Geometric Design and Machining of Variable Pitch Lead Screws with Swinging and Translating Meshing Rollers*

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The design of variable pitch lead screws with swinging and translating meshing rollers is proposed in this study. Mathematical expressions representing the surface geometry of the screw generated by an endmill cutter are derived for the purpose of calculating the meshing points on the threaded surface of the screw. This mechanism is modeled using a Silicon Graphics workstation for motion simulation. Two examples are given. Our results show that constant thickness of screw threads can be attained using the design proposed. Finally, the machining settings of a 5-axis CNC machine are provided for manufacture of the screws.

Key Words: Variable Pitch Lead Screw, Geometric Design, Machining Setting, 5-axis CNC Machine

1. Introduction

The design of variable pitch lead screws with swinging and translating meshing rollers is proposed in this study (Fig. 1). Four meshing rollers are attached to two swinging blocks. Each roller (4a, 4b, 4c, 4d) is conjugated to the threaded surface of the screw (2). The screw is forced to rotate angular displacement ϕ through the engaging surfaces above it when the slider translates linear displacement D . Each swinging block is adjacent to the slider (3) with a revolute pair. The thread thickness is defined as the distance between two points which are the opposite intersection plane of the screw surfaces and the plane established by central axes 4a and 4b or 4c and 4d. Each screw thread has two end points. When the meshing rollers move to the end points, the screw is forced to reverse the motion. β is the angle between the normal cross section of the screw and the plane through central axes 4a and 4b or 4c and 4d. If β is fixed, backlash is generated between the screw surface and the rollers. Backlash is due to wearing, manufacturing, or assembly errors and results in

vibration of the mechanism. To alleviate the above disadvantages and decrease the precision requirement of every dimension without affecting conditions of transmission efficiency, β is designed as a function of D . Due to the existence of swinging angle β , the thread thickness can be maintained constant inhibiting stress concentration on the screw. Furthermore, angle β can be self-adjusted by the positive contact points between the screw and the rollers, and backlash can thus be avoided. Therefore, vibration vanishes. This mechanism has the characteristics of positive-return cam when the slider precisely transforms the rectilinear motion to the screw to rotary motion

Fig. 1 Variable pitch lead screw with swinging and translating meshing rollers

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around the fixed link (1).

Several approaches are available for generating a pair of conjugated surfaces. Weatherburn⁽¹⁾ investigated the differential geometry of three dimensions assuming that the line joining any point on the plane tangent to the point of contact between them is perpendicular to the normal of this contact point. Litvin⁽²⁾ developed the theory of gearing assuming that the line from a point on the instantaneous axis of rotation to a point on the generating surface is parallel to its normal vector. Chakraborty and Dhande⁽³⁾ studied the geometry of spatial cam mechanisms. Their studies were focused on cyclical mechanisms that move continuously along a closed-track. Furthermore, two type double-end cams were developed in which the roller has a reciprocating linear displacement and the cam has a reciprocated angular displacement. This mechanism results in vibrations at each end due to backlash incurred during manufacture and assembly⁽⁴⁾⁻⁽⁶⁾. Recently, similar mechanisms were developed by Yan and Liu⁽⁷⁾ for applications in rapier-type shuttleless weaving looms.

During manufacture, the surfaces of mutually conjugated mechanisms are cut by proper cutters. Sculpture is one of manufacture techniques, but low precision of the surfaces and the inefficiency of the approach makes it infeasible. For conjugated pairs, determining the manufacturing method before designing the surfaces of the mechanisms is critical. Generating method is usually adopted for the manufacture of gears, screws, and cams. The mechanisms can be cut by a cutter, selected, after experiencing correlative kinematics; in addition, the surfaces of the rollers are determined by the rotary surface of the cutter while preserving some tolerances. Next, the lapping procedure is repeated by form grinding instead of use of the edge cutter. To perform these tasks, pre-calculation of the machine and cutter settings in the manufacture of screws is necessary. Formate and Helixform methods for calculation of machine settings have been applied to machine hypoid gears by Litvin and Gutman^{(8),(9)}.

