Optimum Magnetic Bearing Design Considering Performance Limitations*

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Magnetic bearings are subject to three practical performance limitations: the peak force capacity, force slew rate limitation, and displacement sensitivity. In conventional designs, we usually first design a feasible bearing, and then investigate these three limitations. Unfortunately, one usually fails to design the most suitable magnetic bearing in actual cases. In this paper, a methodology for dealing with the performance limitations through the concept of optimum design is presented. The objective functions are to maximize the force slew rate limitation and to minimize the displacement sensitivity under several design constraints. Using the optimization solver software package, we developed a systematic method for determining the characteristics of magnetic bearings. By considering the performance limitations and design constraints, we achieve a better design than that produced using the conventional trial-and-error method.

Key Words: Magnetic Bearings, Electromagnet, Optimum Design, Performance Limitations

1. Introduction

Magnetic bearings (MBs) are engineered systems finding increasing use in a wide range of applications. Such devices have a number of advantages over conventional bearings. These include no contact between the bearing and the shaft (hence no friction and wear), high speed capacity, potential for vibration control, removal of complex oil supply systems, and extension of machinery life through reduction/elimination of wear. Regarding the whole system, two undesirable designs must be avoided: using a smart controller to control a poorly designed plant, or manufacturing an elaborate plant but using a poor controller. Because the magnets are subject to active control, the forces are functions of both the physical geometry and the control parameters. Thus, while it is important to develop robust control strategies, it is also desirable to quantify further the forces exerted by electromagnets on a rotor in order to fully optimize a MB system.

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MBs have been the subject of much research and new applications for MBs continue to be introduced. The investigations reported in the literature can be roughly classified into three topics: the design of MBs⁽¹⁾⁻⁽⁴⁾, vibration control in rigid or flexible rotor systems⁽⁵⁾⁻⁽⁸⁾, and a combination of these two topics⁽⁹⁾⁻⁽¹²⁾. However, most of these studies—even those that treat both design and control—have focused on the control problem. Little research has been conducted on actuator design.

The stiffness of MBs may change considerably as the shaft deviates from the central position. Additionally, and perhaps even more important, the rate at which the bearing force can change is strictly limited. This phenomenon is referred to as the force slew rate limitation. These two phenomena, which represent nonlinearity, are especially apparent because their onset can be quite abrupt. Maslen et al.⁽²⁾ investigated the performance limitations of MBs. Later, Maslen and Allaire⁽¹²⁾ used the open-loop control strategy to identify the optimal control force in a rotor system. Pang et al.⁽³⁾ investigated the optimization of a “pancake” MB which was used as the bearing of a momentum flywheel. There are some defects in their

optimum design methods: the maximum linear range of the objective function does not take account of the inherently nonlinear relationship among the rotor displacement, the control current, and the bearing force; furthermore, their optimization does not take the effect of the controller into consideration. Rao et al.⁽¹³⁾ investigated the stiffness of a MB subjected to static and dynamic loads simultaneously.

In this paper, we focus on the optimum design of MBs in the sense of maximizing the force slew rate limitation and minimizing the displacement sensitivity. The dimension limitation of MBs, the maximum magnetic flux density of the material, and other peripheral equipment restrictions are used as design constraints. We also present approximate design formulas and curves for representing the stiffness capacity as a function of the ratio of dynamic to static load, which make our design realistic and efficient.

Nomenclature

- A_g : effective cross-sectional area of air gap
 B_{sat} : saturation flux density
 c : ratio of the surface area on the air gap A_g to the section area of pole piece A_c
 d_c : diameter of copper wire
 d_p : inner diameter of pole piece
 D_{shaft} : diameter of shaft
 F_{stat}, F_{dyn} : amplitudes of static and dynamic components of external load
 G : air gap size
 G_d, G_s : driver and sensor gains
 I_b : bias current for premagnetization
 $i_{c,max}$: maximum control current
 I_{dyn} : amplitude of dynamic control current
 I_{max} : maximum allowable current
 I_{stat} : static current for rotor weight
 i_x, i_y : control currents in horizontal and vertical directions
 $I_{x,dyn}, I_{y,dyn}$: control currents for dynamic force
 K : nondimensional stiffness factor
 k_{dyn} : dynamic bearing stiffness
 K_i, K_d : current and displacement stiffness
 k_p, k_d : proportional and derivative gains of PD controller
 l : length of pole leg
 L_c : coil inductance
 N : number of turns of winding coil around a pole
 P_{dyn} : amplitude of dynamic bearing force
 $P_{y,dyn}$: dynamic bearing force
 R_c : coil resistance
 s : thickness of the fringe of pole piece
 t_s : thickness of stator
 V_c, V_b : voltages for control current and bias current
 V_p : power supply voltage

