A Study on Sliding Accuracy Characteristics of an Externally Pressurized Gas-Lubricated Guide Way* (Fundamental Sliding Accuracy Characteristics)

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Sliding accuracy characteristics such as the variation of floating height and the pitching-mode angular fluctuation of an externally pressurized gas-lubricated pad bearing traveling on a linear guide way are investigated theoretically with special attention paid to the relationship between the sliding accuracy and the machining error of the guide-way rail, where sinusoidal undulation of the guide-way surface is assumed. The effects of pitch length of the undulation of the rail surface on the sliding accuracy characteristics are discussed. The experimental results verify the validity of the theoretical analysis of the sliding accuracy of the bearing pad.

Key Words: Hydrostatic Bearing, Guide Way, Design Engineering, Accuracy, Gas Lubrication, Groove Compensation, Sliding Accuracy, Machining Error, Precision Design

1. Introduction

Externally pressurized gas-lubricated supporting system, such as externally pressurized journal gas bearing and externally pressurized gas-lubricated linear guide way, are successfully applied to the precision machining tools and the measuring instruments in the field of precision industry. For the applications as these devices, the running accuracy of the supported part is considered to be one of the most important bearing characteristics.

It is empirically known that the order of the running accuracy turns to be much better than that of the machining accuracy of the bearing parts, when we apply the externally pressurized gas-lubricated supporting systems; this feature is called as "averaging effect". However a few papers⁽¹⁾⁻⁽⁴⁾ have been reported concerning theoretical results obtained by analyzing the relationship between the running accu-

racy and the machining error of the parts, thus establishing theoretical background of the precision design from point of view of the running accuracy characteristics of the bearing system. It may be of great importance to study and clarify them in order to achieve a successful design of an externally pressurized gas-lubricated precision bearing having better running performances.

One of the authors has reported several papers concerning theoretical results of the running accuracy characteristics of a rotor supported by an externally pressurized gas-lubricated thrust-collar bearing^{(1),(2)} and a journal bearing^{(3),(4)}, considering the relationship to the machining error of the bearing parts. In this paper, then, the sliding accuracy characteristics of the bearing pad supported by an externally pressurized gas-lubricated linear guide way will be analyzed theoretically where an undulation is supposed to exist on the guide-way rail surface as a machining error.

Figure 1 is a schematic diagram of cross-sectional view of an externally pressurized gas-lubricated linear guide way. The table is supported by three bearing pads surrounding the guide way rail, and it can travel smoothly along the rail.

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Fig. 1 Cross sectional view of linear guide way

Fig. 2 Schematic diagram of bearing pad

Figure 2 is a schematic configuration of the surface of a rectangular pad bearing applied to the linear guide device. Shallow and narrow rectangular grooves are provided on the bearing surface as usually designed at such a kind of application, and they play the role of groove-compensation mechanism.

A supply hole is arranged at the center of the bearing pad, through which compressed gas is supplied into clearance gap. Two cases are considered to be feasible at the practical design; one is the case where an orifice restrictor is provided in the supply hole, which composes compound compensation in cooperation with the compensation grooves, and the other is the case in which no particular restrictor is arranged in the supply line so that only groove compensation is utilized.

On the other hand, an undulation in longitudinal direction on the opposing rail surface is supposed as a machining error in this study, which governs the sliding accuracy characteristics of the bearing pad. As the bearing pad travels on the rail with such an undulation, gas film thickness variation takes place, which produces a change of pressure distribution in the bearing clearance, then a change of gas film force is resulted. If the pad is supported freely without any constraint, the pad may change its attitude, i.e. the floating height and the pitching-mode angular inclination, to restore the equilibrium state compensating the resulted force unbalance. As the pad travels along the rail, the attitude of the pad may fluctuate due to the change of gas film thickness distribution, and a deterioration of sliding accuracy of the pad may be subse-

Fig. 3 Attitude of bearing pad

quently resulted.

