Formulation of Elevator Door Equation of Motion*

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Multiple elevator doors are driven with one servomotor through a multi-degree-of-freedom linkage system. This paper deals with the derivation of the equation of motion of the door mechanical system. The movement of the doors is described by giving various constraint conditions to multiple single-chain and multi-degree-of-freedom rigid body systems. The door motion was simulated based on this formulation. The conclusions are as follows.

[1] It is clarified by computer simulation that the change in the constraint condition caused by integration error is extremely small.
[2] The method of obtaining the motor driving torque, in which a given movement of the door or the motor is regarded as a constraint condition, is proposed.
[3] A mutual conversional relations among the joint torques, door driving force, and motor torque is shown.

Key Words: Mechanism, Simulation, Multi Degree-of-Freedom System, Elevator, Door, Equation of Motion

1. Introduction

Multiple elevator doors are driven with one servomotor through a multi-degree-of-freedom linkage mechanism. The control technique is shifting from DC servomotor with PWM control\(^{13}\) to AC servomotor with inverter control and the use of DSP\(^{23}\) is also reported. In a design of a controller and a mechanism, it is indispensable to solve the kinematical relation between the driving motor and the door panels. Conventionally, a method of using an approximate equation\(^{23}\), introducing a geometrical instantaneous center has been employed in the calculation of the kinematics. However, the kinematics, and the dynamics based on a kinematics are limited to an approximate analysis in these methods\(^{41}\).

Recently, an algorithm for numerical generation of equation of motion of a multi-degree-of-freedom single-chain rigid body system without constraint is actively studied in order to describe the equation of motion for a robot arm. In robotics field, various numerical generation methods are proposed\(^{90-103}\). We are also developing\(^{104}\) a numerical generation software of manipulator dynamics employing a formulation based on the Gibbs-Appell's equation\(^{99,100}\).

This paper proposes a formulation of motion of an elevator door derived from the equation of motion for a multi-degree-of-freedom single-chain rigid-body system without constraint. That is the method to describe the equation of motion for an elevator door by giving various constraint conditions to a multiple single-chain linkage system. The advantage of this method is that both the kinematics and the dynamics are solved simultaneously and accurately. As a result of the investigation described in the text below, it is clarified that the proposed method is successfully applicable to dynamics simulation of a door system.

2. Door Mechanism and Condition of Analysis

Figure 1 shows the opening/closing mechanism of an elevator door which is a subject of investigation in this paper. This is the most complicated mechanism
in the several kinds of door driving systems.

The analyses required for a design of a door system are roughly classified into the following four categories.

[1] Kinematics
  Analysis to find door motions (position, velocity, and acceleration) from angular motion of motor (rotational angle, angular velocity, and angular acceleration).

[2] Inverse kinematics
  Analysis to transform door motion which is given in Cartesian coordinate system into joint coordinate system of motor's rotational angle.

[3] Inverse dynamics
  - Analysis to find motor torque required to move driving motor in accordance with a given trajectory.
  - Analysis to find motor torque required to move door panel in accordance with a given trajectory.

[4] Forward dynamics
  Analysis to find motion of door mechanism when a driving torque of motor is given. This analysis is used for comprehensive simulation of the whole door system including control, driving subsystem and so on, when a driving torque is determined by the performances of the controller and the driving motor.

The above classification is made on our analogy of the nomenclature used in the field of dynamics of a multi-degree-of-freedom rigid body system such as robotics.

3. Method of Analysis

3.1 Constraint conditions

The door mechanism is modeled as a multi-degree-of-freedom linkage system as shown in Fig. 2. In Fig. 2, \( \Sigma_2 \) coordinate system is a basic Cartesian coordinates system, and pulley-2 in the final step in Fig. 1 is modeled as link-1 in Fig. 2. Therefore, the origin of the basic coordinate system is located on axis of rotation of pulley-2 and two variables \( \theta_1 \) and \( \theta_2 \) are equivalent to rotational angle of pulley-2.

As shown in Fig. 2, it is understood that door mechanism can be expressed as a multi-degree-of-freedom linkage system composed of rotational joints with a branch by introducing the following constraint conditions:

[1] Points A and B on the links are immovable, that is, these points are constrained in 2-degree-of-freedom of \( x_b, y_b \) direction.

[2] Points C and D on the tip of linkage system, which are the upper parts of door panels, are constrained in 2-degree-of-freedom of \( \varphi_b \) direction and around \( z_b \) axis.

