The Simplest Identification Model of an Asymmetric Hydraulic System*

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A single-rod cylinder servo system driving a discontinuous load is a 4th-order nonlinear system. It can be simplified as a time-varying 1st-order system for typical low frequency applications. The time-varying parameters of such a system conceal the discontinuous loading effect, uncertainties and transient behavior of a single-rod cylinder and servovalve. To overcome these difficulties, the parameter identification process requires the characteristics of exponential convergence and unbiased parameter estimation. Here, the parameter identification process incorporates the 1st-order ARMAX model with PEDF algorithm. Experiments indicate quantitatively that this identification process is superior and more adaptive than high-order ARMA models with RLS algorithm.

Key Words: Asymmetric Hydraulic System, ARMAX, PEDF

1. Introduction

The single-rod hydraulic actuating system as shown in Fig.1 is a typical asymmetric hydraulic system. Here, the mechanical load includes an inertia and a single-side spring to simulate the effect of the discontinuous loading. The asymmetry implicitly associated with nonlinearity and discontinuity thus reduces the control accuracy and range of applications of the system. So far, there have been only a few studies aimed at solving this problem^{(1)~(3)}.

From the theoretical analysis, this is a 4th-order nonlinear system⁽⁴⁾. Because of its nonlinearity and discontinuity, it is very difficult to implement the identification process. In addition to the requirement that the input signal be sufficiently rich, its numerous parameter estimation can result in slow convergence⁽⁵⁾. In 1990, Watton⁽²⁾ assumed that it was a 2nd-order system, and was identified by a recursive

least square (RLS) method with a constant forgetting factor. If the order reduction is carried out for a high-order system, it might yield bursting in the identification process by RLS analysis due to its time-variety⁽⁶⁾. In 1987, Edge and Figueredo⁽³⁾ assumed a 3rd-order system and developed a robust parameter adaptation law of variable dead-band to directly adjust seven controller parameters. However, the problems of discontinuity, uncertainty and time-variety were not yet overcome.

Fig. 1 A single-rod cylinder servo system driving a discontinuous load

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For dealing with these difficulties and reducing the number of estimated parameters, the 4th-order nonlinear system can be simplified as a 1st-order time-varying and discontinuous system based upon the singular perturbation method⁽⁷⁾. In other words, the identified plant is assumed to be a 1st-order autoregressive moving average process with an exogenous input (ARMAX) model, in which the disturbance term is used to reduce the effect of noise and uncertainties caused by the order reduction.

Due to the features of time-variety and discontinuity of this system, the exponential convergence is a fundamental requirement to avoid bursting in the identification process. Here, the prediction-error directional-forgetting factor (PEDF) algorithm^{(8),(9)} is adopted because it incorporates the directional-forgetting and prediction-error-forgetting algorithms yielding the characteristic of exponential convergence⁽⁸⁾.

In this paper, the identification structure is composed of a 1st-order ARMAX model and a parameter adaptation routine PEDF. The adaptation and precision of this proposed method is verified by a variety of experiments. In addition, the 1st-order ARMAX model leads to less computer load than the 4th-order ARMA model, and is also proven to be the best candidate by comparing it with high order ARMAX models. However, the unbiased parameter estimation of 1st-order ARMAX model is guaranteed only if the posterior prediction error $\varepsilon(k)$ tends toward a white noise. The whiteness validation criterion⁽⁵⁾ is applied to prove the posterior prediction error $\varepsilon(k)$ is uncorrelated.