W_m : width of pole

Y : vibration to gap ratio

y_0 : amplitude of rotor vibration

α : half of angle formed by the two poles of an EM

β : bias indicator

γ : dynamic to static load ratio

μ_0 : $4\pi \times 10^{-7}$ H/m, permeability of free space

ω : rotating speed

2. Magnetic Circuit Analysis

The MB used here is an 8-rib all radial electro-magnetic bearing (EMB) with the configuration shown in Fig. 1. The system of rotor and electromagnets (EMs) shown in Fig. 1 can be analyzed with the magnetic circuit in Fig. 2, where we assume that the reluctance is mainly that of the air gaps and there is no leakage flux or fringing phenomenon. The number of winding coil turns is the same for each EM. Writing the loop equations of loops 1.2.10.9, 2.3.11.10,

Fig. 1 Configuration of the EMB

Fig. 2 Magnetic circuit of the EMB

3.4.12.11, 4.5.13.12, 5.6.14.13, 6.7.15.14, and 7.8.16.15, and the node equation of nodes 1.2.3.4.5.6.7.8, we have

$$\begin{aligned} R_1\Phi_1 - NI_1 - NI_1 + R_2\Phi_2 &= 0 \\ -R_2\Phi_2 + NI_1 - NI_2 + R_3\Phi_3 &= 0 \\ -R_3\Phi_3 + NI_2 + NI_2 - R_4\Phi_4 &= 0 \\ R_4\Phi_4 - NI_2 + NI_3 - R_5\Phi_5 &= 0 \\ R_5\Phi_5 - NI_3 - NI_3 + R_6\Phi_6 &= 0 \\ -R_6\Phi_6 + NI_3 - NI_4 + R_7\Phi_7 &= 0 \\ -R_7\Phi_7 + NI_4 + NI_4 - R_8\Phi_8 &= 0 \\ \Phi_1 - \Phi_2 - \Phi_3 + \Phi_4 + \Phi_5 - \Phi_6 - \Phi_7 + \Phi_8 &= 0 \end{aligned} \quad (1)$$

where

$$\begin{aligned} R_1 &= \frac{G - y \cos \alpha + x \sin \alpha}{\mu_0 A_g}, \\ R_2 &= \frac{G - y \cos \alpha - x \sin \alpha}{\mu_0 A_g}, \\ R_3 &= \frac{G - y \sin \alpha - x \cos \alpha}{\mu_0 A_g}, \\ R_4 &= \frac{G + y \sin \alpha - x \cos \alpha}{\mu_0 A_g}, \\ R_5 &= \frac{G + y \cos \alpha - x \sin \alpha}{\mu_0 A_g}, \\ R_6 &= \frac{G + y \cos \alpha + x \sin \alpha}{\mu_0 A_g}, \\ R_7 &= \frac{G + y \sin \alpha + x \cos \alpha}{\mu_0 A_g}, \\ R_8 &= \frac{G - y \sin \alpha + x \cos \alpha}{\mu_0 A_g}. \end{aligned}$$

$R_1 \sim R_8$ are the reluctances of air gaps, G is the air gap size, y and x are the displacements in the vertical and horizontal directions, respectively, $\mu_0 = 4\pi \times 10^{-7}$ is the permeability of free space, α is half the angle formed by the two poles of one EM, A_g is the effective cross-sectional area of an air gap, N is the number of turns of winding coil around a pole, and I_n is the current through the four electromagnets ($n=1 \sim 4$).

In MB design, the allowable displacement of the rotor is about one-tenth of the air gap size. Thus the extreme values of reluctances R_1 to R_8 of air gaps # 1 to # 4 in Fig. 1 occur at $x=0.1G$ and $y=0.1G$, and we have the fluxes

$$\begin{aligned} \Phi_1 &= \frac{N}{R} [1.050I_b + 1.063i_y + 0.021i_x] \\ \Phi_2 &= \frac{N}{R} [1.142I_b + 1.063i_y + 0.021i_x] \\ \Phi_3 &= \frac{N}{R} [1.142I_b + 1.063i_y + 0.021i_x] \\ \Phi_4 &= \frac{N}{R} [1.050I_b + 1.063i_y + 0.021i_x] \\ \Phi_5 &= \frac{N}{R} [0.955I_b + 1.063i_y + 0.021i_x] \\ \Phi_6 &= \frac{N}{R} [0.878I_b + 1.063i_y + 0.021i_x] \\ \Phi_7 &= \frac{N}{R} [0.878I_b + 1.063i_y + 0.021i_x] \\ \Phi_8 &= \frac{N}{R} [0.955I_b + 1.063i_y + 0.021i_x] \end{aligned} \quad (2)$$

where

$$R = \frac{G}{\mu_0 A_g},$$

and the coil currents are assumed to be in the following form:

$$\begin{aligned} I_1 &= I_b + i_y, & I_2 &= I_b + i_x \\ I_3 &= I_b - i_y, & I_4 &= I_b - i_x \end{aligned} \quad (3)$$

in which I_b is the bias current for premagnetization, and i_x and i_y are the control currents in the horizontal and vertical directions, respectively.

