H^∞ Control of Transverse-Torsional Coupled Vibrations Using Multiple Active Dynamic Vibration Absorbers*

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In this paper we discuss the vibration control of a flexible structure which possesses transverse-torsional coupled vibration modes using two active dynamic vibration absorbers. The H^∞ control theory is adopted to obtain an output-feedback robust controller by which structural vibrations in two lower vibration modes are reduced without spillover. The statements of the control object are organized into the generalized plant, that is, robust stabilization and sensitivity reduction. A method of synthesis of the controller is presented in which balancing of control inputs is taken into consideration. In numerical analysis and control experiments, it is confirmed that the controller achieves considerably high performance with robust stability.

Key Words: Vibration Control, Dynamic Absorber, Robust Control, Coupled Vibration, Flexible Structure, H^∞ Control, Spillover

1. Introduction

In most studies on the vibration control problems, it is assumed that a structure vibrates in only one direction or in multiple directions independently. This assumption simplifies the analysis of a system and the synthesis of a controller. In real structures, however, this assumption is not always appropriate because the structures often possess multidirectional coupled vibration modes and the control performance of the controller is degraded due to parameter variation or spillover induced by the effect of coupling. Moreover, there exist not only transverse vibrations but also torsional vibrations in real structures and they often possess coupling. The controllers have to be designed taking account of the effect of transverse-torsional coupled vibration modes in such cases.

The control problem of structures which possess transverse-torsional coupled vibration modes has

already been examined by some researchers. Arai et al. discussed the flexibility control of a flexible structure⁽¹⁾. Sakawa and Luo reported the case of a flexible arm⁽²⁾. Yoshida and Goshima proposed the application of double tendon mechanisms in a flexible plate⁽³⁾.

In many vibration control problems, it is necessary to control multiple control inputs or actuators such as active dynamic vibration absorbers (ADVAs) simultaneously due to the following reasons.

- (1) Control forces should be increased.
- (2) Multiple vibration modes should be controlled. In such a case, it is more effective to apply multiple control forces at appropriate positions than to apply them at the same point.
- (3) Multiple directions of vibrations should be controlled.

Tanida and Koike⁽⁴⁾, and Kaneko and Mita⁽⁵⁾ utilized two ADVAs and LQ controllers to reduce transverse and torsional vibrations. Ikeda et al.⁽⁶⁾ applied absolute acceleration feedback control. Kuruta and Mita⁽⁷⁾ examined the performance of a fuzzy controller. Nonami et al.⁽⁸⁾ dealt with a magnetic bearing system.

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In such multiactuator simultaneous control, there are many problems caused by interactions among control inputs in addition to the problems inherent in ordinary single-actuator control. Thus, controller synthesis becomes more difficult since problems such as balancing of the control inputs should be taken into consideration.

These two problems, multidirectional coupled vibration control and multiactuator simultaneous control, are closely related to each other. Because of the progress of active vibration control technology, the multidirectional and multimodes simultaneous control using multiactuator system will be required in the near future. However, few studies have dealt with such problems, and the design method and the control performance of such systems have not been investigated in detail.

In this paper we discuss robust vibration control, using multiple ADVAs, of a flexible structure which possesses transverse-torsional coupled vibration modes. The control object is a four-storied towerlike structure on which two ADVAs are mounted. The structure has eight degrees of freedom. The H^∞ control theory is adopted to obtain robust controller, and a design procedure taking account of balancing of control inputs is presented. The purpose of this study is to present general guidelines for the design of a multiactuator active vibration control system for flexible structures which possess transverse-torsional coupled vibration modes through theoretical calculations and control experiments.

2. Control Object

2.1 Structural model

The schematic diagram of a theoretical model of the structure is shown in Fig. 1. The structure has four stories and is towerlike in shape. Each story is modeled so as to have one degree of freedom in the transverse direction (the same direction as the excitation) and another one in the angle of torsion around the centroid of the story. Therefore, the theoretical model of the entire structure has eight degrees of freedom. The base of the structure is attached to a shaker and the structure is subjected to excitation. Meanwhile, two active control forces f_R and f_L are applied to the right and left edges of the structure.

