Robot Design for Stair Navigation*

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Among the several designs of mobile robots, the wheel design is unfavorable for navigation of stairs or over obstacles. The leg design and the creeping design demand very intricate control systems and are not efficient on flat surfaces. The caterpillar design is unreliable and unstable in stair navigation. In the arm-wheel design, the sensor which discriminates between stairs and flat surfaces and the mechanism which alternates between wheel-running and arm-creeping modes are complicated. In this paper, a new type of arm-wheel design for a mobile robot enabling it to navigate stairs is developed, which requires no sensor for discrimination between stairs and flat surfaces of stairs and no mechanism for alternating between wheel-running and arm-creeping modes. Since no sensor is required, the reliability in an extreme environment is high. Moreover, since it has no intricate discrimination mechanism, it can also be applied to a wheelchair, for example, which is not powered. The author has confirmed through experiments using a stair-navigating robot prototype that the mechanism developed here functions as intended.

Key Words: Moving Robot, Machine Element, Gear, Friction, Materials Handling Equipment, Stairs, Environmental Engineering, Human Engineering, Mechanics

1. Introduction

There are several designs for mobile robots. Among these, the wheel design is unfavorable for accommodation of level differences, or for navigation of stairs or over obstacles. The leg design demands very intricate control systems and is not efficient on flat surfaces. The creeping design has similar disadvantages to the leg design. The caterpillar design is unreliable and unstable in stair navigation^{(1),(2)}. Besides these, the arm-wheel design has been developed⁽¹⁾, which has sets of rotating arms and wheels attached to the ends of arms, and facilitates navigation of flat surfaces by wheel-running while navigating stairs by arm-creeping. However, in the previous arm-wheel design, the sensor which discriminates between stairs and flat surfaces and the mechanism which alternates between wheel-running and arm-creeping are complicated. Furthermore, a

mobile mechanism which has sets of a leg and a wheel attached to the end of the leg has been developed⁽²⁾. This mechanism facilitates navigation on flat surfaces using the wheels while stairs are navigated by extension and retraction of the legs. Although this mechanism has advantages in that it is relatively simple and the body remains horizontal, a sensor to discriminate between stairs and flat surfaces and a mechanism to alternate between floor-running and stair navigation are also required.

In this work, a mechanism for a mobile robot enabling it to navigate stairs is developed. This is a new type of arm-wheel design, in which the change between wheel-running and arm-creeping occurs as a purely dynamical phenomenon, and requires no sensor for discrimination between stairs and flat surfaces and no mechanism for alternating between wheel-running and arm-creeping modes. Since no sensor is required, the reliability in an extreme environment is high. Moreover, since it has no intricate mechanism, it can also be applied to a wheelchair, for example, which is not powered. This report deals with the fundamental principles of this mobile robot mechanism, production of the prototype robot by modification of a radio-

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controlled model car on the market, and experimental results obtained by using the robot during navigation tests on flat surfaces and in navigating stairs.

Nomenclature

- b : step depth of stairs
 f_v : normal reaction force of floor to the wheel
 f_θ : normal reaction force of obstacle to the wheel
 G : center of gravity of robot
 h : step height of stairs
 H_G : height of G from driving axle level in horizontal state
 L : distance between driving axles
 L_G : longitudinal distance of G from rear driving axle
 m : sufficient moment of rotation for wheel to rotate onto point p
 M_a : arm moment of rotation
 M_x : driving axle moment of rotation
 n : number of teeth of wheel gear or wheel sprocket
 N : number of teeth of driving gear or driving sprocket
 p : point of contact between wheel and obstacle
 r : radius of wheel
 r_x : radius of driving gear or driving sprocket
 R : radius of arm
 t : navigation time
 W : load acting on an arm-wheel combination
 α : effective inclination of stairs
 θ : angular position of point p on the wheel from grounding point
 μ : coefficient of friction between wheel and floor or between wheel and obstacle
 μ_p : peak value of μ varying with θ
 ϕ : body angle of inclination
 ψ : arm angle of inclination during arm-creeping
 ω_a : angular velocity of arm
 ω_x : angular velocity of driving axle
 ω_w : angular velocity of wheel

2. Characteristics and Fundamental Principles

Figure 1 shows a schematic diagram of the mobile robot. The robot has two driving axes which are located at the front and the rear of the body, similar to a four-wheel-drive car. However, the robot has four arm-wheel combinations instead of four driving wheels. The robot navigates by wheel-running on flat surfaces as well as over obstacles and up and down stairs which have steps smaller than a certain height as shown in Fig. 2(a), while it navigates by arm-creeping over obstacles and up and down stairs which have steps greater than a certain height, as shown in

Fig. 2(b).