We see that the flux in an air gap due to the perpendicular coil current is less than 2% the flux of its own exciting coil current. Therefore, we may neglect the interaction between the magnetic flux in the vertical and horizontal directions. That is, the fluxes in the vertical and horizontal directions are uncoupled for the EMs shown in Fig. 1. To further ensure that the fluxes are uncoupled, we utilize the fact that magnets of the same polarity repel each other and magnets of opposite polarity attract each other in the winding scheme, as shown in Fig. 2. Thus the magnetic flux of the circuit will circulate within the four local circuit (see Fig. 1), as we desire.

3. Magnetic Force Modeling

The fluxes in the four electromagnets (see Fig. 1) are

$$\Phi_n = \frac{\mu_0 A_g N I_n}{G - d \cdot \cos \alpha} \quad (4)$$

where Φ_n is the magnetic flux through the air gap ($n=1 \sim 4$) and d is the displacement in the horizontal or vertical direction. By the principle of virtual work, the bearing force provided by one EM is expressed as

$$F_n = \frac{\mu_0 A_g N^2 I_n^2 \cos \alpha}{(G - d \cdot \cos \alpha)^2}. \quad (5)$$

The relations between magnetic force and vertical and horizontal displacements as well as coil current I_n ($n=1 \sim 4$) are nonlinear. Because a linear control algorithm is used, it is necessary to linearize the magnetic force. The premagnetization scheme is achieved by supplying a large constant current to each coil of the EMs and a small bi-directional varying current can be used to achieve linearization of the bearing force in a certain range. Because of the rotor's weight, additionan bias current must be added in the vertical direction, that is:

$$\begin{aligned} i_y &= I_{\text{stat}} + I_{y, \text{dyn}} \\ i_x &= I_{x, \text{dyn}}. \end{aligned} \quad (6)$$

The resultant forces in the vertical and horizontal directions are

$$\begin{aligned} P_{y, \text{brg}} &= \mu_0 A_g N^2 \cos \alpha \left[\frac{(I_b + I_{\text{stat}} + I_{y, \text{dyn}})^2}{(G - y \cos \alpha)^2} \right. \\ &\quad \left. - \frac{(I_b - I_{\text{stat}} - I_{y, \text{dyn}})^2}{(G + y \cos \alpha)^2} \right] \\ P_{x, \text{brg}} &= \mu_0 A_g N^2 \cos \alpha \left[\frac{(I_b + I_{x, \text{dyn}})^2}{(G - x \cos \alpha)^2} \right. \end{aligned}$$

$$-\frac{(I_b - I_{x, \text{dyn}})^2}{(G + x \cos \alpha)^2} \Big]. \quad (7)$$

Equation (7) shows that magnetic force is a double-variable function of $I_{x, \text{dyn}}$ (or $I_{y, \text{dyn}}$) and x (or y). By Taylor's expansion, we linearize the bearing force as

$$\begin{aligned} P_{y, \text{brg}} &= P_{\text{stat}} + K_i \cdot I_{y, \text{dyn}} + K_{y, d} \cdot y = P_{\text{stat}} + P_{y, \text{dyn}} \\ P_{x, \text{brg}} &= K_i \cdot I_{x, \text{dyn}} + K_{x, d} \cdot x = P_{x, \text{dyn}} \end{aligned} \quad (8)$$

where

$$\begin{aligned} P_{\text{stat}} &= \frac{4\mu_0 A_g N^2 \cos \alpha \cdot I_b I_{\text{stat}}}{G^2} \\ K_i &= \frac{4\mu_0 A_g N^2 \cos \alpha \cdot I_b}{G^2} \\ K_{y, d} &= \frac{4\mu_0 A_g N^2 \cos^2 \alpha \cdot (I_b^2 + I_{\text{stat}}^2)}{G^3} \\ K_{x, d} &= \frac{4\mu_0 A_g N^2 \cos^2 \alpha \cdot I_b^2}{G^3} \end{aligned}$$

Due to the suspension of the rotor, the most critical case will occur in the y direction, and we will consider the y -direction bearing force in the following discussion. Similar results can be obtained for the x -direction bearing force.

4. Dynamic Stiffness of Magnetic Bearings

In most practical cases, the external load consists of a combination of a static load and a superposed dynamic load. Since the sum of the peak static load and peak dynamic load is limited by the saturation flux density of the core material, a larger static load will imply a smaller capacity to resist dynamic loads and vice versa. This dynamic force will affect the stiffness capacity of the bearing directly. Thus the dynamic stiffness capacity of a MB is influenced by the ratio of dynamic to static load. Assume the maximum external force has the following form:

$$F_{\text{ext}} = F_{\text{stat}} + F_{\text{dyn}} \cos(\omega t - \psi), \quad (9)$$

where F_{stat} is the static load (usually the weight of the rotor) and F_{dyn} is the peak dynamic force. Thus the amplitude of vibration is as follows:

$$y = -y_0 \cos(\omega t - \theta). \quad (10)$$

Moreover, in order to investigate the relation between the parameters and bearing stiffness, we consider a simple spring without damping

$$\begin{aligned} I_{y, \text{dyn}} &= -k_0 y \\ &= I_{\text{dyn}} \cos(\omega t - \theta), \end{aligned} \quad (11)$$

where I_{dyn} is the peak dynamic control current.