The matrix equation of motion of the structure is expressed as

$$\mathbf{M}\ddot{\mathbf{x}}_0 + \mathbf{C}\dot{\mathbf{x}}_0 + \mathbf{K}\mathbf{x}_0 + \mathbf{d}\ddot{\mathbf{z}} + \mathbf{b}\mathbf{f} = \mathbf{0} \quad (1)$$

by using the state vector

$$\mathbf{x}_0^T = [x_1 \ x_2 \ x_3 \ x_4 \ \theta_1 \ \theta_2 \ \theta_3 \ \theta_4], \quad (2)$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the inertia, damping and stiffness matrices, $\mathbf{d}$ is the noise matrix for excitation acceleration $\ddot{\mathbf{z}}$ and $\mathbf{b}$ is the input matrix for the

control force $\mathbf{f}$. x_i and θ_i are transverse and torsional displacements of the i th story, respectively. $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, $\mathbf{d}$ and $\mathbf{b}$ have the following forms.

$$\begin{aligned} \mathbf{M} &= \text{diag} [m_1 \ m_2 \ m_3 \ m_4 \ I_1 \ I_2 \ I_3 \ I_4] \\ \mathbf{K} &= \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \\ \overline{k_1 l_1 + k_2 l_1} & -\overline{k_2 l_1} & 0 & 0 \\ -\overline{k_2 l_1} & \overline{k_2 l_2 + k_3 l_2} & -\overline{k_3 l_2} & 0 \\ 0 & -\overline{k_3 l_2} & \overline{k_3 l_3 + k_4 l_3} & -\overline{k_4 l_3} \\ 0 & 0 & -\overline{k_4 l_3} & \overline{k_4 l_4} \\ \overline{k_1 l_1 + k_2 l_1} & -\overline{k_2 l_2} & 0 & 0 \\ -\overline{k_2 l_1} & \overline{k_2 l_2 + k_3 l_2} & -\overline{k_3 l_3} & 0 \\ 0 & -\overline{k_3 l_3} & \overline{k_3 l_3 + k_4 l_3} & -\overline{k_4 l_4} \\ 0 & 0 & -\overline{k_4 l_4} & \overline{k_4 l_4} \\ k_1 l_1^2 + k_2 l_1^2 & -k_2 l_1 l_2 & 0 & 0 \\ -k_2 l_1 l_2 & k_2 l_2^2 + k_3 l_2^2 & -k_3 l_2 l_3 & 0 \\ 0 & -k_3 l_2 l_3 & k_3 l_3^2 + k_4 l_3^2 & -k_4 l_3 l_4 \\ 0 & 0 & -k_4 l_3 l_4 & k_4 l_4^2 \end{bmatrix} \\ \mathbf{d}^T &= [m_1 \ m_2 \ m_3 \ m_4 \ 0 \ 0 \ 0 \ 0] \\ \mathbf{b}^T &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & l_{JR} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -l_{JL} \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} f_R \\ f_L \end{bmatrix} \end{aligned} \quad (3)$$

m_i and I_i are mass and moment of inertia, k_{Ri} and k_{Li} are spring constants of right and left sides of the i th story, l_{Ri} and l_{Li} are distances from the centroid to the springs of right and left sides of the i th story, and l_{JR} and l_{JL} are distances from the centroid of the 4th story to the right and left actuators, respectively. The term $\overline{k_i l_j}$ in the matrix $\mathbf{K}$ denotes the cross term between x_i and θ_j , that is,

$$\overline{k_i l_i} = k_{Ri} l_{Ri} - k_{Li} l_{Li}. \quad (4)$$

Fig. 1 Schematic diagram of theoretical model

The mass distribution of each story is homogeneous and the stiffnesses of the four columns are assumed to be the same in the direction of the excitation for all stories. Under this condition, l_{fR} and l_{fL} are equal and all the cross terms are zero. Then the transverse and torsional vibrations are independent of each other. In this study, however, a lumped load is mounted at the right side of the third story. Therefore l_{fR} and l_{fL} become unequal, the cross terms have nonzero values and the structure possesses transverse-torsional coupled vibration modes.

