Contact Points Identification in Enveloping Grasp*

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An enveloping grasp is stronger than other grasps. This grasp, however, has a disadvantage that it is difficult to find contact points and contact forces. In this paper we discuss the identification of unknown parameters, which are contact points and contact forces, in the enveloping grasp.

Key Words: Robotics, Mechanics, Whole Arm Manipulation, Enveloping Manipulation, Contact Point, Contact Force, Identification

1. Introduction

In recent years, in multi-lot and mixed production systems, parallel-jaw grippers are widely used but they can not grasp and manipulate objects with flexibility. So multifingered hands are studied and developed, because the hands can dexterously grasp and manipulate the objects with complex shape.

Grasping an object by a multifingered hand can be classified into the following two categories as shown in Fig. 1.

- (a) Fingertip grasp
- (b) Inner link grasp

In the grasp (a), only fingertips are in contact with the object. The type of grasp can manipulate the object dexterously, but can not grasp it unless its accurate shape is known. Moreover the grasp is not robust against external forces. In contrast with this,

in the type of grasp (b), inner link mechanisms are in contact with the object. Hence the grasping is possible even if its shape is poorly known. Moreover, the grasp shown in Fig. 1(c) can firmly hold the object and is stable against external forces because the manipulator envelops the object. However, this type of grasp has a disadvantage that it is difficult to identify contact points and contact forces.

In this paper, we propose an identification method of contact points and forces in the enveloping grasp as shown in Fig. 1(c). In general, the identification of contact points between a robot hand and a grasped object are divided into two types. One is the direct

Fig. 1 Grasps by fingertips and inner links

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method using vision or tactile sensors, and another is the indirect method in which contact points and forces are calculated using joint torque and/or angle sensors mounted on robots intrinsically. Since the latter does not require additional devices, it is of great value in practical use. So the paper focuses on the latter approach for identification of the contact points.

In the line of the indirect method, Mimura et al.^{(1),(2)} proposed an identification method for contact points in the type of grasp Fig. 1(b). They have also elucidated the identifiability condition.

Kaneko et al.⁽³⁾⁻⁽⁶⁾ discussed an indirect method for identifying contact points between a manipulator and an object in the enveloping grasp. With the concept of Self-Posture Changeability the contact points are calculated. However, the points are identified approximately in the sense that the calculation is made by deforming grasp configuration. Consequently, the estimated contact points have intrinsic approximation errors.

In contrast to this, the accurate contact points on the object will be determined in this paper. For this purpose, planar enveloping grasps are formulated in section 2. In section 3, we propose a new method for identifying contact points and forces using only the joint torque and angle sensor information and also show the necessary and sufficient condition for identifiability.

2. Formulation of enveloping grasp

In this section, we derive two basic equations for enveloping grasp by a planar n -link manipulator. One is the relation between joint torques and contact forces and the other is the relation between contact forces and external force/moment.

Now, we define the following symbols as shown in Fig. 2.

- Σ_b : base coordinate frame fixed at manipulator base
- Σ_o : object coordinate frame
- θ_i : i -th joint angle
- τ_i : i -th joint torque

- l_i : i -th link length
- l'_i : distance between contact point and joint on i -th link
- f_i : force applied by i -th link at i -th contact point
- r_i : position vector of i -th link
- r_o : position vector of origin of Σ_o with respect to Σ_b
- F_e : external force applied to the object relative to Σ_o
- m_e : external moment applied to the object relative to Σ_o
- n : total number of links or joints
- m : total number of contact points
- $\theta = (\theta_1, \theta_2, \dots, \theta_n)^T$: joint angle vector
- $\tau = (\tau_1, \tau_2, \dots, \tau_n)^T$: joint torque vector
- $w = (F_e^T, m_e)^T$: external force/moment vector

2.1 Relations between joint torques and contact forces

First, we consider the equilibrium state where a manipulator has been brought into contact with an object as shown in Fig. 3(a). So, the joint torques are balanced by the contact forces. A multi-contact grasp as shown in Fig. 3(a) can be divided into single-contact grasps as shown in Fig. 3(b). That is to say, the equations between joint torques and contact forces in the multi contact grasp can be expressed by the sum of the equations in the single-contact grasps.

