate characteristics such that the value of the steady gain is the same as that of 2 WS, the time coefficient of first-order delay is 0.07 sec., and the yawing center coincides with the vehicle's center of gravity. We also set \( k_1 = k_2 = 0 \). The specifications of a typical passenger vehicle are used in simulations, and it is assumed that \( \alpha_0 = 0.6 \) when the braking force and each longitudinal force of right and left wheels are the same.

3.1 Steady circular turning with tractive and braking forces

Figures 3 and 4 show, respectively, the calculated results of \( \beta_1 \), \( \gamma_1 \) and \( \beta_0 \), \( \gamma_0 \) for the steady lateral accelerations \( \alpha_0 \) under cornering with \( R_0 = 80 \) m and \( \alpha_0 g = -0.2 \) G. In the case of 2-wheel-steering, the absolute value of \( \beta_0 \) is relatively large and the vehicle attitude also changes largely for \( \alpha_0 \). The \( \beta_0 \) values of ACT-1 and 4 WS vehicles are small when \( \alpha_0 \) is relatively small; however, \( \beta_0 \) of the ACT-1 vehicle becomes smaller than that of 4 WS with increasing \( \alpha_0 \). The result in Fig. 4 clarifies that the tire attitude control provides both small tire slip angles and large margin to the maximum value of cornering force owing to the effect of tire camber thrust.

\[ \Delta \phi = \left( \frac{\Delta r}{\Delta \theta} \right) = \frac{1}{N} \left( \frac{\alpha_0}{1 + K_1 \alpha_0^2} \right) \] (18)

where \( \Delta \phi \) is the steady yaw rate gain derived from Eq. (16).

Figures 5 and 6 show \( K_1 \) as a function of \( \alpha_0 \) with constant radius of curvature and constant vehicle velocity, respectively, where the steady longitudinal acceleration \( \alpha_0 \) is treated as parameter. The fluctuation in the cornering characteristics of the ACT-1 vehicle is smaller than those of 2 WS and 4 WS. These results suggest that ACT-1 works to increase the handling stability of the vehicle even if \( \alpha_0 \) and \( \alpha_0 \).

3.2 Transient response of vehicle for steering input

Figure 7 shows the calculated frequency response
two vehicle characteristics fall into small when $\alpha_{\theta}$ and $\alpha_{\phi}$ are relatively large, and the damping characteristic of the vehicle behavior becomes worse since the yaw rate gain ratio of the resonance peak gain to the steady gain becomes large. Furthermore, outside the foregoing running conditions, in the case of 2 WS, the gain becomes extremely large and the vehicle motion becomes unstable; however, ACT-1 works to suppress the instability. We have the calculated results for 4 WS, but they are omitted for want of space. We can rank the control methods of ACT-1, 2 WS, and 4 WS in order of good steering response and stability. Thus, it is clear that the tire attitude control improves the maneuverability and stability for steering input even during running with tractive and braking forces.

4. Control Law Considering Acceleration

4.1 Control Law of Tire Attitude Angles

The analytical results show that toe control greatly affects in actual driving region where $\alpha_{\theta}$ and $\alpha_{\phi}$ are relatively small, whereas the subsidiary control of camber angles in addition to the toe control improves the steering response stability in the region where $\alpha_{\theta}$ and $\alpha_{\phi}$ are so large that the nonlinearity of the tire cornering force characteristics is marked. Substituting $\alpha_{\theta}$ and $\alpha_{\phi}$ into the control functions of tire attitude angles, this new control method enables the enhancement of vehicle performance and decreases the actuating values of tire attitude angles.

We consider a modified tire attitude control method (ACT-2) in which control functions are comprised of the toe control functions added to the camber control functions, substituting a function of $\alpha_{\theta}$ and $\alpha_{\phi}$ for constant parameter $k_c$ of camber angle control in Eq.(17).

$$\gamma_i = \gamma(\alpha_{\theta}, \alpha_{\phi}, \beta)$$

(19.a)
where

\[ \gamma_i(\alpha_x, a_y, \beta_i) = -k_i \left( \frac{\alpha_x}{g} \right)^2 \left( 1 + d_i \frac{\alpha_x}{g} \right) \beta_i \]

\[ \gamma_j(\alpha_x, a_y, \beta_j) = -k_j \left( \frac{\alpha_x}{g} \right)^2 \left( 1 + d_j \frac{\alpha_x}{g} \right) \beta_j \]

(19.b)

Here, \( d_i \) is the gain parameter when the sensitivity of \( \alpha_x \) is adjusted.

Analyzing under the quasi-steady state in the same way as for ACT-1, \( K_i \) is obtained from Eq. (18). The increment of \( \gamma_i \) is represented as follows.

\[ \Delta \gamma_i = \left( \frac{\partial \gamma_i}{\partial \alpha_x} \right) \Delta \alpha_x + \left( \frac{\partial \gamma_i}{\partial \beta_i} \right) \Delta \beta_i \]

Substituting Eq.(20) into Eq. (16), the transfer functions of steering response are derived.

In the analytical calculation of ACT-2, we assume that the steady yaw rate gain is nearly equal to that of ACT-1 and the control gains are \( k_l = 0.5 \), \( k_s = 0.6 \), \( d_l = 1.0 \) and \( d_s = 4.0 \) to ensure the vehicle cornering stability under emergency braking.