The purpose of this paper is to introduce the concept of variable pitch lead screws with swinging and translating meshing rollers and to derive the mathematical models for the surface geometry of the screws and rollers. These mechanisms are modeled in order to simulate the kinematic state. The machining settings in a 5-axis CNC machine center are also calculated to provide basic data for NC programming.

Nomenclature

a, b, c : relative translative increment in X, Y , and Z axes

a_0, b_0, c_0 : P_r offsets to P_{m0} in X, Y , and Z axes
 A_n^m : major axis of endmill cutter m represented in coordinate system n
 D : rectilinear displacement of endmill cutter, slider or rollers
 e : offset of axis of endmill cutter from axis Y_{43}
 f : distance between coordinate origins O_{43} (O_{44}) and O_{42}
 l : active machining depth of endmill cutter matching length of meshing rollers
 L : distance between coordinate origins O_{47} (O_{4a}) and O_{46} (O_{45})
 $L_{i,j}$: submatrix obtained by eliminating the last row and the last column of matrix $M_{i,j}$
 $M_{i,j}$: transformation matrix from coordinate system j to system i
 N_n^m : unit normal vector of conjugate points on meshing rollers m in coordinate system n
 P_r : machine relative point
 P_{m0} : machine zero point
 $\dot{Q}$: change relative to time, Q represents ϕ, β or D
 r : radius of neck circle of endmill cutter or rollers
 R_2 : length of crank
 R_3 : length of connected link
 R_n^m : surface on element m represented in coordinate system n
 S_{mn} : coordinate system; m represents number of links, $n=0$ represents coordinate rigidly connected to the link, $n=1$ represents coordinate fixed in an inertia frame, $n=2-9$ represents moving coordinate
 u, φ : curvilinear coordinates of endmill cutter or rollers
 V_n^m : absolute velocity on element m represented in coordinate system n
 $\beta(D)$: swinging angle of rollers relative to axis Y_{44}
 λ : angle between axes Y_{46} and Y_{45}
 δ_1, δ_2 : relative angular increment in A and C axes
 ϕ : rotatory angle of the screw
 ω : rotatory speed of crank
 $4m$: number of endmill cutter, $4m$ represents $4a, 4b, 4c$ or $4d$

2. Generating Method

The surface geometry of the screw is determined by the relative position and velocity between the screw and the rollers. Let the translative velocity along the screw axis be $\dot{D}$ and the swinging velocity about the normal cross section of the screw be $\dot{\beta}$. Then, the screw has an angular velocity $\dot{\phi}$ relative to its axis. The location of roller $4a$ is replaced by cutter $4m$ for generating the conjugated surface of the screw.

Based on these kinematic relations, a set of coordinate systems is defined as shown in Fig. 2.

Coordinate systems $S_{10}(X_{10}, Y_{10}, Z_{10})$ and $S_{4a}(X_{4a}, Y_{4a}, Z_{4a})$ are rigidly attached to the screw and the generating surface $4m$, respectively. Coordinate systems $S_{42}(X_{42}, Y_{42}, Z_{42})$, $S_{43}(X_{43}, Y_{43}, Z_{43})$, $S_{44}(X_{44}, Y_{44}, Z_{44})$, $S_{45}(X_{45}, Y_{45}, Z_{45})$, $S_{46}(X_{46}, Y_{46}, Z_{46})$, and $S_{47}(X_{47}, Y_{47}, Z_{47})$ are six moving coordinate systems. All of them translate displacement along axis X_{41} . Coordinate systems $S_{11}(X_{11}, Y_{11}, Z_{11})$ and $S_{41}(X_{41}, Y_{41}, Z_{41})$ are moving coordinate systems attached to the screw and cutter $4m$, respectively. By rotating the coordinate system S_{4a} -90 degrees around axis X_{4a} , coordinate system S_{47} is obtained. The distance along axis $-Y_{47}$ between origins O_{47} and O_{46} is L , and the distance along axis Z_{45} between origins O_{45} and O_{44} is e . Coordinate systems S_{43} and S_{45} are obtained by rotating S_{44} and S_{46} with $\beta(D)$ and λ degrees around axes $-Y_{44}$ and X_{46} , respectively. Axis X_{43} is parallel to axis X_{42} , the axis of the screw, and a distance f is biased along the $-Y_{42}$ direction. The angle between coordinate system S_{11} and S_{10} is ϕ relative to axis X_{41} .

