Structural Homology Design Using Equality Equations for Linear and Nonlinear Conditions*

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The homology design concept was devised for application to the large telescope structure by S. V. Hoerner. He defined the deformation of a structure as homologous if a given geometrical relation holds for a given number of structural points before, during, and after the deformation. Some researchers have utilized the design theory on structural design with the finite element method. In the present study, a simple method using geometrical equality equations is suggested for performing the homology design. The previous methods are improved for the cost of making simultaneous equations. The homology design technique is expanded to designs for user-specified responses. The basic formulations of the homology design with the optimization concept are described and several practical examples are solved to verify usefulness and validity.

Key Words: Homology Design, FEM Equation, Sensitivities, Equality Equations

1. Introduction

One of the most important goals in structural design is finding a solution which satisfies the imposed constraints. Recently, various methods including optimization have been developed for the automatic design of engineering systems⁽¹⁾. It is difficult to incorporate geometrical constraints to existing methods. For example, the design of a reflector in a large telescope or a satellite antenna structure must satisfy the constraint that its configuration remains paraboloidal before, during, and after deformation.

Homology design, unlike other design methods can incorporate the constraints mentioned above. The concept of homology design was devised for application to a large telescope structure by S. V. Hoerner⁽²⁾. He defined the deformation of a structure

as homologous if a given geometrical relation holds for a given number of structural points before, during, and after deformation. Recently, the need for homology design in structural design has been increasing with increasing requirements for structural precision. Some researchers have developed the homology design theory which is based on the finite element method (FEM)^{(3),(4)}. In the first approach of Ref. (3), total degrees of freedom (DOFs) of a structure are decomposed into independent and dependent ones. The dependency relation for DOFs is defined by a constraint matrix. The FEM governing equation is divided by the decomposed DOFs, and simultaneous equations are solved by the introduction of Moore-Penrose's generalized inverse matrix⁽⁵⁾. By using the matrix, a final solution is obtained as the nearest one from the initial design variables. In the second approach of Ref. (4), the simultaneous equations are derived according to the geometrical constraint for a homologous deformation, while the solution procedures are the same as in the first approach.

In the present investigation, a simple method using geometrical equality equations to obtain homologous deformation is suggested. The geometrical

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equality equations are classified into two types. One is where the dependent and independent variables are related by a function. The other is where the dependent DOFs are specified. This method improves on the first approach described above in that the decomposition of the FEM equation, which is very expensive, is not necessary. The proposed method requires the sensitivities of displacements in the FEM equation. Therefore, the finite element program is directly applied to homology design without any modification. The results of the proposed method are the same as those of the second approach described above. However, the latter requires a coding of the simultaneous equations for each application problem, and can be expensive in practical application. In contrast, the proposed method requires the relation between the independent and dependent DOFs but does not require problem-dependent coding for the simultaneous equations. Thus, the suggested method has some advantages in numerical applications. Also, homology design has been applied to design for user-specified responses.

The basic formulations of homology design using equality equations are described. A program for homology design has been developed and several practical examples are solved to verify its usefulness and validity. A satellite antenna back-up structure is specially designed by the suggested method. The results are compared with those obtained by previous methods. For problems with quadratic geometric constraint, a homology index is defined to measure the performance.

2. Homology Design Using Decomposition of FEM Equation

Homology design is one element of design methodology. It is a technique to determine design variables in order to gain predicted structural responses. The predicted structural responses imply displacements, stresses, natural frequencies, or mode shapes. In this study, homology design is focused on structural deformations. Existing approaches using decomposition of FEM are explained briefly in this section and a simple method using geometrical equality equations is developed in the next section.

The governing equation is stated as⁽⁶⁾

$$[K]\{U\}=\{f\}, \quad (1)$$

where $[K]$: system stiffness matrix

$\{U\}$: displacement vector

$\{f\}$: force vector.

The displacement vector is divided into displacements related to independent DOFs, $\{U_i\}$, and displacements related to dependent DOFs, $\{U_d\}$. Thus, Eq. (1) is decomposed as

$$\begin{bmatrix} K_{ii} & K_{id} \\ K_{di} & K_{dd} \end{bmatrix} \begin{Bmatrix} U_i \\ U_d \end{Bmatrix} = \begin{Bmatrix} f_i \\ f_d \end{Bmatrix}, \quad (2)$$

where $K_{ii}, K_{id}, K_{di}, K_{dd}, f_i$ and f_d : rearranged elements of $[K]$ and $\{f\}$ due to decomposed $\{U\}$. The relationship between $\{U_d\}$ and $\{U_i\}$ can be expressed by introducing a constraint matrix $[C]$ and a constant vector $\{C_0\}$,