Nomenclature

A	: area
B	: damping coeff.
C	: leakage coeff.
d	: clearance
e	: priori prediction error
K	: spring stiffness or gain
M	: mass
P	: pressure
Q	: flow rate
$R(i)$	: autocorrelation function
$RN(i)$	: normalized autocorrelation function
T	: sampling time
u	: control effort
$U(\cdot)$	: step function
V	: volume
x	: position
α	: directional forgetting factor
β	: bulk modulus
Δ	: delta operator
ε	: posterior prediction error

Γ	: tuning parameter
ϕ	: regression vector
μ	: variable forgetting factor
θ	: parameter vector

Subscript

1, 2	: index of control volume
e	: external or extending
c	: command
i	: internal
o	: operating
p	: piston
r	: retracting
s	: supply
sv	: servovalve

Superscript

$\wedge$	: estimation
T	: transpose

2. System Description

The schematic diagram of the hydraulic system driving a discontinuous load associated with a tiny clearance is shown in Fig. 1. The hydraulic system includes a constant pressure source, the electrohydraulic servovalve and the single-rod cylinder, which is asymmetric in its extending and retracting processes. The continuities of the cylinder chambers are^{(2),(4)}:

$$Q_1 = C_i(P_1 - P_2) + C_e P_1 + A_1 \dot{x}_p + \frac{V_{1o} + A_1 x_p}{\beta} \dot{P}_1 \quad (1)$$

$$Q_2 = C_i(P_1 - P_2) - C_e P_2 + A_2 \dot{x}_p - \frac{V_{2o} - A_2 x_p}{\beta} \dot{P}_2 \quad (2)$$

The dynamic of the piston is represented as:

$$M \ddot{x}_p = P_1 A_1 - P_2 A_2 - B \dot{x}_p - K(x_p - d)U(x_p - d) \quad (3)$$

Here, $K(x_p - d)U(x_p - d)$ represents the behavior of single-side and discontinuous spring. Then, the flow rate through the servovalve is given as follows.

$$\left. \begin{aligned} \text{for the extending process } (u \geq 0): \\ Q_1 = K_{sv} u (P_s - P_1)^{1/2} \\ Q_2 = K_{sv} u P_2^{1/2} \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} \text{for the retracting process } (u < 0): \\ Q_1 = K_{sv} u P_1^{1/2} \\ Q_2 = K_{sv} u (P_s - P_2)^{1/2} \end{aligned} \right\} \quad (5)$$

Here, Eqs. (1) through (5) represent the system model. For the position (x_p) control, this model is described as nonlinear 4th-order, and its nonlinear natures include the compliance, flow rate and discontinuous load induced by clearance.

3. Simplified Model

Instead of using the traditional linearization technique, the simplified model is defined to operate at the low frequency. Then, neglecting the leakage effect,

Eqs. (1)–(2) can be simplified and rearranged as:

$$Q_i - A_i \dot{x}_p = \varepsilon_i \dot{P}_i \approx 0, \quad i=1, 2 \quad (6)$$

where

$$\varepsilon_1 = \frac{A_1 x_p + V_{10}}{\beta}; \quad \varepsilon_2 = \frac{A_2 x_p - V_{20}}{\beta}.$$

Here, $\varepsilon_i \dot{P}_i$ indicates the compressibility effect, and exists only at very high pressure rate. In other words, due to operation at low frequency and ignoring the transient behavior of the servovalve, $\varepsilon_i \dot{P}_i$ can thus be eliminated⁽⁷⁾. In general, the load inertia and damping effect in Eq. (3) are also negligible at the low frequency of operation. Then, treating the tiny clearance d as an uncertainty, Eq. (3) can be simplified as:

$$P_1 A_1 - P_2 A_2 - K x_p U(x_p) = 0. \quad (7)$$

Now, we consider the extending process ($u > 0$). Substituting Eq. (4) into Eq. (6), the pressure dynamics are then expressed as algebraic equations.

$$P_1 = P_s - \left(\frac{A_1 \dot{x}_p}{K_{sv} u} \right)^2 \quad (8)$$

$$P_2 = \left(\frac{A_2 \dot{x}_p}{A_{sv} u} \right)^2. \quad (9)$$

Substituting Eqs. (8)–(9) into Eq. (7), the dynamic of the velocity $\dot{x}_p$ becomes:

$$\dot{x}_p = K_{sv} u \left(\frac{P_s A_1 - K x_p U(x_p)}{A_1^3 + A_2^3} \right)^{1/2}. \quad (10)$$