First we consider the case that the i -th link of the manipulator is in contact with the object as shown in

Fig. 2 Symbols

Fig. 3 Decomposition of an enveloping grasp

Fig. 3(b). From the principle of virtual work, the equilibrium equation⁽⁷⁾ is given by

$$\boldsymbol{\tau} = \mathbf{J}_i^T \mathbf{f}_i, \quad (1)$$

where $\mathbf{J}_i$ is Jacobian matrix of the manipulator. Let us define position vector of i th contact point with respect to manipulator base Σ_b as, $\mathbf{r}_i = (x_i(\theta), y_i(\theta))^T$, then $\mathbf{J}_i$ is given by.

$$\mathbf{J}_i = \frac{\partial \mathbf{r}_i}{\partial \boldsymbol{\theta}^T} = \begin{bmatrix} \frac{\partial x_i}{\partial \theta_1} & \frac{\partial x_i}{\partial \theta_2} & \cdots & \frac{\partial x_i}{\partial \theta_n} \\ \frac{\partial y_i}{\partial \theta_1} & \frac{\partial y_i}{\partial \theta_2} & \cdots & \frac{\partial y_i}{\partial \theta_n} \end{bmatrix} \in \mathbf{R}^{2 \times n} \quad (2)$$

In the case of serial link, the position vector is given by

$$\mathbf{r}_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{pmatrix} l_1 c_1 + l_2 c_2 + \cdots + l_{i-1} c_{i-1} + l'_i c_i \\ l_1 s_1 + l_2 s_2 + \cdots + l_{i-1} s_{i-1} + l'_i s_i \end{pmatrix} \quad (3)$$

where

$$s_i = \sin\left(\sum_{k=1}^i \theta_k\right), \quad c_i = \cos\left(\sum_{k=1}^i \theta_k\right).$$

Hence, the transpose of matrix $\mathbf{J}_i$ in Eq.(1) can be expressed by

$$\mathbf{J}_i^T = \begin{bmatrix} -l_1 s_1 - l_2 s_2 - \cdots - l'_i s_i & l_1 c_1 + l_2 c_2 + \cdots + l'_i c_i \\ -l_2 s_2 - \cdots - l'_i s_i & l_2 c_2 + \cdots + l'_i c_i \\ \vdots & \vdots \\ -l_{i-1} s_{i-1} - l'_i s_i & l_{i-1} c_{i-1} + l'_i c_i \\ -l'_i s_i & l'_i c_i \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \quad (4)$$

Note that $\mathbf{J}_i^T$ is in a special form. By subtracting the $(j+1)$ th row from the j th row, Eq.(1) can be simplified as follows.

$$\tilde{\boldsymbol{\tau}} = \tilde{\mathbf{J}}_i^T \mathbf{f}_i, \quad (5)$$

where

$$\tilde{\boldsymbol{\tau}} = \begin{bmatrix} \tau_1 - \tau_2 \\ \vdots \\ \tau_{i-1} - \tau_i \\ \tau_i - \tau_{i-1} \\ \vdots \\ \tau_{n-1} - \tau_n \\ \tau_n \end{bmatrix} \text{ and } \tilde{\mathbf{J}}_i^T = \begin{bmatrix} -l_1 s_1 & l_1 c_1 \\ \vdots & \vdots \\ -l_{i-1} s_{i-1} & l_{i-1} c_{i-1} \\ -l'_i s_i & l'_i c_i \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}$$

Equation (5) denotes the relation between joint torques and contact forces when the manipulator is in contact with the object at one point. Similar equations are formulated at each contact point, and the sum of equilibrium equations in the planar enveloping grasp is given as

$$\tilde{\boldsymbol{\tau}} = \sum \tilde{\mathbf{J}}_i^T \mathbf{f}_i. \quad (6)$$

Equation (6) is the basic equation between joint torques and contact forces in the enveloping grasp.

2.2 Relation between contact forces and external force/moment

We consider the same equilibrium state, which is

already assumed in section 2.1. Then the contact forces $\mathbf{f}_i$ balance external force/moment $\mathbf{w}$ applied to the grasped object, so the following relation holds.

$$\mathbf{w} + \sum G_i \mathbf{f}_i = \mathbf{0} \quad (7)$$

where

$$G_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -(y_i - y_o) & x_i - x_o \end{bmatrix} \in \mathbf{R}^{3 \times 2} \text{ and}$$

$$\mathbf{r}_o = (x_o, y_o)^T$$

and $\mathbf{r}_o$ denotes an arbitrarily fixed point in the object.

3. Identifiability

3.1 Assumptions

In this section, we consider identification of parameters, such as contact points, contact forces and external force/moment, using the basic equations (6) and (7).

We make the following assumptions.

(1) Enveloping grasp is made in the planar plane using an n -link manipulator.

(2) The whole grasping system is in static state, i.e., although unknown external force is applied to the grasped object, contact forces balance the external force.

(3) Each link has at most one point contact with friction.