If the magnetic material, such as a silicon strip, is saturated, the polarization cannot be increased further. In other words, even though we increase the number of ampere turns, the flux density in the magnetic material cannot increase proportionally to the number of ampere turns. There exists a maximum saturation current I_{max} . In general, we will use the unsaturated region, which can be approximated by the linear relation $B = \mu H$. The maximum flux in the

most restricted cross section of the pole piece is

$$\phi_c = B_{\text{sat}} \cdot A_c. \quad (12)$$

On the other hand, the flux in the air gap for the concentric rotor-bearing system with I_{max} provided to the coil is

$$\phi_g = \frac{\mu_0 A_g N I_{\text{max}}}{G}. \quad (13)$$

Since these two fluxes must be equal, we have the maximum number of ampere turns as

$$N \cdot I_{\text{max}} = \frac{G B_{\text{sat}} A_c}{\mu_0 A_g} \quad (14)$$

The corresponding maximum bearing force P_{max} is defined as

$$\begin{aligned} P_{\text{max}} &= \frac{\mu_0 A_g N^2 \cos \alpha}{G^2} [(I_b + i_{c, \text{max}})^2 - (I_b - i_{c, \text{max}})^2] \\ &= \frac{4\mu_0 A_g N^2 \cos \alpha \cdot I_b i_{c, \text{max}}}{G^2}, \end{aligned} \quad (15)$$

where $x, y = 0$ and $I_{\text{max}} = I_b + i_{c, \text{max}}$. Because of the premagnetization scheme, we define the bias indicator β as the ratio of bias current I_b to maximum control current $i_{c, \text{max}}$, i.e.,

$$\beta = \frac{I_b}{i_{c, \text{max}}}. \quad (16)$$

The maximum allowable control current $i_{c, \text{max}}$, the bias current I_b , and the maximum allowable current I_{max} are related by

$$\beta I_{\text{max}} = (\beta + 1) I_b = \beta(\beta + 1) i_{c, \text{max}}. \quad (17)$$

By defining the dynamic to static load ratio γ and the vibration to gap ratio Y as

$$\begin{aligned} \gamma &= \frac{F_{\text{dyn}}}{F_{\text{stat}}} \\ Y &= \frac{y_0}{G}, \end{aligned} \quad (18)$$

and using the requirement that the static component of the bearing force P_{stat} cancel that of the external load F_{stat} and the relations

$$\begin{aligned} P_{\text{max}} &= F_{\text{stat}} + F_{\text{dyn}} \\ i_{c, \text{max}} &= I_{\text{stat}} + I_{\text{dyn}}, \end{aligned} \quad (19)$$

we can express the dynamic bearing force $P_{y, \text{dyn}}$ as follows:

$$\begin{aligned} P_{y, \text{dyn}} &= Y P_{\text{max}} \left[\frac{\gamma}{Y(1+\gamma)} - \frac{[\beta^2(1+\gamma)^2 + 1] \cos \alpha}{\beta(1+\gamma)^2} \right] \\ &\times \cos(\omega t - \theta) \\ &= P_{\text{dyn}} \cos(\omega t - \theta), \end{aligned} \quad (20)$$

where P_{dyn} is the amplitude of the dynamic bearing force in the vertical direction. For small vibrations of a rigid journal, the dynamic bearing stiffness is defined as the ratio of the dynamic component of the bearing force $P_{y, \text{dyn}}$ to the journal displacement y , i.e.,

$$\begin{aligned} k_{\text{dyn}} &= -\frac{P_{y, \text{dyn}}}{y} = \frac{Y P_{\text{max}}}{y_0} \left[\frac{\gamma}{Y(1+\gamma)} \right. \\ &\left. - \frac{[\beta^2(1+\gamma)^2 + 1] \cos \alpha}{\beta(1+\gamma)^2} \right]. \end{aligned} \quad (21)$$

Note that k_{dyn} is independent of y for small rotor vibration. Defining a nondimensional stiffness factor

K as the ratio of the dynamic bearing stiffness k_{dyn} to (P_{max}/G) , we have

$$K \equiv \frac{k_{\text{dyn}}}{(P_{\text{max}}/G)} = \frac{\gamma}{Y(1+\gamma)} - \frac{[\beta^2(1+\gamma)^2 + 1] \cos \alpha}{\beta(1+\gamma)^2}. \quad (22)$$

The bearing stiffness factor K depends on the displacement to gap ratio Y , the dynamic to static load ratio γ , and the bias indicator β .