The matrix C is determined according to the experimental modal analysis.

2.2 Active dynamic vibration absorber model

In this study, two active dynamic vibration absorbers (ADVAs) are utilized in control devices to apply control forces f_R and f_L . Each absorber is composed of an auxiliary mass, a moving coil to drive the mass and a linear bearing to support the coil and the mass. The control forces are generated by applying voltages to the coils. The equations of the circuits are

$$L_R \dot{i}_R + R_R i_R + E_R \dot{x}_{aR} = e_R, f_R = T_R i_R \text{ (right side)} \quad (5)$$

$$L_L \dot{i}_L + R_L i_L + E_L \dot{x}_{aL} = e_L, f_L = T_L i_L \text{ (left side)} \quad (6)$$

where i is current, L is inductance, R is reactance, E is induced voltage coefficient, e is input voltage, f is force and T is thrust constant of each circuit. The suffixes R and L denote the positions of circuits, right side or left side.

In addition, the equations of motion of the absorbers are

$$m_{aR}(\ddot{x}_{aR} + \ddot{x}_4 + l_{fR}\ddot{\theta}_4 + \ddot{z}) = f_R \text{ (right side)} \quad (7)$$

$$m_{aL}(\ddot{x}_{aL} + \ddot{x}_4 - l_{fL}\ddot{\theta}_4 + \ddot{z}) = f_L \text{ (left side)}, \quad (8)$$

where m_a is the auxiliary mass and x_a is the stroke.

By using Eqs.(5)-(8), we obtain matrix differential equations which describe the dynamics of the ADVAs.

2.3 State equation of control object

Combining the above-mentioned matrix equations, the state equation of the control object can be written as

$$\dot{\mathbf{x}}_f = \mathbf{A}_f \mathbf{x}_f + \mathbf{B}_f \mathbf{e} + \mathbf{D}_f \dot{\mathbf{z}} \quad (9)$$

where $\mathbf{x}_f$ is the state vector of the entire control object

$$\mathbf{x}_f^T = [x_{aR} x_{aL} \dot{x}_{aR} \dot{x}_{aL} x_{s1} x_{s2} x_{s3} x_{s4} \theta_{s1} \theta_{s2} \theta_{s3} \theta_{s4} i_{RiL}] \quad (10)$$

and $\mathbf{A}_f$ is the system matrix, $\mathbf{B}_f$ is the input matrix, $\mathbf{D}_f$ is the noise matrix and $\mathbf{e}$ is the control voltage matrix.

All the state variables included in the state vector (13) are observable. In this study, however, the available outputs of the structure are limited to the

strokes of ADVAs and the accelerations of the structure at both edges. Using the output matrix $\mathbf{y}_f$, the output matrix equation of the control object is written as follows.

$$\mathbf{y}_f = \mathbf{C}_f \mathbf{x}_f, \mathbf{y}_f = [x_{aR} x_{aL} \ddot{y}_{4R} \ddot{y}_{4L}] \quad (11)$$

where $\ddot{y}_{4R} = \ddot{x}_4 + l_{4R}\ddot{\theta}_4 + \ddot{z}$ and $\ddot{y}_{4L} = \ddot{x}_4 - l_{4L}\ddot{\theta}_4 + \ddot{z}$ are absolute accelerations of the 4th story at right and left edges, respectively.