(4) Angle and torque at each joint can be measured by sensors.

(5) Each link in contact with an object is known beforehand.

Assumptions (1) and (2) are the conditions for supporting Eqs.(6) and (7). The reason why we make assumption (3) is that when some link has more than one contact points, the contact points and forces can not be identified by the joint torque information only. Assumption (3) is easily satisfied by assuming that the shape of the object to be grasped is convex. The assumption (5) is reasonable because the link in contact with the object can be measured from movement of links and change of joint torque when the manipulator is brought into contact with the object.

3.2 Possibility of identification

To clarify the discussion, we classify parameters into the given and the unknown.

given parameters: l_i, τ_i, θ_i

unknown parameters: $l'_i, \mathbf{f}_i, \mathbf{w}$

The unknown parameter, $\mathbf{w}$ which denotes external force/moment, appears only in Eq.(7). In order to identify $\mathbf{w}$ from Eq.(7), the other unknown parameters $l'_i, \mathbf{f}_i$ in Eq.(7) must be determined from Eq.(6). Namely, $\mathbf{w}$ can be identified only after identifying the l'_i and $\mathbf{f}_i$. Therefore the identification proceeds as

follows. Firstly, contact point l'_i and contact force $\mathbf{f}_i$ are identified by Eq.(6), which is the equation between joint torques and contact forces. Then the external force/moment $\mathbf{w}$ is determined by substituting l'_i and $\mathbf{f}_i$ into Eq.(7). In following subsections, we consider the possibility of identification of l'_i and $\mathbf{f}_i$.

3.3 Identifiability of contact points and forces

The unknown parameters l'_i and $\mathbf{f}_i$ are to be identified. There are 3 unknown parameters, that is, l'_i and $\mathbf{f}_i = (f_{ix}, f_{iy})^T$ at each contact point. When there are m contact points, the total number of unknown parameters is $3m$ in the enveloping grasp. On the other hand, the number of equations in Eq.(6) equals n , which denotes the number of links or joints. The number of equations must be equal to or more than the number of unknown parameters, the following necessary condition is obtained.

$$m \leq \frac{n}{3} \quad (8)$$

Hence, in the case where the number of contact points is more than one third of the number of links, the contact points can not be identified. However, the parameters can not always be identified when Eq.(8) is satisfied. In the following subsections, we discuss the identifiability condition. The one-point contact case is studied first, and then the two-point contact case, and finally the m -point contact case is investigated.

3.3.1 One-point contact case This case is shown in Fig. 1(b). When one contact point exists on the i -th link, ($1 \leq i \leq n$), Eq.(6) becomes

$$\hat{\boldsymbol{\tau}} = \hat{\mathbf{J}}_i^T \mathbf{f}_i \quad (9)$$

Since the $(i+1)$ -th row through the n -th row in Eq.(9) are trivial, these rows are omitted. The 1st through the $(i-1)$ -th rows are linear, and only the i -th equation is nonlinear with respect to the unknown parameters l'_i and $\mathbf{f}_i$. Dividing Eq.(9) into linear and nonlinear equations, we have

$$\boldsymbol{\tau}_A = A \mathbf{f}_i \quad (10)$$

$$\tau_i = [-l'_i s_i \quad l'_i c_i] \mathbf{f}_i \quad (11)$$

where

$$\boldsymbol{\tau}_A = \begin{bmatrix} \tau_1 - \tau_2 \\ \vdots \\ \tau_i - \tau_{i-1} \end{bmatrix}, A = \begin{bmatrix} -l_1 s_1 & l_1 c_1 \\ \vdots & \vdots \\ -l_{i-1} s_{i-1} & l_{i-1} c_{i-1} \end{bmatrix} \in \mathbf{R}^{(i-1) \times 2}.$$

If the rank of matrix A is equal to 2, contact force $\mathbf{f}_i$ can be determined from the linear equation (10). Then l'_i can be determined by substituting $\mathbf{f}_i$ into nonlinear equation (11). In contrast with this, if rank of A is less than 2, the number of independent equations is less than 2, that is, the number of linear and nonlinear equation both are less than 1. Because the number of unknown parameters is more than the number of independent equations, we can not determine the value of unknown parameters.

Therefore, the necessary and sufficient condition for identification of the unknown parameters l'_i and $\mathbf{f}_i$ from Eq.(9) is $\text{rank}(A)=2$.