Figure 8 shows the tire attitude angles of ACT-1.
and of ACT-2. The controlled variables of \( \gamma \) decrease sharply in actual driving. Both the control angles of \( \delta_c \) and \( \delta_s \) increase slightly as \( \gamma \) decreases. On the whole the controlled variables of the tire attitude angles of ACT-2 tend to be less than those of ACT-1.

**Fig. 8** Tire attitude angle characteristics

**Fig. 9** Comparison of stability factors

**Fig. 10** Lane changing simulation
4.2 Vehicle motion characteristics

In the case of ACT-2, the steady circular turning characteristics and the frequency response characteristics are shown in Figs. 9 and 7, respectively. Comparing ACT-2 and ACT-1 in Fig. 7, little difference is found between their motion characteristics.

Next, in order to investigate the control performance of the driver-vehicle system as a closed-loop control system, we carry out computational simulation using the Runge-Kutta method in which the driver is asked to execute lane changes during quick braking. This running situation emulates the maneuverability and the driver's work load during emergency obstacle avoidance. We apply the first-order prediction model using the feedback of lateral error from a desired course to the driver's model. The calculated result is shown in Fig. 10. The running trajectory of the vehicle is at the vehicle's center of gravity. ACT-2 provides the same vehicle motion characteristics as ACT-1, and when steering input is applied, the yaw rate and lateral acceleration generations are sharp and the driver can maneuver the steering wheel naturally and stably. The vehicle's movement is stable immediately after lane changes. Since generations of the body slip angle and the yaw rate are suppressed, the vehicle moves efficiently. Furthermore, it is clarified that the actuating values of the tire attitude angles of ACT-2 tend to be less than those of ACT-1 on the whole.

5. Conclusions

We investigated the maneuverability and the stability of a vehicle with tire attitude control in acceleration and braking, and the following results are obtained.

1. We have shown a simple theoretical analysis model of the vehicle incorporating the attitude angle control of front and rear tires by treating the vehicle behaviors during circular turning with acceleration and braking in the quasi-steady state.

2. The extended stability factor is defined with consideration of the tire camber characteristics, during circular turning with longitudinal accelerations.

3. Tire attitude control improves the steering response stability and provides natural and controllable steering performance during circular turning with acceleration and braking.

4. A modified control method, in which the tire attitude angles are controlled according to lateral and longitudinal accelerations, makes it possible not only to improve the maneuverability and the stability for various steering inputs but also to decrease actuating values of attitude angles in actual driving.

Appendix

\[ d_{11} = \frac{(h_1 b M l + K_0 m a h_1 K_0 l)}{l}; \]
\[ d_{12} = \frac{(h_1 b M l + K_0 m a h_1 K_0 l)}{l}; \]
\[ \xi = a F_{1x} \cos \delta_{10} - b F_{2y} \cos \delta_{20}; \]
\[ \eta = a F_{1x} \cos \delta_{20} + b F_{2x} \cos \delta_{20}; \]
\[ \zeta = F_{1x} \cos \delta_{10} + F_{1y} \cos \delta_{20}; \]
\[ M = M - A_{11} - A_{21}; \]
\[ d_{w} = - a A_{11} + b A_{21}; \]
\[ A_{11} = M b \sin \delta_{10} \sin \delta_{20} + F_{1x} \cos \delta_{10}/2; \]
\[ + F_{1x} d_{12} \cos \delta_{10} \]
\[ A_{12} = \frac{(M a b \cos \delta_{10}/l - M a b \cos \sin \delta_{10})}{l}; \]
\[ \times (\sin \delta_{10} + F_{1x} \cos \delta_{10}/2) \]
\[ + F_{1x} \cos \delta_{10} + M a b \cos \delta_{20}; \]
\[ A_{21} = M b \sin \delta_{20} \sin \delta_{10} + F_{2x} \cos \delta_{20}/2; \]
\[ + F_{2x} d_{12} \cos \delta_{20} \]
\[ A_{22} = \frac{(M a b \cos \delta_{20}/l - M a b (1 - a \cos \sin \delta_{20})}{l}; \]
\[ \times (\sin \delta_{20} + F_{2x} \cos \delta_{20}/2) + F_{2x} \cos \delta_{20} \]
\[ + M a b (1 - a \cos \sin \delta_{20}), \]
\[ F_{1x} = \frac{\partial F_{1x}}{\partial \delta_{10}} \frac{\partial F_{1x}}{\partial \delta_{10}} \]
\[ F_{2x} = \frac{\partial F_{2x}}{\partial \delta_{20}} \frac{\partial F_{2x}}{\partial \delta_{20}} \]
\[ F_{1w} = \frac{\partial F_{1w}}{\partial W_1} \frac{\partial F_{1w}}{\partial W_1} \]
\[ F_{2w} = \frac{\partial F_{2w}}{\partial W_2} \frac{\partial F_{2w}}{\partial W_2} \]
\[ F_{1a} = \frac{\partial F_{1a}}{\partial F_{1x}} \frac{\partial F_{1a}}{\partial F_{1x}} \]
\[ F_{2a} = \frac{\partial F_{2a}}{\partial F_{2x}} \frac{\partial F_{2a}}{\partial F_{2x}} \]

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