Finite Element Analysis of Thermal Behaviour in Metal Machining*

(2nd Report, Determination of Energy Balance and Its Application to Three-Dimensional Analysis)

Katsuhiro MAEKAWA**, Yoshihiro NAKANO***
and Takeaki KITAGAWA****

An inverse analysis approach using the finite element method is proposed to determine the energy balance or the energies flowing into a tool, workpiece and chip per unit volume of material removed in two-dimensional machining. Once the energy flow rates are obtained, a three-dimensional temperature distribution for complicated tools such as an end mill can be analysed easily and efficiently. The method proposed is employed to clarify the effect of thermophysical properties on the energy balance. The energy flowing into the tool is most enhanced when a low-thermal-conductivity workpiece is machined by a high-thermal-conductivity tool with a coolant; wet turning of titanium alloy by a diamond tool is a typical case. It is found that temperature change of a carbide square end mill ranges from 400 to 800 °C in titanium machining at a cutting speed of 314 m/min because of a low energy flow rate into the tool of around 0.04 and the action of a coolant. The suitability of the present method is confirmed by comparison with the Loewen-Shaw model as well as by temperature measurement.

Key Words: Cutting, Cutting Temperature, Inverse Problem, Finite Element Method, Energy Balance, Heat Conduction, Heat Transfer, Cutting Tool, Milling, Cutting Fluid

1. Introduction

Few investigations have been made in terms of three-dimensional thermal behaviour in metal machining. An analytical model has been proposed for the prediction of cutting temperature only in a turning process⁽¹⁾. As the shape of the cutting edges becomes complicated, such as in end milling⁽²⁾, it is difficult to carry out a precise thermal analysis which includes chip formation. Finite element analysis is applicable

to it, but it is worth proposing an approximate method of determining the energy balance during machining^{(3),(4)}. If heat generation and its rate flowing into the tool are specified, thermal analysis of the tool can be separately conducted in a simple manner. A theoretical approach, though desirable, is not straightforward as far as the determination of the heat flow rate in three-dimensional machining is concerned.

In the present paper an inverse analysis using the finite element method is proposed for this purpose, being employed to clarify the relation between the energy balance and thermophysical properties of work and tool materials. Further, the energy balance thus determined is applied to the analysis of three-dimensional thermal behaviour in end milling. The proposed method is validated by comparison with the Loewen-Shaw model⁽³⁾ and temperature measurement.

2. Determination of Energy Balance

Figure 1 shows the definition of heat flow rates

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** Department of Mechanical Engineering, Ibaraki University, 4-12-1 Nakanarusawa, Hitachi 316, Japan

*** Hitachi Koki Co., Ltd., 1060 Takeda, Hitachinaka 312, Japan

**** Department of Mechanical System Engineering, Kitami Institute of Technology, 165 Koen-cho, Kitami 090, Japan

Fig. 1 Definition of heat flow into tool, workpiece and chip

based on the shear plane cutting model: (a) after Loewen and Shaw⁽³⁾ and (b) the present model. The Loewen-Shaw model defines the heat flow rate into the tool as the ratio $1 - R_2$, whose standard is the heat flux consumed on the rake face per unit area and unit time. On the other hand R_t in the present model is the ratio to the specific cutting energy u required for the removal of unit volume. As for the heat flow rate into the workpiece through the shear plane, $1 - R_1$ and R_w are defined for the Loewen-Shaw and present models respectively. Accordingly, the following relations hold between the two models:

$$\left. \begin{aligned} R_t &= (1 - R_2) u_2 / u \\ R_w &= (1 - R_1) u_1 / u \end{aligned} \right\} \quad (1)$$

where u_1 and u_2 are the shear and friction energies per unit removal volume, respectively, and $u = u_1 + u_2^{*1}$. The heat fluxes on the shear plane and on the rake face are respectively given by

$$\left. \begin{aligned} q_1 &= u_1 V \sin \phi / J \\ q_2 &= u_2 Vf / (Jl_c) \end{aligned} \right\} \quad (2)$$

where V is the cutting speed, ϕ is the shear angle, f is the feed, l_c is the chip contact length and J is the mechanical equivalent of heat.

