Optimal Shape Design of Engine Connecting Rod with Special Lumping-Mass Constraint*

Hsing-Juin LEE**, Ming-Chih LIN**, Yeh-Liang HSU*** and Koek-Koe SOON**

In a typical reciprocating internal combustion engine, the connecting rod (conrod) is generally working under high-speed rotation and heavy dynamic loading, with relatively more complicated dynamics behavior than other components. The key issue in this study is to achieve an optimal design shape of the conrod not only with the optimum stress distribution, but also under the special constraint of accurate lumping-mass at piston pin and crankpin centers for optimal dynamic characteristics. Herein, the varied-mass lumping method, the finite element method and the optimization theory are combined to achieve the innovative design objective of dual optimization. This optimization is aided by the sensitivity analysis technique, the finite difference scheme, and the sequential secondary programming method to achieve a conrod shape design with fully stressed boundary curve and desirable dynamics features.

Key Words: Engine conrod, special dynamics constraint, shape optimization.

1. Introduction

For a typical reciprocating mechanism, vibrations are inevitably induced due to an imbalance in the mechanism. The dynamic behavior of a connecting rod (conrod) is relatively more complicated than other engine parts. In conventional treatment, the two-particle lumping-mass method\(^{[1,2]}\) is commonly used in an attempt to lump conrod mass at the crankpin and piston-pin centers; however, this method is usually inaccurate in a dynamical equivalence sense. In order to overcome this difficulty, an ingenious varied-mass lumping method fused with the percussion concept proposed by Lee and Lin\(^{[3]}\) is employed to integrate the dynamics analysis and design of an internal combustion engine (ICE) in a highly mutually beneficial sense. In this study, a special equation for accurate mass-lumping is taken as a constraint in the optimal shape design of the conrod to achieve optimal stress and dynamics conditions.

In the following procedure, a typical conrod is first meshed for finite element analysis, then the horizontal and vertical loads of crankpin and piston pin and the inertia force of each finite element are calculated for stress analysis with the aid of ANSYS software. Moreover, shape variables, objective function and special constraint condition are considered in the sensitivity analysis\(^{[4,5]}\). Finally, optimum design\(^{[7,6]}\) aided by MOST software\(^{[9]}\) is employed to achieve a fully stressed state for the conrod boundary shape. Usually, stress is reduced by varying the boundary shape defined by continuous curves, such as boundary curvature, B-spline, and boundary points polynomial\(^{[2,11,12]}\). Accordingly, the sensitivity analysis technique, the finite difference scheme, and the sequential secondary programming methods are employed to achieve the dual goal of optimal stress and dynamics features.


The piston kinematics of ICE as shown in Fig. 1 is briefly reviewed first. The center of piston pin A can

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Herein, the varied-mass lumping method (VLM) will be briefly reviewed\(^{20}\) to accommodate later processing. In an attempt to lump the mass of conrod into two particle masses \(m_1\) and \(m_2\), so that they are dynamically equivalent to the original conrod, it is necessary to satisfy the following conditions.

1. The total mass remains the same.
   \[
   m = m_1 + m_2. \tag{4}
   \]

2. The center of gravity will not change its position.
   \[
   m_1 \bar{l}_1 = m_2 \bar{l}_2. \tag{5}
   \]

3. The mass moment of inertia remains the same.
   \[
   m_1 \bar{l}_1^2 + m_2 \bar{l}_2^2 = I_\text{ce} = m(k_0 \bar{r}_0^2). \tag{6}
   \]

Here \(m_1\) and \(m_2\) are masses lumped at the piston pin center and the crankpin center, respectively, and their distances to center of gravity are \(l_1\) and \(l_2=\sqrt{(k_0 \bar{r}_0^2)}\) is the radius of gyration of conrod about its center of gravity.