By applying homogeneous coordinates, the transformation matrices for the above coordinate systems can be represented as follows

$$\begin{aligned} [M_{44,4a}] &= \text{Trans}(0, 0, -e) \text{Rot}(X_{46}, \lambda) \\ &\quad \times \text{Trans}(0, L, 0) \text{Rot}(X_{4a}, -90^\circ) \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & S\lambda & -C\lambda & LC\lambda \\ 0 & C\lambda & S\lambda & -LS\lambda - e \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (1)$$

$$\begin{aligned} [M_{41,44}] &= \text{Trans}(D, -f, 0) \text{Rot}(Y_{44}, -\beta) \\ &= \begin{bmatrix} C\beta & 0 & S\beta & D \\ 0 & 1 & 0 & -f \\ -S\beta & 0 & C\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} D \\ [L_{41,44}] - f \\ 0 \\ 0 \ 0 \ 0 \ 1 \end{bmatrix} \end{aligned} \quad (2)$$

Fig. 2 Coordinate systems

$$[M_{10,41}] = \text{Rot}(X_{41}, -\phi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\phi & -S\phi & 0 \\ 0 & S\phi & C\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$\begin{aligned} [M_{41,4a}] &= [M_{41,44}][M_{44,4a}] \\ &= \begin{bmatrix} C\beta & S\beta C\lambda & S\beta S\lambda & -eS\beta - LS\beta S\lambda + D \\ 0 & S\lambda & -C\lambda & LC\lambda - f \\ -S\beta & C\beta C\lambda & C\beta S\lambda & -eC\beta - LC\beta S\lambda \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -eS\beta - LS\beta S\lambda + D \\ [L_{41,4a}] & LC\lambda - f \\ -eC\beta - LC\beta S\lambda \\ 0 \ 0 \ 0 & 1 \end{bmatrix} \end{aligned} \quad (4)$$

$$\begin{aligned} [M_{10,4a}] &= [M_{10,41}][M_{41,4a}] \\ &= \begin{bmatrix} C\beta & S\beta C\lambda & S\beta S\lambda \\ S\beta S\phi & C\phi S\lambda & -C\phi C\lambda \\ -S\beta C\phi & S\phi S\lambda & -S\phi C\lambda \\ 0 & 0 & 0 \\ -eS\beta - LS\beta S\lambda + D \\ LC\phi C\lambda + LS\phi C\beta S\lambda \\ -fC\phi + eS\phi C\beta \\ LS\phi C\lambda - fS\phi \\ -LC\phi C\beta S\lambda - eC\phi C\beta \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} -eS\beta - LS\beta S\lambda + D \\ [L_{10,4a}] & LC\phi C\lambda + LS\phi C\beta S\lambda \\ -fC\phi + eS\phi C\beta \\ LS\phi C\lambda - fS\phi \\ -LC\phi C\beta S\lambda - eC\phi C\beta \\ 0 \ 0 \ 0 & 1 \end{bmatrix} \end{aligned} \quad (5)$$

where $S\lambda = \sin \lambda$, $C\lambda = \cos \lambda$, the notation "Trans" represents pure translation and "Rot" represents pure rotation. The notation $[M_{i,j}]$ represents transformation matrix from coordinate system j to i , and $[L_{i,j}]$ represents a 3×3 submatrix obtained from the upper-left elements of the $[M_{i,j}]$ matrix.