Let us define three models: the whole model consisting of workpiece, tool and chip; the tool model, and the workpiece model comprising work material below the shear plane. The boundary heat

flux $R_t q$ is loaded at the chip contact area in the tool model, whereas $R_w q$ is given on the shear plane in the workpiece model, being defined as

$$\left. \begin{aligned} R_t q &= R_t u Vf / (Jl_c) \\ R_w q &= R_w u V \sin \phi / J \end{aligned} \right\} \quad (3)$$

On the other hand, it is unnecessary to specify the energy balance in the whole model, as shown in Fig. 1 of the preceding paper⁽⁶⁾, as only q_1 and q_2 of Eq. (2) are needed.

The energy flow rate into the tool, R_t , can be determined as follows:

(1) Obtain the temperature distribution inside the tool, θ_i , using the whole model.

(2) Assuming a value of R_t , obtain the temperature distribution inside the tool, θ'_i , using the tool model.

(3) Calculate the total temperature difference, E , according to

$$E = \sqrt{\sum_{i=1}^n (\theta_i - \theta'_i)^2} \quad (4)$$

(4) Repeat procedures (2) and (3) with changing R_t until E becomes smallest.

The same procedures can be applied to the determination of the energy flow rate into the workpiece, R_w , except repeating procedures (2) and (3) using the workpiece model. Temperature calculation can be carried out using either the finite element method or the finite difference. In the present paper the finite element method is employed, so that θ_i and θ'_i are the node temperatures and n is the total node number within the tool for the tool model. The method proposed is not restricted to the shear plane cutting model in conjunction with experimental cutting data, if the cutting energy per unit removal volume can be specified. The temperature θ_i using the whole model can be obtained by the machining simulation method based on material characteristics alone⁽⁷⁾.

The equation to be solved is the energy equation for the whole model, which should take into account heat transfer by the movement of the chip. On the other hand, the tool model only requires the thermal conduction equation without the velocity term. This is an advantage of using the tool model since the solution procedure becomes much simpler. Friction heat flux on the rake face is assumed to have a triangular distribution⁽⁶⁾ increasing from the chip leaving point towards the cutting edge in the whole model, whereas a uniform distribution^{*2} shown in Fig. 1(b) is given in the tool model. The distribution of heat flux on the shear plane, which stems from plastic

*1 Strictly speaking, surface energy and the energy stemming from momentum change at the shear plane should be taken into account. However their contribution to the total cutting energy is negligibly small at less than 1%⁽⁵⁾.

*2 The distribution of heat flux can also be determined using the present method; this will be the concern of a subsequent paper.

deformation, is postulated to be uniform in both models.

3. Influence of Thermophysical Properties on Energy Balance

Analytical results based on the shear plane cutting model in conjunction with cutting data⁽⁶⁾ will be shown in the present study. Figure 2 compares the cutting temperatures obtained using (a) the whole model and (b) the tool model when machining a Ti-6Al-6V-2Sn alloy in air using a K10 carbide tool. The energy flow rate R_t is found to be 0.076. Figure 3 shows the cutting temperatures in the combination of a 0.43%C carbon steel and a HSS tool: (a) represents the whole model and (b) the tool model where R_t is assumed to be 0.013. The data used in these analyses are the same as in Table 1 of the preceding paper⁽⁶⁾. Predicted temperatures using both models are in good agreement despite different combinations of the materials, but some discrepancy can be seen in the vicinity of the cutting edge.