Considering the vibration to gap ratio $Y=0.1$, we plot in Fig. 3 the stiffness factor as a function of the dynamic to static load ratio for different bias indicators ($\beta=1\sim 8$). This figure exhibits how much stiffness can be supplied by the MB for a given dynamic to static load ratio and a specified bias indicator. In Fig. 3, we see that a larger β causes a smaller K for a specified value of γ , and the stiffness curve becomes flat at large γ for a given β .

The stiffness curve in Fig. 3 shows that the stiffness can become zero at some values of γ . This

means that, for a given dynamic to static load ratio, there is a range of allowable bias indicators to make the bearing system stable. This bias indicator, called the critical bias indicator, can be obtained by setting $K=0$ in Eq. (22). It is

$$\beta_{\text{crit}}^{a,b} = \frac{\gamma \pm \sqrt{\gamma^2 - 4Y^2 \cos^2 \alpha}}{2(1+\gamma)Y \cos \alpha}. \quad (23)$$

Figure 4 shows the relationship between γ , Y , and β_{crit} , where we use $Y=0.1$ as the designed vibration to gap ratio. The figure shows that, for a given dynamic to static load ratio and a given vibration to gap ratio, the bias indicators within the two curves are acceptable. Thus we can find the appropriate value of the bias indicator for a given ratio of dynamic to static load and a designed vibration to gap ratio.

The results derived in this section can also be applied to the x -direction bearing force by letting γ approach an infinite value and calculating the limit value of Eq. (22).

5. Design Constraints of EMBs

In the practical design of an EMB, we encounter a number of restrictions due to the peripheral equipment or actuator itself. These restrictions interfere with one another; for example, we may require a large load capacity, but the flux density will be saturated if the number of ampere turns is too large. Also, the number of ampere turns will be restricted by the winding space and copper wire capacity. Therefore, we must find a compromise between these design constraints. In the following paragraphs, we will discuss the practical design constraints in detail.

5.1 Flux density constraint

Assume that the permeability of magnetic material μ is much larger than the permeability of free space μ_0 . Using Ampere's law, we express the flux density in air-gap B_g , with coil current I , as

$$B_g = \frac{\mu_0 NI}{G} \quad (24)$$

Because we ignore the leakage flux from the poles and the fringing effect at the pole face, the flux in the air gap ϕ_g is equal to the flux in the core ϕ_c , i. e.,

$$B_g \times A_g = B_c \times A_c$$

$$B_c = c \frac{\mu_0 NI}{G}, \quad (25)$$

where c is the ratio of A_g to A_c . To avoid saturation of the magnetic material, we have to observe the following constraint:

$$B_c \leq B_{\text{sat}}. \quad (26)$$

5.2 Load capacity constraint

We have shown that the dynamic bearing stiffness depends on the displacement to gap ratio Y , the dynamic to static load ratio γ , and the bias indicator β . We can use Fig. 3 to choose the appropriate K as

Fig. 3 Effect of dynamic to static load ratio on the stiffness of the magnetic bearing

Fig. 4 Critical bias indicator as a function of the dynamic to static load ratio

we set the values of γ , Y , and β . In our design, we have the following specifications:

$$\begin{aligned}\gamma &= 5 \\ Y &= 0.1 \\ \beta &= 6 \\ K &= 0.5,\end{aligned}$$

where β should be large enough to satisfy the requirement of linearizing the bearing force. We set $\gamma=5$ so that the dynamic bearing force will sustain five times the rotor's weight. Thus we have the following constraint:

$$\frac{P_{\text{dyn}}}{P_{\text{stat}}} \geq \gamma. \quad (27)$$

5.3 Winding space constraint

Figure 5 shows the effective slot space in our design (the hatched region). By experience, the most effective winding space is about eighty percent of the effective slot space in which the copper wire can be arranged tightly. We can approximate the most effective winding space as a trapezoid region, which we express as follows:

$$A_w = 0.8 \cdot [(d_p + 2s + l) \sin \alpha - W_m \cos \alpha] \cdot l \cdot \cos \alpha, \quad (28)$$

where d_p is the inner diameter of the pole piece, s is the thickness of the fringing edge of the pole face, l is the length of the pole piece, and W_m is the width of the pole piece. The required winding space A_r is

$$A_r = 2N \cdot \frac{\pi(d_c + t_c)^2}{4}, \quad (29)$$

where d_c is the diameter of the copper wire and t_c is twice the thickness of the wire coating (about 0.05 mm). To achieve a compromise, A_r has to be less than A_w , i.e.,

$$A_r \leq A_w. \quad (30)$$

5.4 Stability constraint

Assume the rotor is a single mass. Then the dynamic equation of the whole system can be expressed as

$$m\ddot{x} = K_i \cdot i + K_d \cdot \dot{x}. \quad (31)$$

Fig. 5 The effective slot space

The control algorithm we use is

$$\begin{aligned}i &= -G_d V_{\text{com}} \\ V_{\text{com}} &= k_p V_{\text{ref}} + k_d \dot{V}_{\text{ref}} \\ V_{\text{ref}} &= G_s x,\end{aligned} \quad (32)$$

where i represents the control current, V_{com} the command voltage to the drivers, V_{ref} the sensor reference input voltage, G_d the driver gain (A/V), G_s the sensor gain (V/mm), and k_p , k_d the proportional and derivative gains of the PD controller. Thus, the control current is

$$i = -G_s G_d (k_p x + k_d \dot{x}), \quad (33)$$

and the dynamic equation can be expressed as

$$m\ddot{x} + K_i G_s G_d k_d \dot{x} + (K_i G_s G_d k_p - K_d)x = 0. \quad (34)$$