2.4 Parameter identification and modal analysis

To obtain the parameters of the system, an excitation experiment is carried out. As a result of parameter identification, the parameters of the structure and the actuators are obtained as shown in Table 1. Moreover, experimental modal analysis is also

Table 1 Identified parameters of experimental structure and absorbers

(a) Structure								
Story	Side	Mass [kg]	Moment of inertia [10^{-2} kg·m]	Stiffness [kN/m]	distance from centroid to each stiffness [cm]	mode	Eigen frequency [Hz]	Damping ratio [%]
4	R	2.33	2.36	15.4	18.3	1	3.70	0.93
	L			15.8	18.3	2	6.26	1.50
3	R	2.35	1.63	4.19	8.99	3	12.8	0.90
	L			4.45	18.6	4	19.7	0.30
2	R	1.80	1.30	4.18	13.8	5	20.8	0.50
	L			4.23	13.8	6	27.8	0.70
1	R	1.79	1.30	6.35	13.8	7	32.0	0.10
	L			6.26	13.8	8	48.3	0.10

(b) Absorbers						
Actuator	m_a [kg]	L_a [mH]	R_a [Ω]	K_f [N/A]	K_e [Vs/m]	L_f [m]
Right	0.112	3.3	2.0	1.6	3.0	0.10
Left	0.112	2.7	2.0	1.6	3.0	0.10

Fig. 2 Modal shapes of the structure

Fig. 3 Frequency response of theoretical and experimental structures

carried out to examine the degree of coupling by observing modal shapes of the structure and to estimate the damping ratios of each mode. The shapes of the first, second, third and fourth modes of the structure are shown in Fig. 2. It is clear that the structure exhibits transverse-torsional coupled vibrations. Moreover, the transfer function from the excitation acceleration $\ddot{z}$ to the right acceleration output $\ddot{y}_{AR}$ is shown in Fig. 3. The theoretical and experimental models show good agreement.

3. Controller Design

3.1 Dynamical characteristics of the control object

In the structural model obtained in the previous section, the gain of the second mode is lower than those of the third, fourth and fifth modes as shown in Fig. 3. This fact indicates that the second mode is cannot easily be controlled using simple proportional output feedback. We also note that the controller should have lower sensitivity in the higher frequency domain to avoid spillover.

Moreover, as the objective system has multiple inputs and multiple outputs (MIMO), the controller must be designed as a MIMO controller, since the application of multiple single input single output (SISO) controllers to a MIMO system often results in the degradation of control performance and stability⁽⁴⁾.

3.2 Adoption of H^∞ control theory

According to the characteristics of the control object, the controller for this system should be designed as a MIMO controller which has frequency-shaped feedback gain characteristics. Frequency shaping design is possible in many control theories; for example, frequency-shaped LQG control^{(9),(10)} is applicable to a MIMO system. In this study, the control object possesses transverse-torsional coupled

vibration modes and the nominal model is presumed to include model errors. Hence, in this study, H^∞ control theory is applied, since the H^∞ controller guarantees robust stability to the uncertainty which is formulated in a generalized plant.

3.3 Design procedure

In the H^∞ controller design procedure, we adopted a "mixed sensitivity design" in order to establish a generalized plant. The basic concept of the mixed sensitivity design is as follows.

(a) Assign weight to state variables in the lower frequency domain such that $G_{\dot{z}-w_L}(s)$ which denotes the closed-loop transfer function from excitation $\dot{z}$ to weighted output W_L satisfies low-sensitivity characteristics in the domain, namely less than a certain level γ . As a result, feedback gains to lower vibration modes are increased and these vibration responses are reduced.

(b) Assign weight to control inputs in the higher frequency domain such that $G_{u-w_H}(s)$ which denotes the closed-loop transfer function from control input u to weighted output W_H satisfies low-sensitivity characteristics in the domain, namely less than 1. As a result, feedback gains to higher vibration modes are reduced and the robustness to model errors is enhanced.