The specific characteristics of the robot mechanism are as follows:

- (1) It can navigate on flat surfaces, up and down stairs, and over obstacles.
- (2) The discrimination between stairs and flat surfaces and the alternation between wheel-running and arm-creeping always occurs as a purely dynamical phenomenon.
- (3) No sensor is required for the discrimination between stairs and flat surfaces.
- (4) No mechanism is required for alternating between wheel-running and arm-creeping.

Figure 3 is a diagram of an arm-wheel combination, which corresponds to a driving wheel in the case of a four-wheel-drive car. The arm, of radius R , is installed on the driving axle so that the center of the

Fig. 1 Diagram showing construction of the mobile robot

Fig. 2 Diagram showing motion of the mobile robot

Fig. 3 Diagram of arm-wheel combination

axle and the midpoint of the arm agree with each other, and the arm can rotate freely being restricted neither by the driving axle nor by the body. The driving gear, or driving sprocket, with N teeth, is attached to the driving axle. Wheels, each of radius r , are installed on both ends of the arms, and wheel gears, or wheel sprockets, each with n teeth, are attached to all wheels. These wheels and gears, or sprockets, rotate freely, unrestricted by the arms. The driving axle transmits rotation of the same direction to both wheels through both gear mechanisms, or both chains. A modified planetary gear⁽³⁾ is composed of a driving gear, arms, and wheel gears. This planetary gear is different from the conventional planetary gear in that the driving gear and wheel gears are not directly in gear with each other but are connected through an intermediate gear, and rotate in the same direction.

Applying the corresponding equation for the velocity ratio of a planetary gear, the following equation is obtained for ω_a , ω_x and ω_w .

$$\omega_x = \left(1 - \frac{n}{N}\right)\omega_a + \frac{n}{N}\omega_w \quad (1)$$

When the arm-wheel combination moves from left to right and the front wheel contacts an obstacle, sufficient moment of rotation m , given by the following equation, is required so that the front wheel rotates onto point p . m is also the moment of rotation about the center of the wheel. According to Eq.(1), the relationships among m , M_a and M_x are given by the following equation.

$$m = \left(\frac{W}{2} + \frac{M_a}{2R}\right)r \sin \theta \quad (2)$$

$$M_a = \left(1 - \frac{n}{N}\right)M_x, \quad M_x = \frac{N}{n}m \quad (3)$$

According to Eqs.(2) and (3), m and M_a increase with θ . However, M_a is limited when the arm begins to rotate as indicated by the following equation.

$$M_a \leq WR \quad (4)$$

By combining Eqs.(2)-(4) the following equation is

Fig. 4 Change of $m/(WR)$ and $M_x/(WR)$ with θ

obtained.

$$m = \frac{nWRr \sin \theta}{2nR - (N-n)r \sin \theta} \quad \left(\sin \theta < \frac{nR}{(N-n)r}\right),$$

$$m = \frac{nWR}{N-n} \quad \left(\sin \theta \geq \frac{nR}{(N-n)r}\right) \quad (5)$$

The upper part of Eq.(5) is the condition for wheel-running and the lower part is that for arm-creeping. Thus, it is clear that the alternation between wheel-running and arm-creeping occurs as a purely dynamical phenomenon. Figure 4 shows the changes in wheel moment of rotation and driving axle moment of rotation (both dimensionless), $m/(WR)$ and $M_x/(WR)$, with θ for several N/n and one R/r . M_x is proportional to m , and increases with θ according to the upper part of Eq.(5) during the wheel-running, and becomes constant according to the lower part of Eq.(5) after changing to arm-creeping. It is clear that at larger N/n , the arm-creeping occurs at lower θ and the moment of rotation is also smaller. Also at smaller R/r , the arm-creeping occurs at lower θ and the moment of rotation is also smaller; this is not shown in the figure.

For the prototype robot in the present report, $R/r = 1.16$ and $N/n = 3.33$ are used and the change from wheel-running to arm-creeping occurs at $\theta = 29.8^\circ$.