Fig. 2 Comparison of tool temperature between whole and tool models
Workpiece: titanium alloy, tool: carbide K10, cutting speed: 50 m/min, feed: 0.1 mm/rev, dry cutting

Fig. 3 Comparison of tool temperature between whole and tool models
Workpiece: carbon steel, tool: HSS, cutting speed: 61 m/min, feed: 0.254 mm/rev, dry cutting

Figure 4 shows the variation of the total error E evaluated by Eq. (4) in titanium machining using various tools in air. There exists a value of R_t which minimises E for each tool. Low-thermal-conductivity tools such as ceramics and high speed steel (HSS) tend to yield large errors. When machining steel, whose thermal conductivity is lower than titanium alloy, E is smaller for every cutting tool. As can be seen in Fig. 2, a complicated temperature gradient close to the cutting edge is not realised in the tool model, and this tendency is very apparent when machining a low-thermal-conductivity workpiece using a low-thermal-conductivity tool. This is partly because heat transfer by a moving chip is neglected in the tool model. Uniform heat flux on the rake face is also a plausible reason for this.

Figure 5 shows the relation between the energy flow rate into the tool R_t and the thermal conductivity of a tool, K , when machining the titanium alloy and carbon steel. The solid lines denote dry machining and the broken lines wet machining where the

Fig. 4 Determination of energy flow rate into tool for various tools: cutting conditions are the same as Fig. 2

Fig. 5 Energy flow rate into tool vs. thermal conductivity of tool

coefficient of heat transfer of the coolant⁽⁹⁾ is assumed to be $5000 \text{ Wm}^{-2}\text{K}^{-1}$. Cutting conditions were the same as in Figs. 2 and 3. In titanium machining the flow rate is doubled compared with steel machining irrespective of the thermal conductivity of the tool, and cutting temperature is also higher⁽¹⁰⁾. The use of the coolant enhances heat transfer into the tool by as much as double that in dry machining, leading to a drop in rake temperature⁽⁶⁾.

The influence of the heat transfer coefficient h on R_t can be investigated further, as shown in Fig. 6, an $h-R_t$ diagram which illustrates the cases of titanium alloy-K10 (solid line) and carbon steel-HSS (broken line). The cutting data are assumed to be unchanged with the variation of h . As expected, R_t rises as h increases, but this rate flattens out above $5000 \text{ Wm}^{-2}\text{K}^{-1}$.

Similarly, the energy flow rate into the workpiece R_w can be determined. Figure 7 compares the cutting

Fig. 6 Energy flow rate into tool vs. coefficient of heat transfer of coolant

Fig. 7 Comparison of tool temperature between whole and workpiece models. Workpiece: titanium alloy, tool: carbide K10, cutting speed: 50 m/min, feed: 0.1 mm/rev, wet cutting

temperatures near the shear plane obtained using (a) the whole model and (b) the workpiece model when machining titanium alloy using a K10 carbide tool with a coolant. The energy flow rate R_w is found to be 0.426, which minimises the total error E of Eq. (4). Temperature distributions below the shear plane as well as on the finished surface coincide in both models. In the case of wet machining of carbon steel using an HSS tool, R_w is 0.672; high thermal conductivity of the steel enhances heat flow into the workpiece. From a computational point of view, the workpiece model is less convenient than the tool model, since the governing equation is the same as the whole model whereas the tool model only requires the thermal conduction equation. However, R_w can be determined uniquely.

Thus the energy flow rates R_t and R_w can be identified by the inverse approach proposed. Accordingly the energy flow into a chip R_c can be obtained as $1 - (R_t + R_w)$.