2.1 Surface equations of the endmill cutter

As shown in Fig. 3, the conical revolution surface, similar to the surface of the meshing elements, is formed by an edge cutter. Coordinate system S_{4a} is defined to model the generating surface. The neck radius of the edge of the endmill cutter is r , the half cone angle is α , and the rotatory angle is φ . Let P express an arbitrary point on the revolution surface of cutter $4m$. Then, its position vector can be expressed as

$$\begin{aligned} \mathbf{R}_{4a}^{4m} &= [(r + u \tan \alpha) \cos \varphi, (r + u \tan \alpha) \sin \varphi, u, \\ &\quad 1]^T, \end{aligned} \quad (6)$$

$$-\pi \leq \varphi \leq \pi, 0 \leq u \leq l.$$

where superscript $4m$ expresses the cutter, and subscript $4a$ expresses the position vector, $\mathbf{R}_{4a}^{4m}$, in coordinate system S_{4a} .

Fig. 3 Curvilinear coordinate of the endmill cutter

The meshing rollers translate linear displacement D along axis X_{41} and swing angular displacement β around axis Y_{43} . According to the above kinematic relations, it is necessary for the cutter to move distance D under velocity $\dot{D}$ along axis X_{41} and to swing angle β under rotational velocity $\dot{\beta}$ around axis Y_{43} in order to produce a screw with precision. Velocity, V_{41}^{4m} , of the cutter in coordinate S_{41} is

$$V_{41}^{4m} = \dot{D} \mathbf{i}_{41} + [L_{41,4a}] \{ \dot{\beta} \mathbf{j}_{44} \times \{ [M_{44,4a}] \cdot \mathbf{R}_{4a}^{4m} \} \}. \quad (7)$$

Based on differential geometry⁽¹⁾, the unit normal vector of P in coordinate S_{4a} is

$$N_{4a}^{4m} = \frac{\frac{\partial \mathbf{R}_{4a}^{4m}}{\partial \varphi} \times \frac{\partial \mathbf{R}_{4a}^{4m}}{\partial u}}{\left| \frac{\partial \mathbf{R}_{4a}^{4m}}{\partial \varphi} \times \frac{\partial \mathbf{R}_{4a}^{4m}}{\partial u} \right|} = C\alpha C\varphi \mathbf{i} + C\alpha S\varphi \mathbf{j} - S\alpha \mathbf{k}. \quad (8)$$

where $C\alpha = \cos \alpha$, $S\alpha = \sin \alpha$, $C\varphi = \cos \varphi$ and $S\varphi = \sin \varphi$. The normal vector N_{4a}^{4m} of the cutter is transformed here from system S_{4a} to system S_{41} by manipulating Eq. (8) with $[L_{41,4a}]$. Consequently, the equation of N_{41}^{4m} becomes

$$N_{41}^{4m} = [L_{41,4a}] \cdot N_{4a}^{4m} = \begin{bmatrix} C\alpha C\beta C\varphi - S\alpha S\beta S\lambda + C\alpha S\beta S\varphi C\lambda \\ S\varphi C\alpha S\lambda + S\alpha C\lambda \\ -C\alpha S\beta C\varphi - S\alpha C\beta S\lambda + C\alpha C\beta S\varphi C\lambda \end{bmatrix}. \quad (9)$$

2.2 Generated surface of the screw

The generated surface of the screw is obtained by the milling cutter by milling along the screw blank. The generated surface must be strictly based on differential geometry subject to two constraints. One restraint is that the neighborhood of the points on the locus of generating surfaces must be tangent with the piece of the generated surfaces. The other restraint is that the piece of the generated surfaces exists as a simple regular surface. The locus and its normal vector in coordinate S_{10} , $\mathbf{R}_{10}^{4a}$ and $\mathbf{N}_{10}^{4a}$, can be derived as

$$\mathbf{R}_{10}^{4a} = [M_{10,4a}] \cdot \mathbf{R}_{4a}^{4m}$$

$$= \begin{bmatrix} D + (r + uT\alpha) \cdot (S\varphi C\lambda S\beta + C\varphi C\beta) \\ + (u - L)S\lambda S\beta - eS\beta \\ - fC\varphi + eC\beta S\varphi - (u - L) \cdot (C\varphi C\lambda + C\beta S\lambda S\varphi) \\ + (r + uT\alpha) \cdot (C\varphi S\varphi S\beta + S\lambda C\varphi S\varphi - C\beta C\lambda S\varphi S\varphi) \\ fS\varphi - eC\beta C\varphi - (u - L) \cdot (S\varphi C\lambda - C\beta S\lambda C\varphi) \\ - (r + uT\alpha) \cdot (C\varphi C\varphi S\beta - S\lambda S\varphi S\varphi - C\beta C\lambda C\varphi S\varphi) \end{bmatrix} \quad (10)$$