The above three equations usually cannot be satisfied simultaneously with two variables \(m_1\) and \(m_2\). However, since a fixed \(m\) in Eq. (4) is not the supreme design objective for optimal dynamics performance, if \(m\) is another variable, it can be easily shown that as long as the mass distribution of conrod is designed to satisfy the following equation, this conrod can be accurately represented by two lumping-masses at the two pin positions in a dynamical equivalence sense\(^{20}\)

\[
\bar{l}_1 l_2 = (k_0 \bar{r}_0)^2. \tag{7}\]

In fact, Eq. (7) suggests that the crankpin center and piston pin center are the center of percussion and the center of oscillation mutually and exchangeably. Then \(m_2\) in pure rotary motion can be dynamically balanced together with the crankshaft. The \(m_1\) is in a pure translational motion in the vertical direction, consequently the engine as a whole will not endure any more horizontal dynamic excitation force due to the engine itself, and the dynamics analysis is considerably simplified and enhanced. As such, the dynamics-analysis and the design of reciprocating mechanism are closely integrated in a highly mutually beneficial sense. Thus, in this study, Eq. (7) can be considered as a special constraint for optimal shape design of a conrod with optimal stress distribution.

We can use the VLM method to accurately lump the mass of conrod points \(A\) and \(B\). The vertical component \(F_\text{av}\) and horizontal component \(F_\text{ah}\) of piston pin \(A\) can be easily calculated as follows:

\[
F_\text{av} = -P + (M_b + m_1) \left[ \bar{r}_1 + (\bar{r}_1 \cdot \cos \phi_1 - \bar{r}_1 \cdot \sin \phi_1) \right] + (\bar{r} - \bar{r}_1 \cdot \cos \phi_1 - \bar{r}_1 \cdot \sin \phi_1), \tag{8}
\]

\[
F_\text{ah} = F_\text{av} \tan (\phi_1) \tag{9}
\]

Here \(P\) represents gas pressure force in the combustion chamber, which acts on the piston as pressure shown in Fig. 2\(^{[14]}\), and \(M_b\) is the mass of the piston. In this manner, the loading state of conrod can be easily determined for later finite element analysis with the
aid of ANSYS software.

3. **Curvature Function of Boundary Shape**

Shape optimization determines the configuration of a mechanical part by minimizing its mass while subject to certain geometric and mechanical constraints such as stress, deformation and natural frequency. Nevertheless, most constraint conditions cannot be accurately expressed by the relationships of boundary shape variables. In the two-dimensional case, suitable boundary curves are used in finite element node locations and geometric function for design analysis. A space curve can be fully described by the two equations of curvature and torsion. Curvature of a point in a plane curve can be written as

$$\kappa = \frac{d\theta}{ds},$$

(10)

where $\theta$ represents the angle between the tangent line and $x$-axis, while $s$ is the arc length. Since most mechanical properties deal with $x$-$y$ coordinates, it is more convenient to replace $\kappa(s)$ by $\kappa(x)$. Then, the curvature function in the $x$-$y$ plane is

$$\kappa(x) = \frac{d^2y/dx^2}{[1+(dy/dx)^2]^{3/2}},$$

(11)

and the possible boundary conditions are

- at $x = x_0$: $y = y_0$, $dy/dx = (dy/dx)_0$;
- at $x = x_f$: $y = y_f$, $dy/dx = (dy/dx)_f$.

(12)

It can be seen that the analytical solution of Eq.(11) with two boundary conditions is difficult to obtain. Therefore, the solution of $y$ can be obtained by a simple numerical integration method corresponding to $\kappa(x)$ and these two boundary conditions.

4. **Relationship between Stress and Curvature of Two-Dimensional Boundary Points**

For a two-dimensional stress problem, the stress value at a boundary point is determined by two factors: nominal stress value and stress concentration effect. Nominal stress value depends on material used and load applied, and generally, nominal stress can be reduced by enlarging the size of the mechanical part concerned. Moreover, stress concentration essentially depends on the smoothness of local geometry, i.e., any abrupt change in geometry can lead to a high stress concentration in a particular region. The stress concentration factor can be defined as

$$K_T = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}},$$

(13)

where $\sigma_{\text{max}}$ represents the true maximum stress value in the concerned region, and $\sigma_{\text{nom}}$ is the nominal stress value. Boundary smoothness is necessary for successful optimization under most conditions. Researchers have used a B-spline or Bezier curve to depict the boundary concerned to obtain a smooth surface. If a clear relationship between the effect of stress concentration and curvature of boundary curve can be determined, then we can apply it to satisfy the smoothness condition for optimization.