To ensure the stability of the whole system, the damping of the dynamic equation must be semipositive definite and the stiffness has to be positive definite. That is,

$$\begin{aligned}K_i G_s G_d k_d &\geq 0 \\ K_i G_s G_d k_p - K_d &> 0\end{aligned} \quad (35)$$

or

$$\begin{aligned}k_d &\geq 0 \\ k_p &> \frac{K_d}{K_i G_s G_d}.\end{aligned} \quad (36)$$

5.5 Power supply voltage constraint

Power supply voltage is often restricted by the peripheral equipment used, such as the current driver. In general, the larger the voltage, the larger the coil current and higher the current slew rate are. Because of our premagnetization scheme and low level of mass imbalance, we consider the following differential equation (see Fig. 6):

$$L_c \frac{dI}{dt} + R_c I = V_b + V_c \sin(\omega t). \quad (37)$$

Here I is the coil current, V_b the bias voltage, V_c the voltage for the control current, L_c the coil inductance, R_c the coil resistance, and ω the rotating speed. The induction of an EM coil is defined as

$$\begin{aligned}L_c &= \frac{2N \cdot \phi}{I} \\ &= \frac{2\mu_0 A_g N^2}{G}\end{aligned} \quad (38)$$

In Eq. (37), we ignore the phase lag due to the controller and current driver. The steady-state current $I_{ss}(t)$

Fig. 6 Simple R-L electronic circuit

of Eq.(37) is

$$I_{ss}(t) = \frac{V_b \tau}{L_c} + B \cos(\omega t - \theta), \quad (39)$$

where

$$B = \frac{V_c / \omega}{L_c \sqrt{1 + \left(\frac{1}{\tau \omega}\right)^2}}$$

$$\tau = \frac{L_c}{R_c}, \quad \theta = -\tan^{-1} \frac{1}{\tau \omega}.$$

Due to the premagnetization scheme (Eqs.(3), (6), (11)), we have

$$V_b = I_b \cdot R_c + I_{stat} \cdot R_c$$

$$V_c = I_{dyn} \cdot (L_c \omega) \cdot \sqrt{1 + \left(\frac{1}{\tau \omega}\right)^2}. \quad (40)$$

In order to reach the required control current at a specific angular speed, the voltage across the two EM coils V_p must satisfy the following constraint:

$$V_p \geq V_b + V_c. \quad (41)$$

Thus V_p is also related to the required angular speed ω , the coil inductance L_c , and the time constant τ .

5.6 Other constraints

In addition to the constraints discussed above, we often encounter other important restrictions, such as the copper wire capacity constraint, linear control current constraint, maximum diameter of stator constraint, and pole width constraint. We express these as follows

(a) Copper wire capacity constraint

$$I_{max} \leq \frac{\pi \cdot d^2}{4} \cdot \zeta \quad (42)$$

where ζ is the maximum current density in the copper wire under normal conditions (6 A/mm²).

(b) Linear control current constraint

$$i_{c, max} \leq G_s \cdot k_p \cdot G_d \cdot (Y \cdot G)_{max} \quad (43)$$

(c) Maximum diameter of stator constraint

$$D_{stator} \leq D_{max}$$

$$D_{stator} = d_p + 2(s + l + 1.1 W_m), \quad (44)$$

where D_{stator} is the outer diameter of the stator and D_{max} is the maximum diameter of the stator.

(d) Pole width constraint

$$0.6 W_m \leq (0.5 d_p \cdot \sin \alpha - 1.5 G \cdot \cos \alpha) \quad (45)$$

This constraint ensures that the poles do not interfere with each other by requiring that the distance between the fringes of the pole faces exceed the air-gap size (here we set the constraint to three times the air-gap size).

6. Performance of Magnetic Bearings

A MB is composed of two basic components: the actuator and the controller. In this research, we focus on designing the actuator of a MB and utilizing an optimum design method to deal with the design problem. To apply MBs effectively, their performance limitations must be understood. The performance

limitations of MBs are quite different from those of conventional bearings. These limitations are due in part to the inherent nonlinearity of the device and in part to its use of electrical circuits. Two important limitations are the force slew rate limitation and the stiffness sensitivity to large displacement.