$$\mathbf{N}_{10}^{4a} = [L_{10,4a}] \cdot \mathbf{N}_{4a}^{4m}, \quad (11)$$

where $T\alpha = \tan \alpha$. To satisfy the aforementioned constraints, $\mathbf{N}_{10}^{4a}$ is set to not equal to zero and the equation of meshing is added, thereby yielding

$$\mathbf{N}_{41}^{4m} \cdot (\mathbf{V}_{41}^{10} - \mathbf{V}_{41}^{4m}) = 0, \quad (12)$$

where $\mathbf{V}_{41}^{10}$ is the velocity of the conjugated point P on the screw. Since the screw is driven of $\dot{\phi}$ degrees per second relative to axis $-X_{41}$, $\mathbf{V}_{41}^{10}$ is expressed as

$$\begin{aligned} \mathbf{V}_{41}^{10} &= \dot{\phi} \mathbf{i}_{41} \times \{ [M_{41,4a}] \cdot \mathbf{R}_{41}^{4a} \} \\ &= \dot{\phi} [(r + uT\alpha)(S\varphi S\beta C\lambda - S\beta C\varphi) \\ &\quad + uC\beta S\lambda - eC\beta - LC\beta S\lambda] \mathbf{j}_{41} \\ &\quad - \dot{\phi} [S\lambda \cdot (r + uT\alpha)S\varphi - uC\lambda + LC\lambda - f] \mathbf{k}_{41} \end{aligned} \quad (13)$$

Substituting Eqs. (7), (9), and (13) into Eq. (12) yields

$$E_{4a}S\varphi + F_{4a}C\varphi + G_{4a} = 0 \quad (14)$$

$$\varphi = 2 \tan^{-1} [(-E_{4a} \pm \sqrt{E_{4a}^2 + F_{4a}^2 - G_{4a}^2}) / (G_{4a} - F_{4a})] \quad (15)$$

where

$$\begin{aligned} E_{4a} &= \dot{D} \left\{ -C\alpha S\beta C\lambda + \frac{d\phi}{dD} [(u - L)C\alpha C\beta \right. \\ &\quad \left. + fC\alpha C\beta C\lambda + (r + uT\alpha)S\alpha C\beta - eC\alpha C\beta S\lambda] \right\} \end{aligned} \quad (16)$$

$$\begin{aligned} F_{4a} &= \dot{D} \left\{ -C\alpha C\beta - \frac{d\phi}{dD} [(u - L)C\alpha S\beta C\lambda \right. \\ &\quad \left. + fC\alpha S\beta + (r + uT\alpha)S\alpha S\beta C\lambda] \right. \\ &\quad \left. + \dot{\beta} [-(u - L)C\alpha S\lambda + eC\alpha - (r + uT\alpha)S\alpha S\lambda] \right\} \end{aligned} \quad (17)$$

$$G_{4a} = \dot{D} \left\{ S\alpha S\beta S\lambda - \frac{d\phi}{dD} [fS\alpha C\beta S\lambda + eS\alpha C\beta C\lambda] \right\}. \quad (18)$$

According to Eq. (14) the screw surfaces are always tangent to the generating surface⁽²⁾. Using the above procedure, the mathematical models of conjugate surfaces on the screw, generated by cutter 4m and set up in S_{4b} , S_{4c} , and S_{4d} , can also be obtained. The corresponding equations can be obtained based on Eqs. (1) to (18) by altering the sign of f , e , β and λ as given in Table 1.