4. Thermal Behaviour in End Milling

Once the energy balance in two-dimensional machining is determined, it can be applied to the analysis of thermal behaviour in three-dimensional machining. End milling as shown in Fig. 8 may serve as an example. A side of a titanium block is machined using a two-flute square end mill (K10), where the edge shape including rake angle, helix angle and core diameter are important from the viewpoints of machining accuracy and tool life. For simplicity, however, it is assumed that the cutting edge and the chip are neglected in the present analysis. Only heat flux is given intermittently at the tool-workpiece interface. The heat flux q is approximated according to Mashiko's empirical equation of the circumferential force $\bar{P}_r$ in slab milling⁽¹¹⁾ as

Fig. 8 Analytical model of end milling

$$\bar{P}_T = \frac{2^{1-\epsilon}}{2-\epsilon} \cdot \frac{bS}{\pi N} \left(\frac{d}{D} \right)^{1-(\epsilon/2)} (p_s)_f \quad (5)$$

and

$$q = \frac{\bar{P}_T V}{J l_a b} \quad (6)$$

where $(p_s)_f$ is the specific cutting force at the feed f , ϵ is the coefficient of $(p_s)_f$, b is the axial cut width, d is the radial cut width, D is the diameter of the end mill, S is the table feed rate, N is the spindle speed (rpm), V is the cutting speed, l_a is the undeformed chip length and J is the mechanical equivalent of heat. In the simplified model shown in Fig. 8, the heat source $(R_t + R_w)q$ acts periodically on the tool-workpiece interface, whereas the tool model requires $R_t q$ acting intermittently on the tool surface. In addition, the table feed per tool rotation is so small that heat transfer through workpiece movement can be neglected. Accordingly, the equation to be solved in the model shown in Fig. 8 is the three-dimensional thermal conduction equation.

Table 1 lists the cutting conditions, experimental data and energy balance used in the analysis, in which titanium alloy is machined using a K10 carbide solid end mill with a coolant. The values of R_t and R_w are revised since the cutting conditions are different from those in Section 3. Thermophysical properties of the work and tool materials are the same as in Table 2 in the preceding paper⁽⁶⁾, and the coefficient of heat transfer of the coolant⁽⁹⁾ is assumed to be 5000 Wm⁻²K⁻¹. A linear tetrahedron is used for the finite element discretization.

Figure 9 represents a snapshot of the cutting temperature when a cutting edge has just finished an action, the spindle speed being 10 000 rpm which gives a circumferential speed of 314 m/min; the cut distance is 3 mm. The temperature changes periodically, as will be shown later, and a maximum temperature of 800°C is attained at the contact area. The maximum temperature is limited to below 900°C due to the

action of the coolant.

Figure 10 shows the temperature distribution using the tool model under the same cutting conditions as Fig. 9. The grey portion denotes the heat source. The temperature profile is fairly similar to Fig. 9(b) which was obtained using the workpiece-tool model, which, however, requires twice the calculation time. Besides, the calculation time was reduced as half as the workpiece-tool model.

Figure 11 demonstrates the variation of temperature at the center of the heat source in the tool model. The abscissa is the number of tool rotations from the beginning of cut. The solid line denotes a result at a circumferential speed of 314 m/min, whereas the dashed line represents the case at 31.4 m/min. As the rotation speed increases, the maximum temperature and the temperature change per cycle are both increased. However, the rate of temperature increase is not proportional to the cutting speed; a tenfold speed increase brings about only a twofold increase in cutting temperature. This is due to the decrease in the energy flow rate into the tool with increasing cutting speed, as shown in Table 1. The dotted line in Fig. 11

Fig. 9 Temperature distribution on workpiece and tool obtained using tool-workpiece model at $V=314$ m/min

Table 1 End-milling data used in analysis		
Workpiece	Ti-6Al-6V-2Sn	
Tool	carbide K10, rake angle = 6°, $D=10$ mm, 2 flutes, helix angle = 0°	
Feed per tooth	$f=0.1$ mm	
Width of cut	$d=0.67$ mm	
Depth of cut	$b=5$ mm	
Coolant	10 l/min, ($h=5000$ Wm ⁻² K ⁻¹)	
Specific cutting force	$(p_s)_{0.1}=2.41$ kN/mm ²	
Exponent	$\epsilon=0.22$	
Average cutting force	$\bar{P}_T=66.9$ N	
Cutting speed	$N=1000$ rpm ($V=31.4$ m/min)	$N=10000$ rpm ($V=314$ m/min)
Cutting time per tooth	60 ms	6 ms
Heat flux	$q=2.68$ W/mm ²	$q=26.8$ W/mm ²
Flow rate into tool	$R_t=0.203$	$R_t=0.036$
Flow rate into workpiece	$R_w=0.478$	$R_w=0.323$

Fig. 10 Temperature obtained using tool model

Fig. 11 Tool temperature vs. number of tool rotations in titanium milling

shows the maximum temperature per cycle at 1000 rpm.