6.1 Force slew rate limitation

The bearing force slew rate is limited because the coils are inductive and the allowable power supply voltage to the driving amplifier is limited. To investigate how fast the bearing force changes with varying current, y is held constant (zero). Thus the slew rate of the magnetic force is

$$\left. \frac{dP_{y, brg}}{dt} \right|_{y=0} = K_i \frac{di_y}{dt}. \quad (46)$$

Considering Eqs.(39) and (46) simultaneously, we obtain

$$\max \left[\left. \frac{dP_{y, brg}}{dt} \right|_{y=0} \right] = K_i \cdot \frac{V_c}{L_c \sqrt{1 + \left(\frac{1}{\tau \omega}\right)^2}}. \quad (47)$$

In order to determine how the force slew rate limitation affects design work, Eq.(47) is converted by way of Eqs.(8), (14), (17), and (38) into

$$\max \left[\left. \frac{dP_{y, brg}}{dt} \right|_{y=0} \right] = \frac{\beta}{(\beta + 1)} \cdot \frac{2 B_{sat} \cos \alpha}{c \mu_0 N} \cdot \frac{V_c}{\sqrt{1 + \left(\frac{1}{\tau \omega}\right)^2}}. \quad (48)$$

In Eq.(48), the two important design parameters for determining the required force slew rate limitation are the power supply voltage, V_c , and the number of coil turns, N .

6.2 Displacement sensitivity

The softening of the bearing stiffness becomes more apparent as the shaft approaches the radial clearance. This phenomenon is a consequence of the nonlinear nature of Eq.(7). Because of the direct relation between this phenomenon and the stiffness of the MB, we consider the following scheme of feedback control (i.e., as a simple spring without damping):

$$i_y = -k_0 y + I_{stat}, \quad (49)$$

where k_0 is the feedback gain and I_{stat} is the bias current for the rotor's weight. The displacement stiffness of the closed-loop rotor bearing system is a derivative of the bearing force with respect to the rotor displacement:

$$K_{y, cl} = 2 \mu_0 A_g N^2 \cos \alpha$$

$$\times \left[\frac{(k_0 G - I_b \cos \alpha - I_{stat} \cos \alpha)(I_b - k_0 y + I_{stat})}{(G - y \cos \alpha)^3} \right.$$

$$\left. + \frac{(k_0 G - I_b \cos \alpha + I_{stat} \cos \alpha)(I_b + k_0 y - I_{stat})}{(G + y \cos \alpha)^3} \right]. \quad (50)$$

For purposes of comparison with $K_{y, cl}$, we define the nominal stiffness, K_{nom} , which is the displacement stiffness of the closed-loop bearing system when the

rotor is at a concentric position. That is,

$$K_{nom} = \frac{4\mu_0 A_g N^2 \cos \alpha}{G^3} [I_b(k_0 G - I_b \cos \alpha) - I_{stat}^2 \cos \alpha] \quad (51)$$

Using Eqs. (50) and (51), we define the displacement sensitivity DS as

$$DS = \frac{|K_{y,cl} - K_{nom}|}{K_{nom}} \quad (52)$$

DS expresses the rate of change of the difference between the displacement stiffness $K_{y,cl}$ and the nominal stiffness K_{nom} with respect to the nominal stiffness K_{nom} . In our MB design, the largest DS occurs at $y=0.5G$, because the clearance of the back-up bearing is half the air-gap size.

7. Optimum Design of Magnetic Bearings

In order to keep the design problem manageable, the optimization should focus on the most important dimensions and design variables of the bearing. Thus, except for dimensions related to the useful flux at the pole face, geometric dimensions are left for further detailed design. The design objectives for the electromagnetic bearings are as follows

Objective 1: the force slew rate limitation should be maximized.

Objective 2: the displacement sensitivity at large displacement should be minimized.

This optimum design is a multiobjective optimization problem. Table 1 lists the design data used here. The design variables we have chosen are generally parameters which have a direct relation to the dimensions and coils. Besides these, the proportional gain of the controller is also chosen as a design variable because

Table 1 Design data of the EMB

Design data	Value
Maximum voltage across EM circuit (V_p)	40 V
Maximum outer diameter of stator (D_{max})	140 mm
Saturation magnetic flux density (B_{sat})	1.2 Tesla
Size of air gap (G)	0.8 mm
Bias indicator (β)	6
Half of angle formed by two poles of an EM (α)	22.5°
Thickness of sleeve (W)	5 mm
Thickness of stator (t_s)	30 mm
Diameter of shaft (D_{shaft})	20 mm
Sensor gain (G_s)	2000 V/m
Amplifier gain (G_d)	0.8 A/V
Rating speed of rotation (ω)	10000 rpm
Thickness of the fringes of pole piece (s)	1.5 mm
Ratio of A_b to A_c (c)	1.2

it has an important effect on the stiffness of the bearing. In summary, there are six design variables in our model of optimum design, as Table 2 shows.

On the basis of the previous analysis⁽¹⁴⁾, an adequate optimum design problem can be formulated as follows:

$$\begin{aligned} &\text{minimum } f_i(\underline{x}) \quad i=1, \dots, m \\ &\text{subject to} \\ &g_i(\underline{x}) \leq 0 \quad i=1, \dots, n \\ &x_i^l \leq x_i \leq x_i^u \quad i=1, \dots, p, \end{aligned}$$

where m , n , and p are the numbers of objective functions, constraints, and design variables, respectively. In this EMB design problem, we have $m=2$, $n=9$, and $p=6$.