Then, based on the linearization technique⁽⁷⁾, the transfer function between the control effort u of the servovalve and the single-rod cylinder position x_p is condensed to

$$\frac{x_p}{u} = \frac{b_e}{s + a_e} \quad (11.a)$$

where

$$b_e = K_{sv} \left(\frac{P_s A_1 - K x_p U(x_p)}{A_1^3 + A_2^3} \right)^{1/2} \Big|_{x_p=x_{p0}} \\ a_e = \frac{K_{sv} K u U(x_p)}{2[(A_1^3 + A_2^3)(P_s A_1 - K x_p U(x_p))]}^{1/2} \Big|_{x_p=x_{p0}, u=u_0}.$$

Applying similar procedures for the retracting process, the simplified model can be represented as:

$$\frac{x_p}{u} = \frac{b_r}{s + a_r} \quad (11.b)$$

where

$$b_r = K_{sv} \left(\frac{P_s A_2 + K x_p U(x_p)}{A_1^3 + A_2^3} \right)^{1/2} \Big|_{x_p=x_{p0}} \\ a_r = - \frac{K_{sv} K u U(x_p)}{2[(A_1^3 + A_2^3)(P_s A_2 + K x_p U(x_p))]}^{1/2} \Big|_{x_p=x_{p0}, u=u_0}.$$

The system order of both the extending and retracting processes is simplified to 1st-order, but their parameters vary with the operating point and the direction. Transferring to the discrete form with zero-order-hold, the simplified model is

$$H_i(z^{-1}) = \frac{x_p}{u}(z^{-1}) = \frac{b_i(1 - e^{-a_i T})z^{-1}}{a_i(1 - e^{-a_i T}z^{-1})} = \frac{b_1 z^{-1}}{1 + a_1 z^{-1}} \quad (12)$$

where

$$a_1 = -e^{-a_i T}$$

$$b_1 = \frac{b_i(1 - e^{-a_i T})}{a_i}$$

$i=e$ represents the extending process

$i=r$ represents the retracting process.

4. Identification Scheme

It is well known that the parametric effects of order reduction and the uncertainties of this simplified model might result in noise which strongly influences the accuracy of its parametric estimation. To achieve an unbiased parameter estimation, a 1st-order ARMAX model is adopted here. The posterior prediction error $\epsilon(k)$ of the ARMAX model is ideally a white noise. Here, it is used to simulate the noise effect caused by order reduction and uncertainties. That is to say, $\phi(k-1)$ and $\epsilon(k)$ are uncorrelated, and this guarantees the unbiased parameter estimation of a system regardless of the presence of noise. The discrete ARMAX model can then be presented as⁽⁵⁾:

$$(1 + a_1 z^{-1})x_p(k) = b_1 z^{-1}u(k) + (1 + c_1 z^{-1})\epsilon(k), \quad (13)$$

The posterior adjustable predictor will be given by

$$\hat{x}_p(k) = \hat{\theta}(k)^T \phi(k-1), \quad (14)$$

and the posterior prediction error is

$$\epsilon(k) = x_p(k) - \hat{x}_p(k) = x_p(k) - \hat{\theta}(k)^T \phi(k-1), \quad (15)$$

in which

$$\hat{\theta}(k)^T = [\hat{a}_1(k), \hat{b}_1(k), \hat{c}_1(k)] \\ \phi(k-1)^T = [-x_p(k-1), u(k-1), \epsilon(k-1)].$$

In the presence of random disturbance corresponding to the ARMAX model (13), $\epsilon(k-1)$ will asymptotically tend toward a white noise, which guarantees the unbiased estimation of a_1 and b_1 ⁽⁵⁾.