5. Discussion

It is postulated that the analytical temperature distribution using the whole model must be correct when using the inverse method proposed to determine the energy balance during metal machining. Rake temperature in the two-dimensional analysis has been validated by comparison with experiment in the preceding paper⁽⁶⁾. A simulation experiment of end milling was performed to check the cutting temperature predicted in Section 4, where many assumptions were introduced. In particular, the energy balance obtained by the two-dimensional analysis is extended to three-dimensional end milling.

Figure 12 shows (a) an experimental setup and (b) the rake temperature measured at a distance of 0.12 mm from the cutting edge. A disc of titanium alloy with different cut lengths is turned in air using a K10 carbide tool. Measurement begins when a constant temperature change is confirmed. Examining the dashed line of Fig. 11 where cutting speed and cutting time per edge are similar to Case A of Fig. 12, we observe that a maximum temperature of 550°C is reached in both instances. Temperature inside the tool is not measured. Case E of Fig. 12(b) denotes the temperature in continuous turning. A short cutting time per edge also plays a substantial role in the decrease in rake temperature during intermittent machining.

Finally, discussion is extended to the verification of the energy flow rates predicted by the present method. Since it is difficult to measure the energy balance precisely during three-dimensional machining, the two-dimensional Loewen-Shaw model⁽³⁾ is employed for comparison. Figure 13 shows the relation between the energy flow rate into the tool and the

(a) Schematic of measurement

(b) Record of temperature

Fig. 12 Local rake temperature in intermittent turning of titanium alloy by K10 carbide tool: $V=30$ m/min, $f=0.2$ mm/rev, rake angle $\alpha=5^\circ$, dry cutting

Fig. 13 Comparison of energy flow rate into tool between present method and Loewen-Shaw model: cutting conditions are the same as Fig. 2

thermal conductivity of the tool when titanium alloy and carbon steel are machined orthogonally in air, where R_2 of Fig. 1(a), obtained by the Loewen-Shaw model, is converted into R_t by Eq. (1). Compared with the corresponding result in Fig. 5, R_t predicted by the Loewen-Shaw model (solid lines in Fig. 13) is much larger. This is partly because the finite element analysis takes into account a real tool shape including an insert clamped in a 0.55% C tool holder, whereas the Loewen-Shaw model postulates a semi-infinite

tool made of a single material. Therefore, the present inverse method is employed again using a tool without the insert. The broken lines in Fig. 13 show the result. Agreement between both models becomes better, but discrepancy is large as the thermal conductivity of the tool is high. This is due to the difference in the dimensions of the tool between the two models.

6. Conclusions

(1) An inverse analysis approach using the finite element method is proposed to determine the energy balance or the energies flowing into a tool, workpiece and chip per unit volume of material removal in two-dimensional metal machining.

(2) The method proposed is employed to clarify the influence of thermophysical properties of work and tool materials on the energy balance. The energy flowing into the tool is most enhanced when a low-thermal-conductivity workpiece is machined by a high-thermal conductivity tool with a coolant.

(3) The propriety of the method proposed is investigated by comparison with the Loewen-Shaw's moving heat source model. The present model shows good agreement for low-thermal-conductivity tools such as carbides and HSS, but yields a smaller energy flow rate into the tool as the thermal conductivity of the tools becomes high.

(4) The energy balance thus determined is applied to the analysis of three-dimensional thermal behaviour in end milling. The suitability of the present model is confirmed by comparison with measured rake temperature.

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