It has been proven in various experiments that the increase in stress concentration factor is accompanied by the decrease in radius of notches or fillets (reciprocal of curvature). In Gao’s recent research on the analytical solution of slightly undulating plane stress concentration, the above relationship has also been proven as follows.

Considering the case of a single wave perturbation on a flat surface as shown in Fig. 3(a), its surface profile can be written as

$$y(x) = \begin{cases} A & |x| > \lambda/2 \\ -A \cos(2\pi x/\lambda) & |x| < \lambda/2 \end{cases}$$

(14)

In Gao’s brief description for plane stress of $|A| < |\lambda|$, the surface stress reaches a maximum at the wave trough ($x=0$) with stress concentration factor

$$K_T = 1 + 14.813(A/\lambda).$$

(15)

From Eq.(11), the curvature $\kappa(x)$ at $x$ can be expressed as

$$\kappa = (2\pi/\lambda)^2 A.$$

(16)

Substituting Eq.(16) into Eq.(15), we obtain

$$K_T = 1 + 2.3576 \sqrt{\kappa_0 A}.$$  

(17)

It can be known from Eq.(17) that for two-dimensional boundary points, the stress concentration factor increases with increasing associated curvature.

5. **Fully Stressed Design and Stress Pointer**

Fully stressed design is widely used as an optimal criterion in structure optimization, and its goal is to find the smallest standard of fully stressed conditions under a given load. Considering a general problem of
two-dimensional shape design with stress flow in the x-direction (Fig. 4), the boundary curve \( \Gamma \) at the interval between \( x = x_0 \) and \( x = x_f \) will be optimized. The objective is to minimize the area enclosed subject to the constraint of tangential stress \( \sigma \), which must be less than or equal to the allowable stress \( \sigma_0 \) along \( \Gamma \). The problem is formulated as follows:\(^{19}\):

Minimize \( A_{req} = \int_{x_0}^{x_f} y \, dx \),

subject to \( \sigma(y, x) \leq \sigma_0 \) for \( x_0 \leq x \leq x_f \),

\[
(18)
\]

According to the discussion in section 4, tangential stress of a two-dimensional boundary point is fully dependent on the value of \( y \) at that point, and it will not ensure a decrease in stress if we merely increase the size concerned without changing the curvature of that point. Therefore, when the positions of two end points are fixed, increasing the value of \( x \) will decrease the curvature as well as stress \( \sigma \), and vice versa. Applying this relationship between stress and curvature, the fully stressed boundary in the curvature region can then be determined.

Moreover, as shown in Fig. 5, \( S \) is a curve of stress distribution, and the stress point \( \Delta \sigma_{obs} \) is defined as the proximity of the curve of stress distribution \( S \) to the optimal curve of stress distribution \( S_{obj}^{(t)} \). Therein, \( S_{obj} \) is a straight line of definite value at \( \sigma = \sigma_0 \), and

\[
\Delta \sigma_{obs} = \frac{1}{\sum \Delta x_i} \cdot \left( \sum (\sigma_i - \sigma_0) \Delta x_i \right), \quad i = \{1, 2, 3, \ldots, n\}.
\]

(19)

It can be learned from Fig. 5 that \( \Delta \sigma_{obs} \) is the sum of areas \( A, B \) and \( C \), divided by total length, and its unit is the same as that of stress. Also \( S \) approaches \( S_{obj} \) as \( \Delta \sigma_{obs} \) approaches zero.

6. Optimal Shape Design

A general mathematical model for shape optimization:

Find minimum \( F(S) \),

subject to

\[
\begin{align*}
\gamma_i(S) & \leq 0 \quad i = 1, NBC; \\
S_k^l & \leq S_k \leq S_k^u \quad k = 1, NSV; \\
\sum d_j S_j & \leq d_j \quad j = 1, NGC,
\end{align*}
\]

where

\( F \): Objective function

\( \gamma \): Design constraint

\( S \): A series of variable vectors which define structural shapes

\( S_k^l, S_k^u \): Upper and lower limits of shape variable

\( NBC \): Number of constraints

\( NSV \): Number of shape variables

\( NGC \): Number of geometric constraints.