It is known that there exists a trade-off between (a) and (b). Therefore, in the design of a H^∞ controller, it is very important to choose the weights so that both requirements are satisfied in each frequency domain. In this study, the controller is designed according to the following procedure.

(1) Reduce the order of the control object.

(2) Choose the state variables to which weights are assigned in order to realize (a), as well as the shape filters which are utilized as frequency-shaped weighting functions taking account of the dynamics of the control object and the model errors that arise from reduction.

(3) Fix the gains of high-pass filters with the aim of achieving (b), and tune the gains of low-pass filters applied to state variables in order to satisfy both requirements.

(4) Fix the gains of low-pass filters, and tune the gains of high-pass filters applied to control inputs in order to balance the control inputs.

3.4 Reduction of order of control object

First, reduction of the state equation of the control object is carried out. In this study, the target of vibration reduction is the first and second modes of the structure. The order of the structural model is reduced to 2 (one transverse and one torsional) degrees of freedom after performing modal transformation and reduction in order to reduce the order of

the controller. The reduced-order state equation is written as

$$\dot{\mathbf{x}}_p = \mathbf{A}_p \mathbf{x}_p + \mathbf{B}_p \mathbf{e} + \mathbf{D}_p \ddot{\mathbf{z}}. \quad (12)$$

3.5 Choosing weighted variables and shaping low-pass filters

In this section, the state variables to be weighted are chosen, and filters for frequency-shaped weighting functions are designed.

With the aim of suppressing feedback gains for uncontrolled vibration modes and maintaining robustness to the change of dynamical characteristics of the control object, the filters are designed as follows.

In the structural model, the control objects are described in the physical coordinate in order to express the physical structure of transverse-torsional coupling. According to this model, two pairs of state variables, transverse and torsional displacements or transverse and torsional velocities, are adopted in order to increase the control effect in the lower modes⁽¹¹⁾. These state variables are weighted using the 4th-order low-pass filter,

$$W_{Ls}(s) = G_{Ls} \frac{1}{(s^2 + 2\zeta_v \omega_v + \omega_v^2)(s^2 + 2\zeta_\nu \omega_\nu + \omega_\nu^2)}, \quad (13)$$

where s is the Laplace operator. The filters are designed on the s -domain and converted to state-space formulae.

To maintain robustness to the change of dynamical characteristics, filters are parametrized so that they possess gentle slopes.

Moreover, in the ADVAs, the length of stroke of an actuator is often limited. Taking account of this fact, strokes of both absorbers are also weighted using the 4th-order low-pass filter,

$$W_{LA}(s) = G_{LA} \frac{\omega_k^2}{s^2 + 2\zeta_x \omega_x + \omega_x^2}. \quad (14)$$

The frequency responses of these filters are shown in Fig. 4. This figure shows the filters for the case that transverse and torsional velocities are weighted.

Fig. 4 Weighting functions for state variables(velocity-weighted case)

3.6 Shaping high-pass filters

In this section, frequency-shaped weighting functions for control inputs are designed.

In a mixed sensitivity problem, weights for control inputs are often designed so that the gains of weights envelope model errors for which robust stability is guaranteed. From the viewpoint of trade-off between nominal control performance and control costs, the weights for control inputs can be represented as weights for performance. Thus the weights must satisfy two conditions simultaneously; high gain for residual vibration modes and low gain for controlled vibration modes.

Moreover, there are many other conditions to be considered in the design of filters; for example, differences in positions and dynamic properties among actuators. However, it is difficult to take into account these conditions explicitly in the H^∞ controller design. Therefore, filters are designed without consideration of such conditions in this step, and tuned in the balancing step, later.