In this study, theoretical model according to this concept is applied to analyze the sliding accuracy characteristics of the bearing pad with relation to the undulation on the opposing guide rail surface. Theoretical analysis in this study is performed under assumption of quasi-static state neglecting the transient effect due to the movement of the pad. As the first step of the study, the undulation on the rail surface is assumed to be only in longitudinal direction and uniform transversely, so that only pitching-mode change of the pad attitude may take place without occurrence of yawing-mode or rolling-mode fluctuations of the pad. We can, hence, neglect the contribution of the side bearing pads in Fig. 1 to the sliding accuracy characteristics of the table, and only the performance of the upper bearing pad is taken into account in the analysis.

2. Analysis

Figure 2 shows an externally pressurized gas-lubricated, rectangular thrust bearing with compound compensation to be analyzed in this paper. The width and depth of the compensation grooves are b_g and h_{g0} , respectively. These dimensions are identical for either section of the grooves. The bearing pad has no machining error in its geometry.

On the other hand, it is assumed that the guide rail opposing to the pad has undulation in its surface as a machining error. The undulation is supposed to be sinusoidal towards the direction of pad movement as shown in Fig. 3. The pitch length and amplitude of the undulation are denoted as p and $\bar{\delta}$, respectively. Due to this undulation, the pad may take a particular attitude on the rail, for example, as shown in Fig. 3. The attitude of the pad can be represented by two quantities; the floating height from the reference line of the rail surface, h , and the pitching-mode tilt angle, α , where α is defined to be positive if the pad inclines so as to make a divergent film thickness distribution towards x -axis (direction of pad movement). In the figure, β is dimensionless traveling distance of the pad (i.e. distance of the pad center from the origin of sinusoidal curve of undulation) normalized by the undulation pitch p .

Divergence formulation (DF) scheme⁽⁵⁾ is applied

to obtain the pressure distribution in the clearance gap. The procedure by this scheme is as follows; discretizing the pad bearing area, and formulating the relating equations between the pressures at the grid points on the basis of the flow continuity condition, over-relaxation method is, then, used to solve the obtained simultaneous equations concerning the gas film pressure. Obtaining the gas film force and its coupling moment by integrating the pressure distribution, then the attitude of the pad is calculated by iteration technique to establish the equilibrium state compensating the deviation of gas film force from the applied external load. At the calculation in the latter section, we assume that a static load, which has no coupling-moment component, is applied to the bearing pad.

The attitude of the pad is formulated by the following expressions;

$$\text{Floating height: } h/h_0 \quad (1)$$

$$\text{Tilt angle: } \alpha L/2h_0 \quad (2)$$

where L is the pad length, and h_0 is the reference floating height, which is the one for the case without rail surface undulation.

Typical dimensions and quantities are assumed as follows in the latter calculations;

$$L/B=1.0, \quad B_1/B=0.8,$$

$$L_1/L=0.8, \quad h_g/B=0.01,$$

$$\delta \equiv \bar{\delta}/h_0=0.1, \quad P_{sg}=(p_s-p_a)/p_a=1.0,$$

where p_s is supply pressure, p_a is ambient pressure, and the other symbols should be referred to Fig. 2.

3. Load Carrying Capacity

Figure 4 is theoretical results of dimensionless load carrying capacity for the case without rail surface undulation, in which

$$W=w/p_a B^2 \quad (3)$$

where w is dimensional load carrying capacity. The abscissa is feeding parameter for the case of com-

Fig. 4 Theoretical load carrying capacity

pound compensation defined by

$$\Gamma = 24 \mu r_s^2 C_d \sqrt{\Re T} / (h_0^3 p_a) \quad (4)$$

where μ : viscosity coefficient, r_s : radius of orifice, $\Re$: gas constant, T : temperature, and C_d : discharge coefficient. The case of groove compensation without contribution of orifice restriction is represented by the limiting value as $\Gamma \rightarrow \infty$. The parameter, H_{g0} , in the figure is dimensionless groove depth defined by

$$H_{g0} = h_{g0}/h_0. \quad (5)$$

The result denoted as $H_{g0}=0$ is a case with an orifice-compensated, single central supply hole and without compensation grooves.