We try to express this door mechanism as a synthesis of a multi-degree-of-freedom linkage mechanism of single-chain. We consider left/right door driving mechanism with a branch to be two independent single chain dynamical systems. These two single-chain dynamical systems are shown in Figs. 3 and 4. Figure 3 shows the driving mechanism of left door and Fig. 4 shows that of right door. In Figs. 3 and 4, left/right doors are identified by subscript 1 and 2 respectively, and the link number of individual single-chain is shown by superscript.

In Fig. 3, further constraint condition that \( \theta_1^a \) and
\( \theta_i \) are constant is added to 4-degree-of-freedom constraint condition in \( \Sigma_0 \) coordinate system as described above. The joints \( \theta_i^a \) and \( \theta_i^b \) are virtual joints assumed on the linkage system. The reason why these virtual joints are assumed is as follows:

It is required to introduce Jacobian matrix at the received point to describe the dynamics when a constraint or an external force is exerted on a multi-degree-of-freedom linkage system. However, Jacobian matrix is usually calculated only at center of gravity and tip of link. This is the reason why we assume the above virtual joints to obtain Jacobian matrix at the received point. We give the constraint condition that there is no displacement at the assumed joint. Above constraint conditions are also applicable to the right door in Fig. 4.

Because degree of freedom of each single chain is 7 and 10 and that of constraint is 6 and 9 respectively, both left and right door mechanisms become one degree-of-freedom dynamical system. Moreover, since one degree-of-freedom constraint is given to two dynamical systems as described below, degree-of-freedom of whole system also becomes one.

In order to express door mechanism, the constraint conditions given to individual single chain are described above. Because link-1 of two single chains is originally identical to pulley-2, the constraint condition to connect two single-chains is expressed as the following equation:

\[
\theta_i = \theta_i^1.
\]

### 3.2 Forward dynamics

We begin with derivation of the equation of motion for driving mechanism of left door based on equation of motion for a multi-degree-of-freedom single-chain linkage system without constraint. It is described by forward dynamics formulation as shown below:

\[
(\dot{\theta}_i) = ([W_i]^{-1}) (\tau_i + Y_i - V_i^T) ,
\]

where

- \( \theta_i \in R^7 \): Joint rotational angle
- \( \tau_i \in R^7 \): Joint driving torque
- \( W_i \in R^{7x7} \): Inertia matrix
- \( Y_i \in R^7 \): Gravity force term
- \( V_i \in R^7 \): Centrifugal and Coriolis force term

and superscript

- 'T': Derivative with respect to time
- 'T': Transpose

The given constraint conditions are constant displacement of all constrained points, and described by vector formulation as follows:

\[
(x_A, y_A, \theta_i, \theta_i^1, y_c, \phi_c)^T \; \text{constant}.
\]

Both Cartesian and joint coordinate systems are used in the above constraint condition. The coordinate system is unified in order to make simultaneous equations with Eq. (2). Transformation to the joint coordinate system of the first differential of the constraint condition is given as follows:
\[
(\ddot{x}, \ddot{y}, \ldots, \ddot{\phi}_c)^T = [E_i]([\dot{\theta}_i]), \quad (4)
\]

Then, the second differential becomes;
\[
(\dddot{x}, \dddot{y}, \ldots, \dddot{\phi}_c)^T = [E_i][\ddot{\theta}_i] + [E_i][\dot{\theta}_i], \quad (5)
\]

where
\[
E_i = \begin{bmatrix}
\frac{\partial x_i}{\partial \theta_1} & \frac{\partial x_i}{\partial \theta_2} & \cdots & \frac{\partial x_i}{\partial \theta_n} \\
\frac{\partial y_i}{\partial \theta_1} & \frac{\partial y_i}{\partial \theta_2} & \cdots & \frac{\partial y_i}{\partial \theta_n} \\
\frac{\partial \phi_c}{\partial \theta_1} & \frac{\partial \phi_c}{\partial \theta_2} & \cdots & \frac{\partial \phi_c}{\partial \theta_n}
\end{bmatrix},
\]

and \(\frac{\partial x_i}{\partial \theta_1}, \frac{\partial x_i}{\partial \theta_2}, \ldots, \frac{\partial \phi_c}{\partial \theta_n}\) are elements of Jacobian matrix.

Lagrange multiplier of the following equation is introduced as constraint force/torque exerted on constraint point to satisfy constraint condition:
\[
\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_r)^T. \quad (7)
\]

The following equation is obtained, when the above constraint force is added to the equation of motion described as Eq. (2):
\[
(\dddot{\theta}_i) = [W_i]^{-1}\left[\tau + Y_i - V_i + [E_i]^T(\lambda_i)\right]. \quad (8)
\]

The equation of motion for left door driving mechanism is formulated as constrained dynamical linkage system by rearranging Eq. (5) of constraint conditions and Eq. (8) of the equation of motion in consideration of constraint forces:
\[
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 \\
\end{bmatrix} = \begin{bmatrix}
W_1 & -E_1 & 0 \\
-E_1 & 1 & 0 \\
1 & 0 & 1
\end{bmatrix}^{-1}\begin{bmatrix}
\tau \\
Y_1 \\
-\dot{\theta}_1
\end{bmatrix}. \quad (9)
\]

Acceleration of each joint and constraint forces are obtained simultaneously by solving the above equation.