3. Conditions Required for Functioning

3.1 Wheel coefficient of friction

The wheel moment of rotation in Eq.(5) is maintained under the condition that the wheel does not slip; thus μ must be sufficiently large. Figure 5 shows the relationships among forces acting on p for the cases of wheel-running (a), and arm-creeping (b). The smallest required μ for the case of wheel-running [Fig. 5(a)] is determined from normal and frictional forces on p , $\{W/2 + M_a/(2R)\}\cos \theta$ and m/r , using Eq.(2), as

$$\mu = \tan \theta \quad \left(\sin \theta < \frac{nR}{(N-n)r}\right). \quad (6)$$

This is similar to μ required for the normal slope of angle θ . For the case of arm-creeping [Fig. 5(b)], f_v and f_θ are determined from the balance of forces

Fig. 5 Relationships among forces acting on p

acting on the grounding point and p . Then, the smallest required μ is determined from these f_v and f_θ , using the relation $\mu(f_v + f_\theta)r = m$ and Eq.(5), as a root of a quadratic equation:

$$\mu = \frac{-b_7 + \sqrt{b_7^2 - 4a_7c_7}}{2a_7} \quad \left(\sin \theta \geq \frac{nR}{(N-n)r} \right),$$

$$a_7 = (N-n)r(1 - \cos \theta) - nR \sin \theta,$$

$$b_7 = (N-n)r \sin \theta,$$

$$c_7 = -nR \sin \theta. \quad (7)$$

Figure 6 shows the changes of μ with θ according to Eqs.(6) and (7) at several N/n and one R/r . μ increases with θ according to Eq.(6) during wheel-running, reaches a peak point, $\mu = \mu_p$, at the changing point, and decreases gradually according to Eq.(7) after reaching the changing point. It is clear that smaller μ is required at larger N/n . For the prototype robot with $R/r = 1.16$ and $N/n = 3.33$, the peak value $\mu_p = 0.57$ is required at $\theta = 29.8^\circ$ and the value $\mu = 0.41$ is required at $\theta = 90^\circ$ in normal stair navigation. In an experiment in which a rubber tire and a wooden plate were used, it was obtained the measured value of the tire coefficient of static friction $\mu = 1.0$, which fully satisfies the above values $\mu_p = 0.57$ and $\mu = 0.41$.

Figure 7 shows the changes of required μ_p with N/n at several R/r . While it has already been pointed

Fig. 6 Change of smallest required μ with θ

Fig. 7 Change of μ_p with N/n

out that smaller μ_p is required at larger N/n , it is clear that smaller μ_p is also required at smaller R/r . The two curves with $R/r = 0.50$ and $R/r = 1.00$ shown in Fig.7 are only for reference; it is difficult to introduce $R/r \leq 1.0$ in design.

3.2 Excess rotation of arm caused by load

The moments of rotation required for rotation of the arm, indicated by horizontal parts of curves in Fig. 4, are valid when the arm is horizontal. When the arm inclines, at the point at which the wheel runs over the obstacle as shown in Fig. 2(a) or the arm rotates as shown in Fig. 2(b), these moments of rotation change with ψ . Figure 8 shows the change of $M_x/(WR)$, $M_a/(WR)$ and $m/(WR)$ with ψ . In the range of ψ from <0 , the initial position of the arm-creeping stairs, to $\leq 90^\circ$, the moments of rotation are positive and reach a maximum at $\psi = 0$. The moments of rotation are negative at $\psi > 90^\circ$.

It is problem that the moments of rotation are negative, due to the following: When m is positive, the tendency for the wheel to rotate forward is obstructed by the obstacle and $\omega_w = 0$ is retained; thus, ω_a can be determined from driving motion ω_x according to Eq.(1). In contrast, when m is negative, the wheel rotates backwards almost freely because there are no obstacles blocking it from the rear. In this case, the arm rotates excessively, and at the same time, the wheel rotates backwards. Thus, arm-creeping cannot occur.

There may be two ways to prevent the excess rotation of the arm: one way is to give the driving axle sufficient angular velocity to counteract the excess rotation of the arm, and the other way is to give the arm a sufficient resistance moment to prevent the moment of the arm caused by load. The former method may have several disadvantages in that it may not be suitable for the robot, and the angular velocity may be insufficient. An efficient way of preventing

Fig. 8 Change of $M_x/(WR)$, $M_a/(WR)$ and $m/(WR)$ with ψ

excess rotation is to apply resistance force using plate springs to the arm, as mentioned in Section 4.3.