For such a problem of shape optimization, the boundary of conrod is divided into \( n \) sections, in order to find a series of curvatures, \( \kappa_i, i = 1, 2, 3, \ldots, n - 1 \), as design variables. Using Eqs. (11) and (12), we can determine the coordinates of the boundary point \( (x_i, y_i) \). Subsequently, stress point \( \Delta \sigma_{obs} \) is taken as an objective function to be minimized subject to the accurately lumping mass constraint\(^{10}\). The mathematical model is as follows:

To find \( \kappa = (\kappa_1, \kappa_2, \kappa_3, \ldots, \kappa_{n-1}) \),

minimize

\[
\Delta \sigma_{obs},
\]

subject to

\[
\kappa_i(x) = \frac{d^2 y}{d x^2} \bigg| \left( 1 + \left( \frac{d y}{d x} \right)^2 \right)^{3/2},
\]

(21)

with boundary conditions and

\[
\frac{h l_x^2}{l_x} - 1 = 0.
\]

(22)

Herein, Eq. (21) is the governing equation which must be satisfied at each instant, and Eq. (22) is the equation of accurate mass lumping which is the prime condition of dynamics optimization for a conrod. The optimization proceeds with aid of MOST software\(^{10}\).

The procedure of shape optimization for conrod is shown as follows.

Input data:

- Initialize a series of boundary curvatures \( \kappa = (\kappa_1, \kappa_2, \kappa_3, \ldots, \kappa_{n-1}) \)
- Angle, angular velocity, angular acceleration of crankshaft
- Pressure force of combustion chamber
- Compute angle, angular velocity and angular acceleration of conrod
- Compute \( (x, y) \)
• Compute inertia force, \( F_{AH} \) and \( F_{AV} \)
• Compute \( l_1, (k_0)_{(x)} \)
• Input data for ANSYS program
• ANSYS programming
• Read out the data output from ANSYS and use in MOST analysis
• Compute cost value \( \sigma_{min} \) and constraint value \( l_1/b_i(k_0)_{(x)}r_{e}-1 \)
• MOST programming
• Produce new design variable, \( k_i \).

Iterative portions:
• New design variable, \( \kappa \)
• Compute \((x_i, y_i)\)
• Compute inertia force, \( F_{AH} \) and \( F_{AV} \)
• Compute \( l_i, (k_0)_{(x)} \)
• Input data for ANSYS program
• ANSYS programming
• Read out the data output from ANSYS for use in MOST analysis
• Compute cost, \( \sigma_{min} \) and constraint value \( l_1/b_i(k_0)_{(x)}r_{e}-1 \)
• MOST programming.

This iterative process will be repeated until optimization is reached.

7. Numerical Results and Discussion

Associated research on torque arm has been described by Ding\(^{11}\), Braibant and Fleury\(^{49}\), Haftka and Grandhil\(^{12}\), Rajan and Belegundu\(^{17}\), Yang et al.\(^{18}\) and Bennett and Bitkiri\(^{19}\). The initial design and load of conrod are shown in Fig.6, and it is constrained around the circumference of the larger hole. This conrod model is different from a conventional conrod which can be separated into two parts, rod and cap. For the sake of simplicity in the following optimization processing, we neglect the complex behavior at the junction of the rod and cap, such as deformation and stress concentration at the corner of bolt head seat surface. The objective function and governing equation of conrod are Eqs.(20) and (21), respectively. Further, the constraint of tangential stress \( \sigma \) must be lower than the allowable stress \( \sigma_{o} \). Next, we minimize the area under boundary curve \( \Gamma \) between \( C \) and \( D \) points, and finally proceed with the shape optimization with the special lumping mass constraint.