2.3 Swinging angle

The swinging angle β can be proposed as a linear or nonlinear function of D . For some special purposes, for example, to design a screw with constant thread thickness, β can be calculated by rearranging Eq. (14) as follows

$$f(\beta) = E'S\beta + F'C\beta + G'\dot{\beta} = 0. \quad (19)$$

where

Table 1 Contrasting parametric signs in systems 4a, 4b, 4c and 4d

Parameters	Roller 4a	Roller 4b	Roller 4c	Roller 4d
f	+	+	-	-
e	+	-	+	-
β	+	+	-	-
$\cos\lambda$	+	+	-	-
$\sin\lambda$	+	-	+	-

$$E' = \dot{D} \left\{ -CaC\lambda S\varphi - \frac{d\phi}{dD} [(u-L)CaC\lambda + (r+uTa)SaC\lambda] C\varphi + fCa + SaS\lambda \right\} \quad (20)$$

$$F' = \dot{D} \left\{ -CaC\varphi + \frac{d\phi}{dD} [(u-L)Ca + fCaC\lambda + (r+uTa)Sa - eCaS\lambda] S\varphi - \frac{d\phi}{dD} [fSaS\lambda + eSaC\lambda] \right\} \quad (21)$$

$$G' = C\varphi [-(u-L)CaS\lambda + eCa - (r+uTa)SaS\lambda]. \quad (22)$$

In order to control the instant enveloping line always passes through a point on the revolution surface of the cutter, several sets of (u, φ) for a known D can be selected. Here, one set of (u, φ) set is chosen and substituted into Eq. (19) in order to find the instant swinging angle β for every D . These β 's are substituted into Eq. (15) to calculate the enveloping φ for every u . Then, the surface point coordinates of the screw can also be found from Eq. (10) corresponding to each of the above-mentioned $(D, \beta(D), u, \varphi)$ sets.

3. Design Examples

An ordinary slider-crank mechanism is applied in this study to drive the rollers as shown in Fig. 4. The length of the crank, rotating under fixed rotatory speed ω around a fixed pivot, is R_2 . The length of the connecting link, which causes the slider and rollers to slide as shown in Fig. 1, is R_3 . Following the sliding facilitated by the connecting link, the screw is forced to rotate by the rollers and follows a specified motion curve.

A number of motion curves are employed based on consideration of the working conditions, i.e., linear impulse, momentum, inertia force and vibration. Angular speed $\dot{\phi}$, translation displacement D , and velocity $\dot{D}$ are known given that the motion curve is specified. Since the derived mathematical model can be fitted to any motion curve, the distributive poiots on different types of screws can be calculated. Two motion curves, modified trapezoid (MT) and modified sine (MS) curves, are selected in the following examples to demonstrate the proposed design.

Fig. 4 Slider-crank mechanism

Example 1. Design a variable pitch lead screw with swinging rollers subject to the following conditions. The motion curve is modified trapezoid (MT). Design parameters of the cutter and the screw are:

- R_2 (length of crank) 130.00 mm
- R_3 (length of connecting link) 357.00 mm
- ω (rotatory speed of crank) 500.00 rpm
- r (neck radius) 8.00 mm
- α (conical angle) 5.00 degrees
- f (offset distance) 37.87 mm
- e (offset distance) 6.77 mm
- L (offset distance) 27.00 mm
- λ (Offset angle) 20.00 degrees
- l (active depth of cutter) 17.5 mm

The coordinates and normal vectors of meshing points are computed by substituting the above data into Eqs. (1) to (18). Since the pitches vary on the screw, the thread thicknesses are not uniform and stress concentration is difficult to avoid. In order to maintain the thread thickness at a fixed value, swinging angle β must vary with linear displacement D of the cutter. To accomplish the above, Eq. (19) is used to determine the β 's. To control that the instant enveloping line always passes through a point on the revolution surface which result in maintenance of the thread thickness, a central point of active depth on the cutter, $u=l/2$ and $\varphi=90^\circ$, is selected in order to form a linear function of β and it is substituted into Eq. (19) to find β since $\cos 90^\circ$ equals zero. Thus, it can be simplified as

$$E''_{4a}S\beta + F''_{4a}C\beta = 0, \quad (23)$$

and hence

$$\beta = \tan^{-1}(-F''_{4a}/E''_{4a}), \quad (24)$$

where

$$E''_{4a} = -CaC\lambda + SaS\lambda \quad (25)$$

$$F''_{4a} = \frac{d\phi}{dD} \{ [(0.5l-L)Ca + fCaC\lambda + (r+0.5lTa)Sa - eCaS\lambda] S\varphi - (fSaS\lambda + eSaC\lambda) \}, \quad (26)$$