The equation of motion for right door is obtained in the same way as the left door by changing subscript 1 to 2. The equations of motions for two separate single-chains are connected by the constraint condition described as Eq. (1). By differentiating Eq. (1) twice and introducing a Lagrange multiplier \(\lambda\), the following equation of motion for the whole door mechanical system is obtained:

\[
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3
\end{bmatrix} = \begin{bmatrix}
W_1 & -E_1 & 0 \\
-E_1 & 1 & 0 \\
1 & 0 & 1
\end{bmatrix}^{-1}\begin{bmatrix}
\tau \\
Y_1 \\
-\dot{\theta}_1
\end{bmatrix}. \quad (9)
\]

Hereafter, Eq. (10) is simply rewritten as Eq. (11) to avoid the complexity of description:
\[
(\dot{\theta}) = [\dot{W}]^{-1}([\tau]). \quad (11)
\]

### 3.3 Inverse dynamics

In a design of an elevator door, it is required to find not only geometric relation of movement, but also the driving torque necessary to move the motor along a desired trajectory. The following two analytical methods classified into inverse dynamics are proposed, because kinematics relations are inevitably obtained from this dynamics analysis.

1. [1] Analysis to find movement of door and motor driving torque when a motor trajectory is given.

2. [2] Analysis to find both movement of motor and motor driving torque simultaneously when a door trajectory is given.

#### 3.3.1 When a motor trajectory is given

Motor driving torque \(\tau\) is obtained from inverse of Eq. (11) by giving movement of each joint \(\theta\) and constraint force \(\tau\). However, independent movements of all joints are not allowed, because there are restricted conditions that movement of each joint must satisfy constraint condition and that torques of passive joints must be zero. Therefore, this paper proposes the following analytical method. A given motor trajectory is regarded as constraint condition, which is introduced into Eq. (11).

The motor trajectory, that is, rotational angle of pulley \(2\), is assumed to be given as a function of time:
\[
\theta(t) = \theta_0(t). \quad (12)
\]

Lagrange multiplier \(\lambda\) is introduced as constraint force/torque to satisfy the given motor trajectory. The introduced variable \(\lambda\) is equivalent to motor driving torque, because given constraint condition is rotational angle of motor. The equation of motion can be expressed as a dynamical system in which the further constraint condition of 1-degree-of-freedom
is added to the equation of motion for the whole door mechanical system:
\[
\begin{pmatrix}
\dot{\vartheta} \\
\dot{\lambda}_i
\end{pmatrix} =
\begin{pmatrix}
\dot{W} & -1 \\
0 & \ddot{\xi}_i(t)
\end{pmatrix}
\begin{pmatrix}
\vartheta \\
\lambda_i(t)
\end{pmatrix}.
\] (13)

The motor driving torque is obtained as \(\lambda_c\) of Eq.(13).

3.3.2 When a door trajectory is given

A given door trajectory is regarded as a constraint condition in the same way as previous section. The movement of left door panel \(x_c\) is assumed to be given as a function of time:
\[
x_c = x_c(t).
\] (14)

Lagrange multiplier \(\lambda_c\) is introduced as constraint force/torque to satisfy the given door trajectory. Lagrange multiplier \(\lambda_c\) is equivalent to the driving force exerted on the upper part of door panel. The equation of motion under this condition is as follows:
\[
\begin{pmatrix}
\dot{\vartheta} \\
\dot{\lambda}_i
\end{pmatrix} =
\begin{pmatrix}
\dot{W} & -E_x^T \dot{1} \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
\vartheta \\
\lambda_i(t)
\end{pmatrix},
\] (15)

where \(E_x\) is given by the following equation:
\[
E_x = \left( \frac{\partial x_c}{\partial \theta_1}, \frac{\partial x_c}{\partial \theta_2}, \ldots, \frac{\partial x_c}{\partial \theta_i} \right).
\] (16)

3.3.3 Transformation of door driving force into motor driving torque

Driving force to move the door in accordance with the fixed trajectory is obtained as \(\lambda_c\) of Eq.(15). Then, the transformation of door driving force \(F\) into motor torque \(\tau_m\) is described below.