For this system, it drives a discontinuous load and then yields the phenomenon of a breakpoint in the identification process. The classical RLS algorithm continuously forgets the old information depending on a forgetting factor, while the new dynamics information is received. Such behavior makes the RLS algorithm sensitive to noise and impracticable for the estimation of time-varying and discontinuous parameters^{(6),(8)}. But these two defects can be easily overcome by the combination of 1st-order ARMAX and PEDF. The PEDF suitably incorporates the features of the directional-forgetting and prediction-error-forgetting algorithms. Its characteristic of exponential convergence thus implies tracking capability for a time-varying and discontinuous system.

The priori predictor $\hat{x}_p(k)$ and the priori prediction error $e(k)$ are given by

$$e(k) = x_p(k) - \hat{x}_p(k) = x_p(k) - \hat{\theta}(k-1)^T \phi(k-1) \quad (16)$$

and the estimation of the parameter vector can be

recursively performed with the PEDF algorithm^{(8),(9)}.

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1)\phi(k-1)}{1+r(k)}e(k) \quad (17)$$

$$r(k) = \phi(k-1)^T P(k-1) \phi(k-1)$$

$$\mu(k) = \max \left\{ \mu_0, 1 - \frac{e(k)^2}{(1+r(k))\Gamma} \right\}, \mu_0 \in (0, 1) \quad (18)$$

$$\alpha(k) = \begin{cases} \mu(k) - \frac{1-\mu(k)}{r(k)} & \text{if } r(k) > 0 \\ 1 & \text{if } r(k) = 0 \end{cases} \quad (19)$$

$$P(k) = P(k-1) - \frac{P(k-1)\phi(k-1)\phi(k-1)^T P(k-1)}{\alpha(k)^{-1} + r(k)} + \omega I. \quad (20)$$

Here, $\mu(k)$ is a variable forgetting factor⁽⁶⁾ and Γ is a tuning parameter. The exponential convergence can be achieved by adding a small positive term ω to the covariance matrix equation (20)⁽⁸⁾.

Due to the exponential forgetting and the variable forgetting factors, all directions of the parametric adaptation space are equally considered. However, if the incoming information or observation vector is nonuniformly distributed, it is advisable to perform a selective amplification of $P(k)$ by means of Eqs. (19)–(20) to compensate for the possible non-uniform distribution of information conveyed by fresh data. The combination of the directional-forgetting and prediction-error-forgetting algorithm would thus improve the transient estimation⁽⁹⁾.

5. Experimental Studies

The different identification algorithms are tested at the closed-loop position control system as shown in Fig. 2 and its typical parameters are $M=28$ kg, $P_s=4.41$ MPa, $K=4.91 \times 10^4$ N/m, $A_1=1.256 \times 10^{-3}$ m², $A_2=1.001 \times 10^{-3}$ m² and the clearance d is approximately 0.01 m. All of the various identification algorithms cannot be simultaneously implemented. To evaluate and compare their performance and convergence speeds fairly, all of the various parameter adaptation algorithms are performed off-line. Alternatively, each of them can perform on-line parameter estimation. The regressor vector of the identification process is filled with the input/output data as Fig. 4(a), which is obtained from an experiment on a simple

Fig. 2 Block diagram of hydraulic control system

proportional controller as Fig. 2.

For investigating the effectiveness of the proposed identification algorithm, the performance index is defined as the summation of the absolute posterior prediction error $J_\epsilon = \sum_{k=1}^n |\epsilon(k)|$, which will attenuate the effect of the initial guess. Then, the proposed algorithm would compare with 1st-, 2nd- and 4th-order ARMA models incorporating the classical recursive least squares (RLS) and a forgetting factor of 0.97⁽¹⁰⁾. The typical parameters of the PEDF and $\Gamma=0.4$, $\mu_0=0.8$ and $\omega=0.002$ ⁽⁹⁾.

The identification structure presented is implemented on an Intel 80486 microcomputer system and the sampling time is 7 ms. To evaluate the computational efficiency, Table 1 shows the computer times of various algorithms, and it indicates that the computer time of 1st-order ARMAX with PEDF is much less than that of unreduced 4th-order ARMA with RLS. Next, the following cases are performed to verify the proposed algorithm.