For the purpose of unification, a definite working condition and data of conrod model are selected and listed as follows,

- Length of crankshaft, \( r = 0.05 \) m
- Length of conrod, \( l = 0.2 \) m
- Mass of piston, \( M_p = 0.68 \) kg
- Diameter of piston, \( d = 0.076 \) m
- Angle of crankshaft, \( \phi_1 = 90^\circ \)
- Angular velocity of crankshaft, \( \phi_1 = 3000 \) rpm
- Pressure force, \( P = 4990 \) N
- Material of conrod: AISI 1050
- Young's modulus, \( E = 207.5 \) GPa
- Yielding strength, \( S_\sigma = 0.8 \) GPa
- Allowable stress, \( \sigma_0 = 0.8 \) GPa
- Density, \( \rho = 7810 \) kg/m\(^3\).

Conrod model (1) : We first consider the initial design of conrod in Fig.6. Its finite element model is shown in Fig.7. The boundary between points \( C \) and \( D \) is divided into ten line segments, each of which has a length of 0.02 m. Then, we proceed with shape optimization of each segment. By using ANSYS programming, we obtain the initial stress distribution of conrod in Fig.8. After eight iteration with MOST analysis, the optimal shape design is obtained as shown in Fig.9, and its stress distribution is shown in Fig.10. The numerical results of three representative iterations are as follows.

- Number of design variables = 10
- Number of objective functions = 1
- Number of equality constraints = 1

![Fig. 6 Initial design of conrod model (1)](image)

![Fig. 7 Finite element model for initial design of conrod model (1)](image)

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Number of inequality constraints = 0
Maximum number of iterations = 25
Gradient calculation indicator = 1
Tolerance in constraint violation for optimum condition = 1.000e-02
Convergence parameter value = 1.000e-02
Delta for finite difference (gradient) calculation = 1.000e-01
Acceptable change in cost value = 1.000e+00
Upper limit of design variables = 1.100e+00
Lower limit of design variables = -1.100e+00

Iteration No. = 1


Cost value, $e_{\text{cost}} = 0.27878 + 0.03$ M/Pa
Constraint value = $2.24780 + 0.00$
Maximum violation = $2.24780 + 0.00$
Convergence parameter value = $1.000e+00$

Iteration No. = 7


Cost value, $e_{\text{cost}} = 0.21379 + 0.03$ M/Pa
Constraint value = $2.14418 + 0.00$
Maximum violation = $2.14418 + 0.00$
Convergence parameter value = $1.4692 + 0.01$

Iteration No. = 8


Cost value, $e_{\text{cost}} = 0.21379 + 0.03$ M/Pa
Constraint value = $2.14418 + 0.00$
Maximum violation = $2.14418 + 00$
Convergence parameter value = $1.4692 + 01$

In the last iteration, the line search limit is exceeded for the RQP method, and the resulted cost value may be taken as optimum solution. Further, the variations of cost value, mass of conrod and constraint value are shown in Figs. 11-13, respectively. It can be realized from Fig. 10 that the final design has reached the fully stressed state, and the cost value in Fig. 11 theoretically must be close to zero. The off-zero result is due to the fact that the stresses near two
end points, C and D, are unable to approach the allowable stress. Figure 12 shows that in the process of iteration, the mass of conrod is reduced from 0.235265 kg to 0.209538 kg. It can be concluded that in the investigation of conrod, its boundary can achieve a fully stressed state by applying the optimal theory as proposed. Furthermore, the initial design for a thin plate is proposed without considering the accurately lumping mass constraint first, and the optimization takes place subject to this constraint by adjusting the external shape of the conrod in a two dimensional-plane with uniform thickness. The results show that the constraint value is improved, but still not fully satisfied. Therefore, we propose another conrod model to solve this problem.

Conrod model(2): Here, we consider the initial design of conrod in Fig. 14 which is set to satisfy the accurately lumping mass constraint as much as possible, and then we perform the same procedure of shape optimization as described in last section.

The initial stress distribution of conrod is shown in Fig. 15, and after 11 iterations, the optimum design is obtained as shown in Fig. 16. Its stress distribution is shown in Fig. 17, also the variations of objective value, mass of conrod and constraint value are shown in Figs. 11–13, respectively. The numerical results for three representative iterations are as follows.