Since output angle ϕ of the screw is a function of displacement D , only one swinging angle, β , can be

found from Eq. (19) for every D . As shown in Fig. 5, β varies continuously from 2.6 to 35.4 degrees, then decreases to 1.2. Regarding this value of parameter β as the instantaneous rotatory angle of the endmill cutter around the Y_{44} axis, the contact points on the screw can be computed at this time. Figure 6 shows the meshing graph of this screw with the MT curve. The pitch has a tendency to gradually increase, and then it decreases fitted with an opposite trend toward the swinging angle.

Figure 7 shows the thickness of the thread which varies between 15.01 and 9.54 mm when angle β is fixed at 35° degrees. Additionally, the thickness of the thread is fixed in 9.54 mm in $u=l/2$ when angle β is varied specifically. Figure 8 shows the solid modeling of this example.

Example 2. Design a variable pitch lead screw with swinging rollers. the modified sine (MS) curve is selected as the motion curve. The design parameters are the same as in Example 1.

Figure 9 shows the swinging angle varied from 40.1 to 22.4 degrees. The meshing graph, Fig. 10, shows that the pitch increases continuously, but is opposite to the varying trend in the swinging angle. The thread thickness of the screw s also maintained at 9.54 mm in $u=l/2$ as shown in Fig. 11; however, the other case the thread thickness varies from 9.54 to 10.9 mm. Figure 12 shows the solid modeling of this

Fig. 7 Thread thickness (MT motion curve)

Fig. 8 Solid modeling (MT motion curve)

Fig. 5 Swinging angles β of rollers relative to axis Y_{44} (MT motion curve)

Fig. 6 Meshing graph (MT motion curve)

Fig. 9 Swinging angles β of rollers relative to axis Y_{44} (MS motion curve)

Fig. 10 Meshing graph (MS motion curve)

Fig. 11 Thread thickness (MS motion curve)

Fig. 12 Solid modeling (MS motion curve)

design.

4. Machining

The machining process must be efficient and accurate. Consequently, a five-axis universal machine, MAHO MH800C, as shown in Fig. 13, is used for manufacture of this screw. Two swivel motions, *A* and *C*, around *X* and *Z* axes, respectively, are provided by this machine for rotating the screw blank and the spindle. The worktable permits the blank to shift along transverse (*X*) and longitudinal (*Z*) axes; in addition, the spindle set cal also move in the vertical direction (*Y*). The machine zero point *P_m*, located on the valid machining border of the machine, is specified as the origin of coordinate *S_{m0}*. Coordinate system *S_{m1}* offsets to *S_{m0}* by *a*₀, *b*₀, and *c*₀ in *X_{m0}*, *Y_{m0}*, and *Z_{m0}* axes, respectively. *S_{m2}* and *S_{m5}* are two moving coordinate systems attached to the screw blank and the endmill cutter, respectively. Auxiliary coordinate systems *S_{m3}* and *S_{m4}* are defined by the rotatable axis. The machine relative point *P_r*, i.e., the common origin of coordinates *S_{m1}* and *S_{m2}*, lies on the

Fig. 13 Coordinate systems of the five-axis machine

axis of the screw blank. Coordinate system *S_{m3}* offsets to *S_{m2}* by *a*, *b*, and *c* in *X_{m2}*, *Y_{m2}*, and *Z_{m2}*, respectively. The angle between *X_{m1}* and *X_{m2}* axes is π around the *Z_{m1}* axis. The angles between *S_{m3}* and *S_{m4}* axes, and *S_{m4}* and *S_{m5}* axes are δ_1 and δ_2 clockwise with respect to *X_{m3}*-axis and *Y_{m4}*-axis, respectively. The transformation matrices of coordinate systems can be derived as