It is possible that there are model errors in this system; for example, reduction errors, parameter variations, and unmodeled dynamics. The reduction error can be expressed as an additional error which is defined as the difference in the transfer functions between the full-order model $\mathbf{P}_f$ and reduced-order model $\mathbf{P}_r$ in this problem. That is,

$$\Delta \mathbf{P}(s) = \mathbf{P}_f(s) - \mathbf{P}_r(s). \quad (15)$$

In this study, we neglect other errors and shape high-pass filters so that they maintain robustness to the errors which are a little larger than $\Delta \mathbf{P}(s)$. The weighting function is realized using the 4th-order high-pass filter:

$$W_H(s) = G_H \frac{(s^2 + 2\zeta_r \omega_r + \omega_r^2)(s^2 + 2\zeta_s \omega_s + \omega_s^2)}{(s^2 + 2\zeta_a \omega_a + \omega_a^2)(s^2 + 2\zeta_\beta \omega_\beta + \omega_\beta^2)}. \quad (16)$$

The frequency responses of $\Delta \mathbf{P}$ and designed filters are shown in Fig. 5.

3.7 Realization of generalized plant and H^∞ controller

According to the procedure shown in former sections, the generalized plant which includes the dynamics of the control object and weighting functions is developed. The generalized plant is formulated to the following expressions.

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A} \mathbf{x}(t) + \mathbf{B}_1 \mathbf{w}(t) + \mathbf{B}_2 \mathbf{u}(t) \\ \mathbf{z}(t) &= \mathbf{C}_1 \mathbf{x}(t) + \mathbf{D}_{12} \mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}_2 \mathbf{x}(t) + \mathbf{D}_{21} \mathbf{w}(t) \end{aligned} \quad (17)$$

The schematic diagram of the organization of the obtained generalized plant is shown in Fig. 6. In this figure, transfer functions from $\ddot{\mathbf{z}}$ to W_{Ls} and W_{LA} denote the sensitivity functions for (a), $G_{\ddot{\mathbf{z}}-w_L}(s)$, and transfer functions from $\mathbf{u}$ to W_H denote the

Fig. 5 Reduction errors and weighting functions for control inputs

Fig. 6 Schematic view of the generalized plant

sensitivity functions for (b), $G_{u-wH}(s)$.

Applying Doyle et al.'s method⁽¹²⁾ to the generalized plant, a H^∞ controller

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{B}y(t) \\ e(t) &= \hat{C}\hat{x}(t) \end{aligned} \quad (18)$$

which satisfies the H^∞ norm condition,

$$\left\| \begin{bmatrix} G_{z-wL}(s) \\ G_{u-wH}(s) \end{bmatrix} \right\|_\infty \leq 1, \quad (19)$$

is obtained.

3.8 Tuning filter gains and balancing control inputs

To obtain a controller, the coefficients G_H , G_{LS} and G_{LA} are tuned according to the procedure shown in section 3.3.

First, while the values of G_H are fixed, the values of G_{LS} and G_{LA} are tuned using the closed-loop response of the controller and the control object in order to achieve the highest performance.

Then, while the values of G_{LS} and G_{LA} are fixed, the values of G_H are tuned in order to balance the levels of control inputs as mentioned in section 3.3. As the filters are designed without consideration of the difference in dynamics between the right and left

side, there is a possibility of concentration of control load to an absorber. To prevent the concentration and to balance inputs simply, the values of G_H are tuned by comparing the root mean square (R.M.S.) responses of control inputs with random excitations. By using R.M.S. values instead of frequency responses, the difficulty of balancing is greatly reduced because R.M.S. values are obtained simply by solving a co-variance equation.

The R.M.S. responses are obtained by solving the co-variance equation of the closed-loop system,

$$A_X V + V A_X^T + D_X w D_X^T = 0, \quad (23)$$

where

$$\begin{aligned} A_X &= \begin{bmatrix} A_f & B_f \hat{C} \\ \hat{B} C_f & \hat{A} \end{bmatrix}, D_X = \begin{bmatrix} D_f \\ 0 \end{bmatrix}, \\ V &= E \left\{ \begin{bmatrix} x_f \\ \hat{x} \end{bmatrix} \begin{bmatrix} x_f^T & \hat{x}^T \end{bmatrix} \right\}. \end{aligned} \quad (24)$$

w is the intensity of the white noise and V denotes the mean squared response matrix.