4. Design Parameters and Prototype

4.1 Diameters of wheels and arms

Figure 9(a) shows an arm-wheel combination in which both wheels contact the ground on each step of the stairs. As the condition that the driving gear or the driving sprocket does not contact the upper step, the following equation is given.

$$r \geq \frac{h}{2} + r_x \quad (9')$$

On the other hand, from the condition that the rear wheel perfectly contacts the ground on the lower step as shown in Fig. 9(b), the following equation is obtained.

$$4R^2 \leq h^2 + (2b - r)^2 \quad (8)$$

From Eq. (8) and the condition that the two wheels do not contact each other, $r < R$, the following equation is obtained.

$$r < \frac{-2b + \sqrt{16b^2 + 3h^2}}{3} \quad (9'')$$

Thus, the condition for the radius of the wheel is

$$\frac{h}{2} + r_x \leq r < \frac{-2b + \sqrt{16b^2 + 3h^2}}{3} \quad (9)$$

With typical dimensions of actual stairs, $h=175$ mm and $b=280$ mm, and a driving sprocket radius with conforms to these values of h and b , $r_x=51$ mm, it is determined from Eq. (9) that $138.5 \text{ mm} \leq r < 200$ mm. Even if r_x is neglected, $r \geq 87.5$ mm is required as a minimum value. However, the prototype robot was constructed by modifying a radio-controlled model car on the market, and $r=77.5$ mm was already fixed. Therefore, the dimensions of the stairs must be altered to carry out the navigation test. From $r_x=25$ mm for the prototype and the left-hand side of Eq. (9), $h \leq 105$ mm is obtained. Thus, a 60% scale wooden model of stairs with dimensions $h=105$ mm and $b=168$ mm was constructed and used for the navigation test. From the dimensions and Eq. (8), $R < 139.5$ mm is obtained. Since a smaller R/r is more favorable for required μ_p , as mentioned in Section 3.1, $R=90$ mm was selected. As a consequence, the prototype can navigate stairs with dimensions $h \leq 105$ mm and $b \geq$

Fig. 9 Determination of r and R

112 mm, and the maximum effective slope angle is 43° .

Thus, when a 100% scale robot is fabricated, the dimensions must be $r_x=42$ mm and $r=129$ mm. For the driving sprocket radius with conforms to the actual values of h and b , $r_x=51$ mm as mentioned above and $r \geq 138.5$ mm is required.

4.2 Prevention of turning upside down of the robot

Figure 10 shows a diagram of the robot climbing up stairs. ϕ changes with ψ for both the front and rear axles, and also changes with the horizontal distance of G from the rear axle. Here, G must always be in front of the forward wheel of the rear arm-wheel combination, or the robot body will turn upside down. Thus, it is necessary to select an appropriate position of G and value of L . The method for selecting these has been examined by Takano et al.⁽⁴⁾, but the method is complex. Thus, in the present paper, examination by simple graphical analysis based on Fig. 10 is used. Figure 10 indicates that ϕ is almost maximum when $L=650$ mm, under the condition that the geometrical dimensions are proportional to the values determined in Section 4.1 and $\alpha=32^\circ$, $L_G=L/2$ and $H_G=70$ mm. It is clear from the graphical analysis based on Fig. 10 that the horizontal position of G is located in front of the forward wheel composing the rear arm-wheel combination. Although not shown in the figure, it is confirmed from the graphical analysis that the robot can climb stairs with $\alpha=43^\circ$ which is the maximum permitted value of α determined from r and R obtained in Section 4.1.

4.3 Production of prototype

As the body of the prototype robot, a radio-controlled model car on the market (TAMIYA: CLOD BUSTER) is used. Four driving axles of the model are modified, so that four arm-wheel combinations are set up. Eight wheels are used, each of which is from the original model and is 105 mm wide. Chains (TSUBAKIMOTO: RS 11 SS) are used for the transmission of rotation from the driving axle to the

Fig. 10 Diagram of the robot for graphical analysis of L

wheels, and the driving sprocket and wheel sprocket are 1.25 mm module and have 40 teeth and 12 teeth, respectively (TSUBAKIMOTO: RS NT 40 and RS NT 12). Total diameter of the driving sprocket around which the chain is wound, $2r_x$, is about 50 mm. Two driving sprockets are attached to a driving axle and two wheel sprockets are independently driven simultaneously, for the purpose of preventing the chain from coming off the sprocket.