It is noted that this figure is considered to be used to evaluate the reference floating height, h_0 , which can be obtained from the corresponding feeding parameter value, when the applied load W is known.

The load carrying capacity increases with increase of groove depth in the dominant groove-compensation region, that is the region with comparatively large Γ value. Though the calculated results of the bearing fundamental characteristics, such as parallel-mode and pitching-mode bearing stiffnesses and damping coefficients, are omitted to be shown in the paper, we can point out that the bearing stiffness takes its peak value at a certain Γ value, and this peak value of bearing stiffness increases with increase of groove depth, while the damping coefficient decreases there.

From the results of these bearing characteristics as well as the experience in practical design, we choose a typical groove depth as $H_{g0}=4.0$, and the feeding parameter for compound compensation case as $\Gamma=7.0$, which corresponds to the optimum value for maximum bearing stiffness under the proposed groove depth. The corresponding dimensionless load carrying capacities are $W=0.408$ and $W=0.298$ for groove compensation case and for compound compensation case, respectively. This means that we will analyze the sliding accuracy characteristics in the next section on the supposition that the assigned load is applied to the bearing.

4. Sliding Accuracy Characteristics of Linear Guide Way

Figure 5 is an example of schematic representation of variation of attitude of the pad, when it travels for one pitch distance of the rail surface undulation, where $p/L=1.0$ and $\delta=0.1$ are supposed as a geometry of undulation. In the figure, the pad is shown by reducing its length into a 1/10 scale.

The general feature of the pad attitude variation can be summarized as follows including the results for the case with $p/L=1.0$ shown in Fig. 5. When $\beta=0$,

Fig. 5 Variation of attitude of bearing pad

the floating height of the pad is nearly equal to the reference gap height, but the pitching-mode tilt angle becomes maximum there. When $\beta = \pi/2$ or $\beta = 3\pi/2$, on the other hand, the pad does not tilt because of symmetrical distributions of gap height and pressure in consequence, while the parallel-mode deviation of the pad becomes maximum taking the maximum or the minimum floating height value. Whether the maximum/minimum floating height is encountered when $\beta = \pi/2$ or when $\beta = 3\pi/2$ depends upon the pitch length of the undulation.

According to these results, we evaluate the magnitude of variation of parallel-mode floating height by those when $\beta = \pi/2$ and $\beta = 3\pi/2$, and the magnitude of pitching-mode fluctuation of the pad by that when $\beta = 0$.

The amplitudes of floating height variation and pitching-mode fluctuation of the pad changes very much depending on the value of p/L , that is, the ratio of pitch length of undulation to the pad length. Figures 6(a) and (b) show the results of amplitudes of floating height variation and angular fluctuation versus p/L ratio for the case with compound compensation. It can be seen in Fig. 6 that, at a particular value of p/L , floating height deviation decreases to be sufficiently small, but the tilt of the pad is enhanced there, and vice versa. As the value of p/L ratio varies, a case with an enhanced floating height deviation and a case with an enhanced angular fluctuation are encountered by turns. In the other words, if the rail surface has certain undulation, either floating height deviation or angular fluctuation of the pad is inevitably encountered, depending on the pitch length of the undulation.

In Fig. 6, it can be seen that the amplitude of floating height deviation becomes less than that of the undulation (i.e. $\delta = 0.1$), and the amplitudes of pad fluctuations decrease with decrease of pitch length of undulation. It should be noted that these results may be quantitative evaluation of so-called "averaging effect", but that the order of this effect seems to be not so remarkable for the case of the externally pressurized gas-lubricated linear guide way. The chained

Fig. 6 Amplitudes of fluctuation of pad attitude (Compound compensation case)

curves in the figures are envelopes obtained by connecting the maximum deviation points of the pad fluctuation curves, and they may offer a qualitative measure of reduction of fluctuation amplitude by the averaging effect. The amplitude of pad attitude fluctuation decreases with decrease of p/L ratio until $p/L \approx 0.1$, but in the region with a smaller pitch length of undulation, the envelopes connecting the peak points decreases their amplitudes not monotonously but oscillatorily as p/L ratio decreases. Thus, the tendency of the sliding accuracy characteristics is not so simple but rather complicated especially when the pitch length of undulation becomes comparatively small.