• Step command

A simple proportional controller is adopted as shown in Fig. 2. Using the step command and obtained output from the experiment, the transient prediction performances of various identification algorithms can be observed and evaluated. Fig. 3(a) shows the actual system response of the experiment and the prediction responses of various identification algorithms. These results prove that the 1st-order ARMAX with PEDF provides the best agreement with the system transient response of the experiment. In the steady state, their performance has no significant difference among all identification algorithms.

The corresponding error indices J_ϵ are shown in Fig. 3(b). In the period of 0.05–0.2 sec, the piston moves at full speed and the control effort is almost constant. Hence, during this period, the dynamics of $\dot{P}_1$ and $\dot{P}_2$ in Eqs. (1)–(2) are negligible^{(4),(10)}. The order of the overall system is then reduced to 2nd-order. Fig. 3(b) proves that the 2nd-order ARMA model would accumulate less J_ϵ than the 4th-order ARMA model. However, in the period of 0–0.05 and 0.2–0.25 sec, no specific term can be neglected. Thus, no order reduction is available and a 4th-order model is more reliable. The tendency shown in Fig. 3(b)

Table 1 Comparisons of computational requirement

model	computational requirement
1st order ARMA & RLS	0.5 ms/step
2nd order ARMA & RLS	0.8 ms/step
4th order ARMA & RLS	1.62 ms/step
1st order ARMAX & PEDF	0.63 ms/step

Fig. 3 Performance of step command, (a): system response and estimated responses of various models, (b): J_ϵ . ($\circ \circ \circ$: system response, $\cdots$: 1st-order ARMA model, $-\cdot-: 2nd-order ARMA model, $---$: 4th-order ARMA model, $-$: 1st-order ARMAX model)$

Fig. 4 Performance of 0.2 Hz square wave command (for symbols please see Fig. 3)

proves this viewpoint. In summary, the 1st-order ARMAX with PEDF is suitable for application to the above situations and Fig. 3 reveals its capability for accurate prediction.

● Periodic command

This experiment is to verify the accuracy and convergence of estimated parameters. The measured input and output data of a case with a square wave command are used as the regressor vectors of the identification routine and shown in Fig. 4(a). From Fig. 4(b), it is evident that the J_ϵ accumulation of ARMAX is more sluggish than those of ARMA models. The main reason is that, at the discontinuous command, the system parameters are strongly time-dependent, and the ARMA models thus cannot follow the discontinuity correctly and cause significant errors.

Figs. 5(a) and 5(b) show the corresponding estimated parameters of the 1st-order ARMAX model and ARMA models. The estimated parameters a_1 and b_1 are strongly time-varying. In particular, when the servovalve is reversed or the piston passes through the contact of the single-side spring, this will result in the discontinuous system parameters. As shown in Figs. 5(a) and 5(b), a_1 and b_1 do show a significant jump at 0.58 sec or 5.58 sec. Then, comparing the convergence, the ARMAX model with PEDF is much faster than the ARMA model with RLS.

Fig. 5 Estimated parameters of 1st-order ARMAX and ARMA models (for symbols please see Fig. 3)

Regarding the accuracy of estimated parameters, a_1 of both ARMA and ARMAX models is nearly -1 at steady state. However, since the cylinder is asymmetric, there exists a tiny and negative control effort to keep the piston immovable. Thus, from Eqs. (11)–(12), $a_1 \lesssim -1$ when the piston is steady at the extending end and $a_1 \gtrsim -1$ when the piston is steady at the retracting end; the estimated a_1 of the ARMAX model in Fig. 5(a) supports this explanation, but the ARMA model cannot demonstrate this asymmetric behavior completely.