Number of design variables = 10
Number of objective functions = 1
Number of equality constraints = 1
Number of inequality constraints = 0
Maximum number of iterations = 25
Gradient calculation indicator = 1
Fig. 16 Finite element model for optimal design of conrod model (2)

<table>
<thead>
<tr>
<th>Iteration No.</th>
<th>1</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>m = 0.265411</td>
<td>( \phi = 0.5870416 ), ( (b)_{xy} = 0.0836010 ) m</td>
<td>m = 0.265411</td>
<td>( \phi = 0.5870416 ), ( (b)_{xy} = 0.0836010 ) m</td>
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<tr>
<td>( a_1 ) = -1.100000 - 01 rad/m</td>
<td>( a_1 ) = -1.100000 - 01 rad/m</td>
<td>( a_1 ) = -1.100000 - 01 rad/m</td>
<td>( a_1 ) = -1.100000 - 01 rad/m</td>
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<td>( a_4 ) = -1.100000 - 01 rad/m</td>
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<td>( a_4 ) = -1.100000 - 01 rad/m</td>
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<tr>
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<td>( a_5 ) = 3.090000 - 01 rad/m</td>
<td>( a_5 ) = 3.090000 - 01 rad/m</td>
<td>( a_5 ) = 3.090000 - 01 rad/m</td>
</tr>
<tr>
<td>( a_7 ) = 1.100000 - 01 rad/m</td>
<td>( a_7 ) = 1.100000 - 01 rad/m</td>
<td>( a_7 ) = 1.100000 - 01 rad/m</td>
<td>( a_7 ) = 1.100000 - 01 rad/m</td>
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<tr>
<td>( a_9 ) = 9.994410 - 01 rad/m</td>
<td>( a_9 ) = 9.994410 - 01 rad/m</td>
<td>( a_9 ) = 9.994410 - 01 rad/m</td>
<td>( a_9 ) = 9.994410 - 01 rad/m</td>
</tr>
<tr>
<td>( a_{10} ) = 3.090000 - 01 rad/m</td>
<td>( a_{10} ) = 3.090000 - 01 rad/m</td>
<td>( a_{10} ) = 3.090000 - 01 rad/m</td>
<td>( a_{10} ) = 3.090000 - 01 rad/m</td>
</tr>
</tbody>
</table>

Cost value, \( e_{ac} \) = 0.250371 + 03 MPa
Constraint value = 4.085500 - 01
Maximum violation = 4.085500 - 01
Convergence parameter value = 3.29371 + 05

Cost value, \( e_{ac} \) = 0.105501 + 02 MPa
Constraint value = 3.413700 - 01
Maximum violation = 3.413700 - 01
Convergence parameter value = 8.2500e + 03

In the last iteration, the line search limit is exceeded for the RQP method, and the resulted cost value may be taken as optimum solution. It is obvious that the accurately lumping mass constraint has been markedly improved from Fig.13, therefore, the conrod shape has been reformed to achieve the dual optimization goal of both stress state and dynamics characteristics.

8. Conclusion

The number of engine connecting rods currently in use worldwide has exceeded one billion; thus, the importance of their optimal design is unspoken. In this study, an original design criterion of an accurately lumping mass is used as a constraint for shape optimization, so as to achieve an optimal condition for both stress state and dynamics characteristics. Accordingly, we develop a preliminary conrod model (2) described in section 7 with Eq. (7) in mind, and then proceed with the optimization steps described in section 6. This conrod model (2) approach is a suitable scheme for achieving the dual optimization.
objective. It can be recognized from Figs. 11-17 that conrod model (2) can efficiently attain reasonably accurate results for dual optimization. Incidentally, the buckling effect is also an important issue affecting the conrod shape design; thus, it can be considered as another constraint for future detailed shape optimizations. Moreover, different design variables, boundary conditions and other variations may be considered in the optimization process as appropriate. Hopefully, the two-facet optimization scheme described herein will be useful for designing a connecting rod with improved overall performance.

References