The following relation applies by the principle of virtual work between door driving force and motor torque:
\[
\tau_m = \frac{dx_c}{d\theta_i} f.
\] (17)

Because the position of door \(x_c\) is a function of joint rotational angles:
\[
\theta = (\theta_1, \theta_2, \ldots, \theta_i)^T,
\] (18)

the following equation is obtained by developing Eq. (17):
\[
\tau_m = \left( \frac{\partial x_c}{\partial \theta_1}, \frac{\partial x_c}{\partial \theta_2}, \ldots, \frac{\partial x_c}{\partial \theta_i} \right) f.
\] (19)

In the dynamical linkage system with constraint, the following matrix \([B]\) is proposed111 as transformation matrix to extract the variables as many as the number of independent degree-of-freedom:
\[
B = \frac{1}{E_x E_x^T}.
\] (20)

where matrices \(E_x, E_x\) are a minor matrix obtained by the following separation of constraint condition matrix \(E_x\):
\[
E = \begin{pmatrix} E_1 & \ldots & E_r \end{pmatrix} \begin{pmatrix} E_1 & \ldots & E_r \end{pmatrix}^T, \quad r = 6
\] (21)

Because joint displacement \([\delta]\) is expressed as follows by using matrix \([B]\):
\[
[\delta] = [B][\vartheta],
\] (22)

it is understood that matrix \([B]\) becomes equal to the following equation:
\[
[B] = \begin{pmatrix} 1, \frac{\partial x_c}{\partial \theta_1}, \frac{\partial x_c}{\partial \theta_2}, \ldots, \frac{\partial x_c}{\partial \theta_i} \end{pmatrix}^T.
\] (23)

On the other hand, door driving force vector \((F) = (f, 0, \ldots, 0)^T\), (24)

is transformed into joint driving torque \([\tau]\) by multiplying transpose of Jacobian matrix:
\[
\tau_n = [J]^T(F)
\] (25)

It is understood that the following equation applies by comparing Eqs.(19),(23) and (25):
\[
\tau_n = [B][J][J]^T(F).
\] (26)

Since matrix \([B]\) has a role of transformation to extract the variables as many as the number of independent degree-of-freedom in dynamical system with constraint, motor driving torque cannot be obtained by only transforming external force \((F)\) into each joint torque using Jacobian matrix. However, in order to obtain motor torque, these joint torques must be transformed again into 1-degree-of-freedom system composed of motor’s rotational angle.

4. Example of Analysis

Tables 1, 2 show dimensions of the mechanical system used in the computer simulation according to coordinate systems of analytical models shown in Figs. 3, 4. In these Tables, rotational angle is expressed as a relative value, that is the angle between coordinate systems of adjacent links. The rotational angle of link-1 changes from 160° to 20° in the process of opening.

Taking an example from the simulation results, the motor trajectory and driving torque are obtained when the door trajectory is given. The given door trajectory is shown as a function of time \(t\) by the following equation:
\[
x_c = \frac{6}{(f_t)^2} (P_t - P_t) \left( -\frac{1}{3} t^3 + \frac{t_t}{2} t^2 \right) + P_5, \quad 0 \leq t \leq f_t.
\] (27)

where \(P_5 = -0.006518\) m, \(P_t = -0.62690\) m, and \(t_t = 2.5\) sec. are the initial and the terminal positions of left door, and terminal time, respectively. Figure 5 shows the change of link configuration in this process from closing to opening.

It is understood that there occurs little change in constraint condition during the dynamics simulation.
Concerning the displacement of each constraint point estimated from simulation result, the amount of the change of the constraint points increases linearly with computed time, although it remains to be 0.025 mm at maximum. Moreover, Fig. 6 shows the relation between door trajectory and motor trajectory, and Fig. 7 shows the door driving force and motor driving torque during operation. In Fig. 7, door driving force is plotted by its value of 1/20.

5. Conclusion

Equation of motion for an elevator door was formulated by introducing constraint conditions into several multi degree-of-freedom single-chain rigid-body systems without constraint, and the door motion was simulated based on this formulation.

As a result, the following conclusions are obtained.

1. This paper adopts the analytical method by which the constraint conditions are differentiated twice. In this method, although the change in the constraint condition caused by integration error is a matter of concern, it is clarified by computer simulation that the change in the constraint condition is extremely small. It is also clear that global constraint
condition can be expressed by modification of local constraint condition for each computation.

[2] This paper proposes the method of obtaining the motor driving torque from constraint force or torque which satisfies the given motion of door or motor regarded as constraint condition.

[3] Even under many constraint conditions like an elevator door, external force can be transformed into each joint torque by multiplying transpose of Jacobian matrix. However, in order to obtain motor torque, these joint torques must be transformed again into 1-degree-of-freedom system composed of motor rotational angle.

References