We shall discuss what is the physical meaning of $\text{rank}(A)=2$. $\text{rank}(A)=2$ holds only if the matrix A must have at least 2 rows. Since the number of rows of matrix A corresponds to the number of uncontacted links, at least two uncontacted links are required between the manipulator base and the contact point. However, this condition is insufficient for the identification. Because, for example, when the k -th joint is in singular configuration, i.e., $\theta_k=0^\circ$ or $\theta_k=180^\circ$, matrix A becomes as follows.

$$A = \begin{bmatrix} -l_1 s_1 & l_1 c_1 \\ \vdots & \vdots \\ -l_{k-1} s_{k-1} & l_{k-1} c_{k-1} \\ -l_k s_k & l_k c_k \\ \vdots & \vdots \\ -l_{i-1} s_{i-1} & l_{i-1} c_{i-1} \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 s_1 & l_1 c_1 \\ \vdots & \vdots \\ -l_{k-1} s_{k-1} & l_{k-1} c_{k-1} \\ -l_k s_{k-1} & l_k c_{k-1} \\ \vdots & \vdots \\ -l_{i-1} s_{i-1} & l_{i-1} c_{i-1} \end{bmatrix} (\theta=0^\circ)$$

$$= \begin{bmatrix} -l_1 s_1 & l_1 c_1 \\ \vdots & \vdots \\ -l_{k-1} s_{k-1} & l_{k-1} c_{k-1} \\ l_k s_{k-1} & -l_k c_{k-1} \\ \vdots & \vdots \\ -l_{i-1} s_{i-1} & l_{i-1} c_{i-1} \end{bmatrix} (\theta=180^\circ)$$

In these cases, the k -th row depends linearly on the $(k-1)$ -th row. Hence, when the joint angle is $\theta_k=0^\circ$ or $\theta_k=180^\circ$, the number of independent equations is decreased.

Therefore, the condition of $\text{rank}(A)=2$ can be rewritten as follows.

When the number of contact point is one, there must be required at least two uncontacted links which are not in singular configuration between the base of the manipulator and the contact link as shown in Fig. 4. In terminology of joints, identifiability requires at least 3 nonsingular joints between the base of the manipulator and the contact point.

After the contact points and forces are determined, the unknown external force/moment can be easily determined from Eq.(7).

3.3.2 Two-point contact case In this section, the case of two-point contact is considered.

When the i -th and the j -th ($1 \leq i < j \leq n$) links are in contact with the object, Eq.(6) becomes

$$\hat{\boldsymbol{\tau}} = \hat{\mathbf{J}}_i^T \mathbf{f}_i + \hat{\mathbf{J}}_j^T \mathbf{f}_j \quad (12)$$

Fig. 4 Identification possibility in case of one-point contact

In a similar way as the previous section, dividing Eq. (12) into linear and nonlinear equations yields

$$\begin{pmatrix} \tau_A \\ \tau_B \end{pmatrix} = L \begin{pmatrix} f_i \\ f_j \end{pmatrix} \quad (13)$$

and

$$\begin{pmatrix} \tau_i - \tau_{i-1} \\ \tau_j \end{pmatrix} = \begin{bmatrix} -l'_i s_i & l'_i c_i & -l_i s_i & l_i c_i \\ 0 & 0 & -l'_j s_j & l'_j c_j \end{bmatrix} \begin{pmatrix} f_i \\ f_j \end{pmatrix}, \quad (14)$$

where

$$\tau_A = \begin{pmatrix} \tau_1 - \tau_2 \\ \vdots \\ \tau_{i-1} - \tau_i \end{pmatrix}, \quad \tau_B = \begin{pmatrix} \tau_{i+1} - \tau_{i+2} \\ \vdots \\ \tau_{j-1} - \tau_j \end{pmatrix},$$

$$L = \begin{bmatrix} A & A \\ 0 & B \end{bmatrix} \in \mathbf{R}^{(j-2) \times 4},$$

$$A = \begin{bmatrix} -l_i s_i & l_i c_i \\ \vdots & \vdots \\ -l_{i-1} s_{i-1} & l_{i-1} c_{i-1} \end{bmatrix} \in \mathbf{R}^{(i-1) \times 2}, \text{ and}$$

$$B = \begin{bmatrix} -l_{i+1} s_{i+1} & l_{i+1} c_{i+1} \\ \vdots & \vdots \\ -l_{j-1} s_{j-1} & l_{j-1} c_{j-1} \end{bmatrix} \in \mathbf{R}^{(j-i-1) \times 2}.$$

Equation (13) is linear, while Eq. (14) is nonlinear with respect to the unknown parameters. When $\text{rank}(L)=4$, f_i and f_j can be identified from Eq. (13). Substituting the estimated forces f_i and f_j into Eq. (14), l'_i and l'_j can be determined. When $\text{rank}(L) < 4$, the number of independent equations is less than the number of unknown parameters, so the unknown parameters can not be identified.