To prevent excess rotation of the arm as mentioned in Section 3.2, to each arm a bar is attached at the radial position of 60 mm, and a plate spring 100 mm long (a hardened thickness gauge) is attached to the top of the driving-axle bearing. The bar rotates with the arm, contacts the plate spring and bends it, and is resisted by the beaded plate spring (Fig. 11). The bar contacts the plate spring at an angle of inclination of the arm from the body of about 90° , and the bending of the plate spring increases with the arm angle of inclination in order to increase the positive

resistance moment. This counteracts the negative moment occurring at $\phi > 90^\circ$, as shown in Fig. 8, and excess rotation of the arm caused by the load will be

Fig. 11 Part of driving axle in arm-wheel combination. Two driving sprockets are attached to the driving axle. Two bars attached to each arm contact a plate spring attached to the top of the driving-axle bearing

Fig. 12 Navigation test using prototype robot (navigation started at $t=0$)

prevented. When the arm and the body are almost parallel, the bar no longer contacts the plate spring and the resistance moment vanishes.

The value $L=650$ mm is much larger than that of the model car; thus extension frames were substituted for the body of the model.

5. Navigation Test

The navigation test using the 60% scale prototype robot and 60% scale wooden model stairs was carried out, which covers flat-surface running and navigating stairs. Here, as the steering mechanism of the radio-controlled model car could not accommodate the running speed of the robot with eight wheels, it was not used during the navigation test.

Figures 12(a)~(l) are video navigation scenes, where the start time of navigation is $t=0$. The running distance from $t=0.33$ s [Fig. 12(a)] to $t=0.63$ s [Fig. 12(b)] was about 320 mm, and the speed was about 1.1 m/s. The change from wheel-running to arm-creeping took place at (b), and the arm rotated through (c) and (d) to finish at (e). The time taken for rotation of the arm from (b) to (e) was 0.7 s. In the interval from $t=1.33$ s, (e), to $t=1.73$ s, both wheels of the arm-wheel combination were in contact with the vertical and horizontal surfaces of the first step, and at $t=1.73$ s the front wheel of the arm-wheel combination contacted the vertical surface of the second step, and then the arm-creeping was repeated as seen in (f). At $t=3.03$ s, 2.70 s after the front wheel of the front combination contacted the stairs, the front wheel of the rear combination contacted the stairs, and then arm-creeping by both the front and rear combinations took place, as seen in (g) and (h). At $t=4.90$ s, 1.87 s after the rear combination contacted the stairs, the front combination arrived at the top horizontal surface of the stairs. At 1.87 s the robot body had ascended almost one step (193 mm in distance), and the creeping speed of the full arm-creeping by the front and rear axles was about 0.1 m/s, which was 1/10 the speed of wheel-running on the flat floor.

The front arm-wheel combination did not perfect-

ly finish the arm-creeping even after arriving at the top horizontal surface, as may be seen in (i) and (j), due to the influence of the arm-creeping of the rear arm-wheel combination. After (j), $t=8.87$ s, the robot navigated on the top horizontal surface of the stairs by wheel-running, and then descended the stairs as seen in (k) and (l). Here, it was confirmed that the robot descended the stairs not by arm-creeping but by wheel-running as seen in (k) and (l), where the arms rocked and the impact of the wheel drop was reduced. The speed of the robot descending the stairs was about 1 m/s.

6. Conclusions

A mechanism for a mobile robot enabling it to navigate stairs was developed using a new type of arm-wheel design, and through navigation tests the following conclusions were drawn.

(1) It is theoretically proven and experimentally confirmed that the alternation between wheel-running and arm-creeping occurs as a purely dynamical phenomenon.

(2) It is clarified with (1), that this mobile mechanism requires no sensor for discrimination between stairs and flat surfaces and no mechanism for alternating between wheel-running and arm-creeping modes.

(3) The robot ascends stairs by arm-creeping, but descends stairs by wheel-running, during which rocking of the arms reduces the impact of the wheel drop.

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