As shown in Fig. 6, where the abscissa is taken to be p/L , the wave length of the sliding accuracy characteristic curves reduces as p/L ratio decreases, but, if we transform the results to show them against L/p ratio as abscissa, we can find that the characteristic curves turn to have an identical wave length irrespective of the pitch length of the undulation.

Figures 7(a) and (b) are results of the sliding

(a)

(b)

Fig. 7 Amplitudes of fluctuation of pad attitude (Groove compensation case)

accuracy characteristics for the case with groove compensation without orifice restrictor in the supply hole. It can be seen that, as for the floating height deviation (Fig. 7(a)), the amplitude is generally larger than that of compound compensation case, and, especially in the region where the pitch length of undulation is comparatively large, the order of fluctuation amplitude becomes nearly the same as that of undulation amplitude, and that the reduction in the amplitude of pad attitude fluctuation with decrease of p/L ratio is less than that in Fig. 6, though the qualitative feature of the sliding accuracy characteristics is quite similar to that of the compound compensation case.

As for the angular fluctuation in Fig. 7(b), the feature is quite the same even quantitatively compared with that in Fig. 6(b). It is considered to be due to the fact that the angular fluctuation may be resulted by the asymmetric distributions of gas film thickness and pressure in the clearance gap in consequence, and it is rather independent on the restriction of gas flow

Fig. 8 Experimental apparatus

in the supply hole.

It is also noted that the effect of supply pressure to the sliding accuracy characteristics is confirmed to be rather small, though the figures of the results are omitted in the paper.

5. Experiments

Some experiments were performed to verify the theoretical results obtained above.

Figure 8(a) is a schematic diagram of experimental apparatus, in which the bearing pad is stationary, while the opposing rail is movable. The bearing part is composed of four plates; the front plate is the test bearing pad with groove compensation, the back plate is to apply the load to the square shaft (rail) by introducing pressurized air in the recess arranged on the plate, and the side plates are externally pressurized thrust bearings to guide the shaft smoothly in the bearing assembly.

The square shaft floating in the bearing assembly corresponds to the rail in the practical guide way construction. In this apparatus, it can move in vertical direction relatively to the bearing pad on the front plate by adjusting the length of the string, which is connected to the upper side of the shaft. The main dimensions of the test bearing are such that $L=B=29$ mm, $L_1=B_1=21$ mm, $b_g=0.7$ mm, $h_{g0}=51\sim 52$ μ m and $2r_s=2.01$ mm. The dimensions of loading pad on the back plate are 29×29 mm, and those of the recess are 24×24 mm.

We have to produce a certain flatness error on the surface of the square shaft opposing to the test bearing, in order to verify the theoretical results obtained in the previous section. From point of view of simplicity and reliability of manufactured geometry of the rail surface undulation, we applied rectangular shallow steps on the rail as shown in Fig. 8(b), instead of processing a sinusoidal undulation assumed in the theoretical analysis. Three shafts were prepared;

one was a flat shaft without steps for the reference, and the others had undulation steps with pitch lengths of $p=20$ mm and 30 mm, respectively. The depth of the steps was $\bar{\delta}=18\sim19\text{ }\mu\text{m}$. The clearance between the square shaft and the surrounding assembly of the bearing parts was about $50\text{ }\mu\text{m}$.

Floating height of the shaft from the bearing pad was measured by a couple of contact-type electronic displacement gages installed at the upper and the lower positions of the bearing pad, and the floating height at the bearing center and the tilt angle of the shaft were calculated from the measured results of the shaft position. The contribution of contact forces exerted by the displacement gages were taken into account as a part of external load.