Therefore, the necessary and sufficient condition for identifying two contact points and forces is that $\text{rank}(L)=4$. However, $\text{rank}(L)$ can be rewritten as

$$\begin{aligned} \text{rank}(L) &= \text{rank} \begin{pmatrix} A & A \\ 0 & B \end{pmatrix} = \text{rank} \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \\ &= \text{rank}(A) + \text{rank}(B) \end{aligned}$$

So, $\text{rank}(L)$ is the sum of $\text{rank}(A)$ and $\text{rank}(B)$. Moreover, $\text{rank}(A)$ and $\text{rank}(B)$ are at most 2. Hence, it is obvious that $\text{rank}(A)=\text{rank}(B)=2$ must be satisfied for $\text{rank}(L)=4$. From the discussions above, this is possible only if there are at least two links in nonsingular configuration.

Summarizing the discussions above, we have the

Fig. 5 Identification possibility in case of two-point contact

following results. In the case where the manipulator is in contact with the object at two points, at least two nonsingular links are required between the manipulator base and the 1st contact point, and at least two nonsingular links are required between the 1st contact point and the 2nd contact point, as shown in Fig. 5. In terms of joints, "at least two nonsingular links" can be replaced by "at least three nonsingular joints".

When the contact points and forces are determined, the external force/moment can be determined by substituting the contact points and forces into Eq. (7).

3.3.3 m -point contact case In this subsection, the case of m contact points is discussed.

Now, we suppose that the manipulator is in contact with the object at the $i, j, \dots, k$ -th link. Then equilibrium equation (6) is transformed into

$$\tilde{\tau} = \tilde{J}_i^T f_i + \tilde{J}_j^T f_j + \dots + \tilde{J}_k^T f_k. \quad (16)$$

By picking up linear equations with respect to the unknown parameters, we have from Eq. (16)

$$\begin{bmatrix} \tau_A \\ \tau_B \\ \vdots \\ \tau_C \end{bmatrix} = M \begin{bmatrix} f_i \\ f_j \\ \vdots \\ f_k \end{bmatrix} \quad (17)$$

where

$$M = \begin{bmatrix} A & A & A & \dots & A \\ 0 & B & B & \dots & B \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \dots & 0 & C \end{bmatrix} \in \mathbf{R}^{(k-m) \times 2m}$$

When, $\text{rank}(M)=2m$, Eq. (17) can be solved with respect to contact forces. Hence, substituting the obtained contact forces into the nonlinear equation, the contact positions can be determined. If $\text{rank}(M) < 2m$, then the number of unknown parameters is greater than the number of equations. So the contact points and forces can not be determined. Therefore, the necessary and sufficient condition for identification of m contact points and forces is that

$$\text{rank}(M)=2m$$

Moreover, $\text{rank}(M)$ can be rewritten as

$$\begin{aligned}
\text{rank}(M) &= \text{rank} \begin{bmatrix} A & A & A & \cdots & A \\ 0 & B & B & \cdots & B \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 0 & C \end{bmatrix} \\
&= \text{rank} \begin{bmatrix} A & 0 & 0 & \cdots & 0 \\ 0 & B & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 0 & C \end{bmatrix} \\
&= \text{rank}(A) + \text{rank}(B) + \cdots + \text{rank}(C)
\end{aligned}
\tag{18}$$

Since the rank of matrix $A, B, \dots, C$ are respectively at most 2, $\text{rank}(M) = 2m$ requires $\text{rank}(A) = \text{rank}(B) = \cdots = \text{rank}(C) = 2$. From the same discussion as in the previous subsections, we can conclude that at least two nonsingular links are required for this.

That is, the necessary and sufficient condition for identification of m contact points and forces are

$$\text{rank}(A) = \text{rank}(B) = \cdots = \text{rank}(C) = 2.$$

In other words, the contact points and forces in enveloping grasp can be identified if there are at least two nonsingular links (three nonsingular joints) between adjoining contact points, provided that the manipulator base is recognized as a contact point.

This is the necessary and sufficient condition for identifiability of contact points and forces in the planar enveloping grasp.

4. Conclusion

In this paper, we have formulated the relationship between joint torques and contact forces in a planar enveloping grasp and proposed an identification method of contact parameters using torque and force sensors. Then we have investigated the condition for identifiability of contact points and forces, and obtained the following results.

In the planar enveloping grasp, in order to iden-

tify the contact points between the manipulator and the object, at least two nonsingular links (three nonsingular joints) are required between adjoining two contact points.

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