In this study, we examined R.M.S. values of absorber strokes x_{ar} and x_{al} , absolute accelerations $\dot{y}_{4R}$ and $\dot{y}_{4L}$, and control voltages e_R and e_L for balancing. The values of G_H are tuned in order to

maintain the constant control performance (x_{ar} , x_{al} , $\dot{y}_{ar}$ and $\dot{y}_{al}$) throughout tuning and to reduce the differences between e_R and e_L .

4. Theoretical Calculations

4.1 Obtaining controller

To obtain the controller, numerical calculations are carried out according to the procedure mentioned above.

First, the gains of low-pass filters are tuned. It is found that the control performance becomes saturated as the values of G_{LS} and G_{LA} are gradually increased, and finally the system becomes unstable. Thus, to maintain robustness, the gains of low-pass filters are set at a level at which saturation begins. The obtained parameters of filters are shown in Table 2.

Then, the values of G_H are tuned in order to balance the levels of control inputs. It is found that the control input and stroke of the right ADVA are greater than those of the left ADVA by about 17% without tuning of G_H . After tuning, the value of G_H for the left ADVA is decreased to 8% of the value for the right ADVA, which corresponds to the decrease of gain by 0.72 dB. As a result, these inputs are almost balanced and the acceleration responses of the structure are also reduced. The change of responses is shown in Table 3.

4.2 Theoretical control performance

Prior to experiment, theoretical calculations of

Table 2 Parameters of filters

(a) High-pass filters

ω_α [rad/s]	3.6	ω_γ [rad/s]	15.0
ζ_α [-]	0.6	ζ_γ [-]	0.3
ω_β [rad/s]	6.1	ω_δ [rad/s]	20.0
ζ_β [-]	0.6	ζ_δ [-]	0.3
G_H	1800 (before balancing)		

(b) Low-pass filters

State Variables			
ω_v [rad/s]	(x) 3.6 (θ) 4.6	ω_w [rad/s]	(x) 6.5 (θ) 6.5
ζ_v [-]	(x) 0.5 (θ) 0.5	ζ_w [-]	(x) 0.5 (θ) 0.5
GLS			
disp.	(x) 4.8×10^{-8}	(θ) 1.0×10^{-8}	
velo.	(x) 2.4×10^{-7}	(θ) 5.0×10^{-6}	
Actuator Stroke			
ω_κ	3.6 [rad/s]	ζ_κ	0.7
$G_{L\kappa}$	25		

Table 3 Theoretical effect of balancing on control characteristics

Weighted variables	R.M.S.	$\sqrt{x_{ar}^2}$ [10^{-5}]	$\sqrt{x_{al}^2}$ [10^{-5}]	$\sqrt{\dot{y}_{ar}^2}$ [10^{-2}]	$\sqrt{\dot{y}_{al}^2}$ [10^{-2}]	$\sqrt{e_R^2}$ [10^{-3}]	$\sqrt{e_L^2}$ [10^{-3}]
Displacement	before	1.75	1.49	1.06	0.86	1.32	1.13
	after	1.64	1.61	0.97	0.79	1.22	1.22
Velocity	before	1.83	1.57	1.24	1.01	1.41	1.24
	after	1.71	1.69	0.99	0.81	1.27	1.27

Intensity of white noise : 10^{-8}

closed-loop frequency responses are carried out to evaluate the control performance of the controllers. As a result, it is confirmed that the controllers exhibit little spillover in the higher modes due to frequency-shaping regardless of whether state variables, displacements or velocities are weighted.

The theoretical responses of the lower modes are shown in Fig. 7. The frequency responses of the acceleration at the top right edge of the structure to the excitation acceleration. In the figure, the dotted line shows the case of no control. The maximum response of the first mode exceeds 30 dB. Thin and thick solid lines denote the cases with the velocity-weighted controller before and after balancing, respectively. The peak responses of the first mode are reduced to 12 dB in both cases.