8. Design Result

Three typical multiobjective optimization techniques—goal programming, compromise programming, and the surrogate worth trade-off method⁽¹⁵⁾ are available in the literature. We adopt goal programming as the method for searching for the optimum solution. The finite difference method is used to calculate the gradient of the objective and constraint functions. Table 2 shows the design result for the EMB. The results show that the constraints on the flux density, the maximum diameter of the stator, the winding space limit, and the copper wire capacity are active. It causes no surprise that the flux density constraint is active in our EMB design, because the two significant factors which affect the force slew rate, that is, the number of turns of winding coil around a pole and the voltage across EM coils, are the major reasons that

Table 2 Optimum design result of the EMB

Design variables:				
D.V.	I.V.	Final design	L.B.	U.B.

W_m (m)	1.20000e-02	1.40905e-02	1.00000e-02	1.60000e-02
N (turns)	3.50000e+02	3.99000e+02	1.00000e+02	1.00000e+02
I (m)	1.00000e-02	2.18500e-02	5.00000e-03	6.00000e-02
I_{max} (A)	1.35000e+00	1.38717e+00	5.00000e-01	1.60000e+00
d (m)	4.00000e-04	6.00000e-04	3.00000e-04	6.50000e-04
k_p	1.00000e+00	1.72723e+00	1.00000e+00	2.00000e+00

Subject to 9 constraints:				
No.	Constraint (normalized)	Final Value	Active/inactive	

g_1	Flux density	-1.46830e-04	active	
g_2	Load capacity	-2.74965e+00	inactive	
g_3	Diameter of stator	-6.24337e-08	active	
g_4	Pole width	-1.49968e-01	inactive	
g_5	Winding space	-1.06526e-07	active	
g_6	Copper wire capacity	-2.64248e-10	active	
g_7	Proportional gain	-9.99514e-01	inactive	
g_8	Linear control current	-1.23049e-01	inactive	
g_9	Power supply voltage	-2.27062e-01	inactive	

Minimum cost:				
objective 1 (Force slew rate limitation)			= 1.99399e+05	
objective 2 (Displacement sensitivity)			= 2.51428e-03	

saturation occurs. That is, in general, a larger number of turns of winding coil or a larger voltage across the EM coils will cause saturation to occur more easily. The geometric limitations for the winding space and the outer diameter of the EMB are inherent

to the design of the system, so the restrictions are difficult to eliminate physically.

Figures 7(a) to 7(f) show the values of the design variables during the optimization process when we optimize the objectives of force slew rate

(a) pole width

(b) turns of winding coil

(c) pole piece length

(d) maximum allowable current

(e) diameter of copper wire

(f) proportional gain of PD controller

Fig. 7 Values of design variables during the optimization process

Fig. 8 Values of objective functions during the optimization process

limitation and displacement sensitivity using goal programming (i.e., by finding a compromise between the two objectives). From the results we can see that the design variables of pole width, number of turns of winding coil, and pole piece length are closely related to the force slew rate; the design variable of the proportional gain of the PD controller affects the displacement sensitivity directly; and the other design variables, such as the maximum allowable current and the diameter of the copper wire, are closely related to both objectives. The proportional gain of the PD controller affects the stiffness of the EMB directly, so it is dominant for the objective of displacement sensitivity.

Figure 8 shows the values of the objective functions during the optimization process. The force slew rate limitation increases quickly and monotonically to its optimum value, while the value of the displacement

sensitivity exhibits fluctuation. The value of the proportional gain of the PD controller fluctuates also (see Fig. 7(f)). This also shows the close relation between the displacement sensitivity and the proportional gain of the PD controller.

9. Conclusion

The configuration of an EMB has been analyzed using lumped models. From analysis of the entire magnetic circuit, we see that, in the linear region, the flux in an air gap due to the perpendicular coil current is less than 2% of the flux of the exciting coil current. That is, we can ignore the coupling phenomenon in the linear region. For optimum design of the EMB, we use the force slew rate limitation and the displacement sensitivity at large rotor displacement as the objective functions. We also impose practical restrictions such as a flux density constraint, load capacity constraint, maximum diameter of stator constraint, pole width constraint, winding space limit, copper wire capacity, proportional gain constraint, and linear control current constraint on our design. The design result shows that the design variables of the width of the pole piece, number of turns of winding coil, and pole piece length have an important effect on the force slew rate of the bearing; the proportional gain of the PD controller dominates the displacement sensitivity; and the maximum allowable coil current and the diameter of the copper wire affect both objective functions. In conclusion, this study provides a systematic method for determining the geometric dimensions of EMBs. By considering the performance limitations and design constraints, we had a better design than that which can be obtained using the conventional trial-and-error method.

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