In order to understand the transient behavior of the servovalve, the cosine wave command is adopted. Fig. 6 shows that J_ϵ of the 1st-order ARMAX model is still the smallest, and it proves that the 1st-order ARMAX model can exactly describe the dynamic behaviors of this system even if the servovalve is discontinuously varied. In addition, it indicates that RLS with a constant forgetting factor applied for the identification of a time-varying system might result in bursting, which occurs in the 4th-order ARMA model.

6. Validation of Simplified Model

Here, a 1st-order ARMAX model is proposed to predict system response, which includes both the transient and steady state behavior. In other words, the

Fig. 6 J_ϵ of 0.2 Hz cosine wave command (for symbols please see Fig. 3)

Fig. 7 The decay rates of 4th-order ARMAX model's poles

only pole of the 1st-order ARMAX model is expected to dominate system performance. To validate this assumption, it is necessary to examine whether the dominant pole of a high order ARMAX model is similar to the pole of the 1st-order ARMAX model.

Fig. 7 indicates the decay rates of the corresponding 4th-order ARMAX model's poles, by mapping z -domain into s -domain as $z = e^{sT}$. It is evident that there is a significant dominant pole relatively close to the origin, and may be interpreted that the dynamics of the remaining 3 poles can be ignored in comparison with the dominant pole. Fig. 8 shows the dominant poles of 1st-, 2nd- and 4th-order ARMAX models. They unanimously confirm that the 1st-order ARMAX model is valid for both transient and steady state behavior.

Then, to verify the characteristic of unbiased parameter estimation of the 1st-order ARMAX model, a biased noise is added to the control effort as shown in Fig. 2. This is to simulate and enhance the effect of noise caused by the order reduction and servovalve transient dynamics. The noise is randomly distributed between 0 and 1 volt., and the noise/signal ratio is 0.0894. Here, ΔJ_ϵ is defined as the difference of J_ϵ with and without noise. Fig. 9 shows ΔJ_ϵ of various models, and indicates that ΔJ_ϵ of the ARMAX model is much smaller than ΔJ_ϵ of the ARMA models. The error $\epsilon(k)$ of the ARMAX model in Eq. (13) conceals a moving average of white noise; thus, its estimation would match the true system response even if the external biased noise existed.

If the posterior prediction error $\epsilon(k)$ is a white noise, it means that $\epsilon(k)$ is not correlated with any other variables. It implies:

$$\lim_{k \rightarrow \infty} E\{\epsilon(k)\epsilon(k-i)\} = 0, \quad i = 1, 2, \dots, n_A.$$

Thus, the whiteness test is performed. The associated autocorrelation functions are defined as:

Fig. 8 The dominant poles of ARMAX models (—: 1st-order ARMAX model, ---: 2nd-order ARMAX model, ...: 4th-order ARMAX model)

Table 2 The whiteness tests of various models

Autocorrelation functions	1st-order ARMAX & PEDF	1st-order ARMA & RLS	2nd-order ARMA & RLS	4th-order ARMA & RLS
$R(0)$	0.0016	0.0758	0.0318	0.0244
$RN(0)$	1.00	1.00	1.00	1.00
$RN(1)$	0.0772	0.6467	0.022	0.0312
$RN(2)$	0.0329	0.6083	0.0476	0.0634
$RN(3)$	0.0420	0.5635	0.0994	0.0574
$RN(4)$	0.0379	0.488	0.0493	0.0403

Fig. 9 ΔJ_ϵ of 0.2 Hz square wave command with and without biased noise (for symbols please see Fig. 3) Fig. 10 J_ϵ of 1 Hz square wave command (for symbols please see Fig. 3)

$$R(i) = \frac{1}{N} \sum_{k=1}^N \epsilon(k) \epsilon(k-i)$$

$$RN(i) = \frac{R(i)}{R(0)}, \quad i=0, 1, 2, \dots, n_A \quad (21)$$

where

n_A : degree of $A(z^{-1})$

N : the number of samples

$RN(i)$: the normalized autocorrelation.