Clearly, balancing hardly affects the responses of control modes. This result conflicts with the results in Table 3. From further calculations, it is clarified that the responses of uncontrolled modes are reduced by about 10% after balancing. That is, balancing can enhance control performance without a verse effects on uncontrolled modes. This conclusion is also supported by the results for the displacement-weighted case.

5. Control Experiment

5.1 Organization of experimental system

A photograph of the experimental structure is shown in Fig. 8, and the schematic diagram of the control system is shown in Fig. 9. The accelerations of the structure are measured by accelerometers and the strokes of absorbers are measured by noncontact gap sensors. The measured signals are sent to a Digital signal processor (DSP) unit. The unit is equipped with an A/D converter where the signals are discretized. The H^∞ controller is incorporated into the DSP and feedback voltages are generated according to the discretized signals. The feedback voltages

Fig. 7 Theoretical evaluation of control performance

are converted to continuous signals by a D/A converter built into the unit, amplified by power amplifiers and applied to the moving coils.

5.2 Frequency Responses

Control experiments are carried out for the controllers obtained by theoretical calculations. First, the controllers are discretized with a sampling time of 1 millisecond. Then they are incorporated into the DSP via a personal computer attached to it.

The results of experiment are shown in Fig. 10 by thick lines. The controllers obtained in section 4.1 are stable and achieve high control performances, although experimental control performances are degraded by about 3 dB due to experimental errors, in comparison with theoretical results. To confirm robustness, changes in the lumped load on the third story are made. The closed-loop system remains stable with changes in mass (addition of 100 g) and in

position (moving mass 5 cm toward the center of the third story).

To confirm the effect of balancing of control inputs, R. M. S. responses of the actuator strokes to

Table 4 Effect of balancing on experimental responses of strokes

stroke balancing	$\frac{x_{aR}}{z}$	$\frac{x_{aL}}{z}$
(a) before	0.64	0.43
(b) after	0.59	0.48

Fig. 8 Photograph of the experimental structure

Fig. 9 Schematic diagram of the experimental system

(a) No control

(b) Controlled (displacement weighted)

(c) Controlled (velocity weighted)

Fig. 10 Frequency responses of the structure (Control effects to 1st and 2nd modes)

Fig. 11 Impulse responses of the structure

random excitation are measured by a fast Fourier transformation (FFT) analyzer. The results in the case with velocity-weighted controllers are shown in Table 4. The imbalance of the stroke between right and left absorbers is obviously decreased. The control performances of both controllers are also measured and confirmed to be the same.

5.3 Impulse Responses

Impulse excitation experiments are also carried out to verify the effect of active control in the time domain. The acceleration responses of the roof of the structure are shown in Fig. 11. Figure 11-(a)-(1) shows transverse response and Fig. 11-(a)-(2) shows torsional response in the case of no control. The vibrations last for over 15 seconds in this case. Figures 11-(a)-(2) and 11-(b)-(2) show the case with the displacement-weighted controller. Vibrations converge to zero within 6 seconds in these figures.

6. Conclusions

In this paper, vibration control of a flexible structure which possesses transverse-torsional coupled vibration modes by using two active dynamic vibration absorbers was discussed. The controller was designed based on the H^∞ control theory. Theoretical calculations and control experiments were carried out and the following conclusions were obtained.

(1) The multiple active dynamic vibration absorber system is effective for reducing vibration of the structure which possesses transverse-torsional coupled vibration modes. Using the H^∞ controller, robustness to spillover and small parametric changes of control objects is achieved.

(2) The performance of the controller is improved by the proposed simple method for balancing control inputs using R. M. S. responses.

(3) The results hold regardless of whether the controller is designed with displacement weighting or velocity weighting.

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