$$\begin{aligned}
 [M_{m2,m5}] &= \text{Rot}(X_{m2}, -\delta_1) \text{Trans}(a, b, c) \text{Rot}(Z_{m4}, -\delta_2) \\
 &= \begin{bmatrix} C\delta_2 & -S\delta_2 & 0 & a \\ C\delta_1 S\delta_2 & C\delta_1 C\delta_2 & -S\delta_1 & bC\delta_1 - cS\delta_1 \\ S\delta_1 S\delta_2 & S\delta_1 C\delta_2 & C\delta_1 & bS\delta_1 + cC\delta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} a \\ [L_{m2,m5}] & bC\delta_1 - cS\delta_1 \\ & bS\delta_1 + cC\delta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (27)
 \end{aligned}$$

$$[M_{m1,m3}] = \text{Rot}(Y_{m1}, 180) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (28)$$

where $C\delta_1 = \cos \delta_1$, $S\delta_1 = \sin \delta_1$, $C\delta_2 = \cos \delta_2$ and $S\delta_2 = \sin \delta_2$. Two constraints are approached to the generation of machine-modeled screw, as mentioned in the previous paragraph. The position vectors, $\mathbf{R}_{m5}^{4a}$, and the major axis vector, $\mathbf{A}_{m5}^{4a}$, of the endmill cutter viewed in coordinate *S_{m2}* must be equal. Therefore, we have

$$[\mathbf{M}_{10,4a}] \cdot \mathbf{R}_{4a}^{4a} = [\mathbf{M}_{m2,m5}] \cdot \mathbf{R}_{m5}^{4a} \quad (29)$$

$$[\mathbf{L}_{10,4a}] \cdot \mathbf{A}_{4a}^{4a} = [\mathbf{L}_{m2,m5}] \cdot \mathbf{A}_{m5}^{4a} \quad (30)$$

Rearranging Eqs. (29) and (30), we have

Fig. 14 Variable pitch lead screw in machining

Fig. 15 Screw Prototype

$$\begin{aligned}\delta_1 &= \tan^{-1} \left(\frac{S\phi S\lambda + C\phi C\beta C\lambda}{C\phi S\lambda - S\phi C\beta C\lambda} \right) \\ \delta_2 &= \sin^{-1}(-S\beta C\lambda) \\ a &= -L \cdot S\beta S\lambda - e \cdot S\beta + D \\ b &= C\delta_1 \{ C\phi(L \cdot C\lambda - f) - S\phi(-L \cdot C\beta S\lambda - e \cdot C\beta) \} \\ &\quad + S\delta_1 \{ S\phi(L \cdot C\lambda - f) + C\phi(-L \cdot C\beta S\lambda - e \cdot C\beta) \} \\ c &= -S\delta_1 \{ C\phi(L \cdot C\lambda - f) - S\phi(-L \cdot C\beta S\lambda - e \cdot C\beta) \} \\ &\quad + C\delta_1 \{ S\phi(L \cdot C\lambda - f) + C\phi(-L \cdot C\beta S\lambda - e \cdot C\beta) \}.\end{aligned}\quad (31)$$

In Eq. (31), a , b , c , δ_1 and δ_2 , are expressed as a function of L , e , f , λ , D , ϕ , and β in X , Y , Z , A , and C axes, respectively. Furthermore, an NC program is completed using the $[M_{m1,m3}]$ matrix to transform all data in the machining coordinate. Figure 14 shows the screw in the machining process, and Fig. 15 reveals the screw prototype.

5. Conclusions

The concept of variable pitch lead screws with swinging and translating meshing rollers is proposed. Mathematical models of the proposed screws and rollers are derived. The meshing graphs of two screws are established in which the lining stand erect grids are the contact lines during transmission and the

adjacent contact lines represent the sliding locus. Solid models of the screws and rollers are developed in order to perform motion simulation. Furthermore, the machining settings in a 5-axis CNC machine center are also calculated to provide basic data for NC programming and then for machining the screws. Our results demonstrate that the proposed design is feasible and the thread thickness of the screw can be constant. This design has tremendous potential for application to design of rapier-type shuttleless textile machines.

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