Figure 9 shows the experimental results of floating height by using the reference shaft without undulation steps, where the pressure in the back pad and hence the applied load are varied. The obtained results coincide well with theoretical ones shown by thick curves, which may certify the validity of the analyzing method of pressure distribution in the bearing clearance. It is mentioned that the diameter of the supply hole in the experimental apparatus is fairly large compared with the practical design of generally used bearing pad, hence the supply hole serves as an inherent compensation restrictor. Then, a compound compensation with contributions of compensation grooves and inherent restrictor is supposed at the theoretical calculation for the succeeding figures.

Figure 10 is the experimental results of floating height and angular fluctuation obtained by using the shaft with step pitch length of $p=20$ mm. The abscissa β is traveling distance normalized by the step pitch so as to $\beta=2\pi$ corresponds to one pitch-length movement. The regions that $0<\beta<\pi$ and $\pi<\beta<2\pi$ are for the cases where the center of the pad opposes to

the land part and to the stepped part of the shaft surface, respectively. The reference gap heights h_0 are assigned to those obtained by theoretical calculation. They are shown in Table 1, where dimensionless supply pressure is assumed as $P_{sg}=2.0$, which corresponds to the experimental condition.

The experimental results coincide fairly well with theoretical ones at least in qualitative sense, though there exists some deviations between them. One factor for these deviations may be attributed to the fact that the steps manufactured on the shaft surface are likely not so exact geometrically as assumed in the theoretical analysis but may be somewhat blunt at their edges, and the other is that the estimation of pressure distribution in the back loading pad may

Table 1 Theoretical values of reference gap height

Supply pressure P_{sg}	Dimensionless load capacity W	Reference gap height $h_0(\text{ }\mu\text{m})$
2.0	0.3	40.0
	0.4	32.9
	0.5	27.7

Fig. 9 Experimental results of load carrying capacity

Fig. 10 Experimental results of pad attitude (I)

Fig. 11 Experimental results of pad attitude (II)

possibly be erroneous when the shaft inclines in the bearing assembly. It is also noted that some discontinuities can be seen at $\beta=0$ and $\beta=\pi$ in the theoretical results in the figures, while the experimental results are rather continuous, and that the step-shaped undulation affects on the fluctuation characteristics of the pad attitude more sensitively than the case with sinusoidal undulation. The latter may be one reason for the deviations between experimental results and theoretical ones in Fig. 10.

Figure 11 shows the effects of pitch length of the steps. The feature of fluctuation of pad attitude is closely related to the pitch length of undulation as clarified in the previous section. It can be seen that the experimental results in Fig. 11 agree well with those theoretical predictions.

We can conclude then that the experimental results confirms the validity of theoretical analysis, though some quantitative deviations are seen between them.

6. Conclusions

In this paper, the sliding accuracy characteristics of a bearing pad of externally pressurized gas-lu-

bricated linear guide way with considering the relationship to the machining error such as undulation of the guide way rail surface. The conclusions obtained can be summarized as follows:

(1) When the bearing pad travels along the guide rail with undulation on its surface, floating-height variation and pitching-mode angular fluctuation take place. The feature of fluctuation is closely dependent on the ratio of pitch length of undulation to the pad length, p/L .

(2) When the value of p/L ratio varies, a case with an enhanced floating-height variation and a case with an enhanced angular fluctuation are encountered by turns.

(3) The amplitudes of fluctuations of the pad attitude decrease with decrease of the pitch length of undulation. This "averaging effect" is not so remarkable for the application of externally pressurized gas-lubricated bearing system to the linear guide way.

(4) Compound compensation yields a smaller amplitude of floating-height variation compared with groove compensation case. Either compensation system results in the same order of amplitude of angular-mode fluctuation.

(5) Experimental results verify the validity of the theoretical treatments developed in this paper.

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