If the value of $RN(i)$ is close to zero, it means that the posterior prediction error $\epsilon(k)$ asymptotically tends toward a white noise and a practical validation criterion⁽⁵⁾ is given as :

$$|RN(i)| \leq \frac{2.17}{\sqrt{N}}, \quad \text{for } i=0, 1, 2, \dots, n_A. \quad (22)$$

That is to say, the violation of the whiteness test criterion (22) represents unsatisfactory and biased parameter estimations.

Table 2 shows that the mean average $R(0)$ of 1st-order ARMAX model with PEDF is smaller than any order ARMA model with RLS. The normalized autocorrelation functions $|RN(i)|$ of 1st- and 2nd-order ARMA models violate the criterion. Thus, the appearance of biased estimation is expected, and the uncertainties of order reduction strongly affect the accuracy of estimation. Thus, $\epsilon(k)$ of the 1st-order ARMAX model is very similar to a white noise and is uncorrelated to other variables.

Because of its lower computational load, order validation, unbiased estimation and uncorrelated behavior, the 1st-order ARMAX model is shown to be

the best estimation model for identification.

7. Parametric Studies

In order to investigate the adaptation of 1st-order ARMAX model with PEDF, various experiments are performed.

● Effect of operating frequency

For the simplified model, the main restriction is to operate at low frequency ; thus, the transient behaviors of the servovalve and inertia are negligible. The experiments indicate that the system response cannot follow the command correctly when the operating frequency is greater than 1 Hz. Due to the tracking limitation, Fig. 10 examines the performance indices J_ϵ of various models at 1 Hz. The results demonstrate that J_ϵ has significant increasing for all models and further proves that the 1st-order ARMAX model is still the most suitable for system estimation.

● Effect of load inertia

The inertia term is also neglected in the simplified model. To realize its adaptation, Fig. 11 shows J_ϵ of 4th-order ARMA and 1st-order ARMAX models for $M=28$ and 128 kgs. Large inertia always reduces the system stability ; thus, the response has obvious oscillation. Although the 4th-order ARMA model includes the inertia effect, the RLS with a fixed forgetting factor cannot trace the transient oscillation correctly. Therefore, the estimated response of 4th-order ARMA with RLS thus cannot match the system response, and induces significant error. The data in Fig. 11 numerically prove that the 1th-order ARMAX

Fig. 11 J_ϵ of various inertia, $M=28$ kg and $M=128$ kg (for symbols please see Fig. 3)

Fig. 12 J_ϵ of various clearance $d=10$ mm and $d=30$ mm (for symbols please see Fig. 3)

with PEDF is more adaptive than the 4th-order ARMA with RLS for various inertia.

● Effect of clearance

The clearance is a typically discontinuous factor of a mechanical driving system, and is included in the nonlinear 4th-order model. However, the simplified model treats the clearance as an uncertainty. To examine the adaptation of this treatment, Fig. 12 shows the performance indices J_ϵ of 4th-order ARMA and 1st-order ARMAX models for various clearances. The data in the figure numerically prove that the influence of both models are similar, and thus, this uncertainty treatment would not reduce the adaptation of 1st-order ARMAX model.

8. Conclusions

The 1st-order ARMAX model with PEDF was applied for the identification of a single-rod cylinder driving a discontinuous load. Theoretical analysis and experimental results prove the effectiveness and adaptation of this algorithm. Based upon experimental studies, the following major conclusions can be drawn:

1 The prediction capability of the 1st-order ARMAX model with PEDF yields good agreement with the system response, in comparison to any order

ARMA model with RLS.

2 This model is fairly insensitive to noise and variation of various operating conditions. Also, the unbiased parameter estimation and exponential convergence of estimated parameters are verified.

3 The computational requirement of this algorithm is less than that of 2nd- and 4th-order ARMA models.

Thus, the 1st-order ARMAX model with PEDF is very useful for the on-line identification of the single-rod cylinder driving a discontinuous load, and due to its order reduction, it will be very useful for future controller design.

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