$$\{U_d\}=[C]\{U_i\}+\{C_0\} \quad (3)$$

The constraint matrix and the constant vector are constructed by the geometrical relation. By substituting Eq. (3) into Eq. (2), the following equations are derived:

$$[K_s]\{U_i\}=\{F_i\} \quad (4)$$

$$[K_r]\{U_i\}=\{F_d\}, \quad (5)$$

where

$$[K_s]=[K_{ii}]+[K_{id}][C] \quad (6)$$

$$[K_r]=[K_{di}]+[K_{dd}][C] \quad (7)$$

$$\{F_i\}=\{f_i\}-[K_{id}]\{C_0\} \quad (8)$$

$$\{F_d\}=\{f_d\}-[K_{dd}]\{C_0\}. \quad (9)$$

Equations (4) and (5) must be satisfied to realize the homology design. However, since Eq. (5) is not generally true for $\{U_i\}$ calculated using Eq. (4), it is necessary to have a procedure for updating the solution until $[K_r]\{U_i\}-\{F_d\}$ is almost zero. Therefore, Eq. (5) is replaced by Eq. (10) which is expanded by Taylor's series considering only first order terms.

$$\sum_{j=1}^M ([K_r]\{\bar{U}_i\} + [\bar{K}_r]\{U_i\})\delta b_j = \{F_d\} - [\bar{K}_r]\{\bar{U}_i\}, \quad (10)$$

where M : number of design variables
' : first order differentiation term
— : value at current design variable

Fig. 1 Flow of homology design with decomposed FEM equation

δb_j : variation of design variables.

The solution, δb_j , is determined by using Moore-Penrose's generalized inverse matrix, which can be used to obtain the nearest design from the initial design⁽⁵⁾. Then, the design variables are replaced as follows:

$$b_j = b_j + \delta b_j, \quad j=1, \dots, M. \quad (11)$$

The existing approach above is summarized in Fig. 1.

3. Homology Design Using Equality Equations

The homology design using decomposition of FEM equation is characterized by a procedure which decomposes total DOFs into dependent and independent ones, and utilizes Moore-Penrose's generalized inverse matrix for solving decomposed FEM equations. However, it is inefficient for dividing FEM equations and evaluating the sensitivities of the decomposed stiffness matrix and displacement vector with respect to design variables. In the approach of Ref. (4), the simultaneous equations are set up as Eq. (12) to eliminate the inefficiency of decomposing FEM equations.

$$\{H(U)\} = \{0\} \quad (12)$$

In Eq. (12), N_{dep} (number of dependent DOFs) equations are included to represent the homologous deformation. Since Eq. (12) is not expressed in general form, the coding process is required depending on the problem. Therefore, an efficient approach to obtain homologous deformation is required.

A method is presented to overcome the shortcomings of the existing methods, which treats the relationship between the dependent DOFs and the independent DOFs as geometrical equality equations. Since the procedure of decomposing the stiffness matrix is not required, the sensitivities are evaluated more easily than in the first previous method, and the sensitivity information is the same as the conventional optimization. Therefore, it leads to the possibility that homology design can be combined with other design methodologies such as structural optimization.

Homology design can be classified either into case I where nodal coordinates are functionally related or case II where one or more nodal displacements are specified after deformation, if any geometrical shape is set up as constraints.

3.1 Case I : Functionally related geometrical constraints

Any structure may have a geometrical constraint such as a linear or a quadratic function between the coordinates. Then, the equality equations, $\mathbf{h}$, considering geometrical relations are defined as⁽⁷⁾

$$\mathbf{h} = \{U_d\} - [C] \{U_i\} - \{C_0\}, \quad (13)$$

where the size of the constraint matrix $[C]$ is $N_{dep} \times N_{ind}$ (number of independent DOFs). Since the equal-

ity equations must still be satisfied after small changes in design variables, Eq. (13) can be rewritten as

$$\mathbf{h} + \delta \mathbf{h} = 0. \quad (14)$$

Substituting Eq. (13) into Eq. (14), Eq. (14) becomes

$$\begin{pmatrix} \frac{\partial U_{d1}}{\partial b_1} & \dots & \frac{\partial U_{d1}}{\partial b_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial U_{N_{dep}}}{\partial b_1} & \dots & \frac{\partial U_{N_{dep}}}{\partial b_M} \end{pmatrix} - [C] \begin{pmatrix} \frac{\partial U_{i1}}{\partial b_1} & \dots & \frac{\partial U_{i1}}{\partial b_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial U_{N_{ind}}}{\partial b_1} & \dots & \frac{\partial U_{N_{ind}}}{\partial b_M} \end{pmatrix} \delta \mathbf{b} = [C] \{U_i\} + \{C_0\} - \{U_d\} \quad (15)$$

and, simplifying Eq. (15),

$$[A] \delta \mathbf{b} = \{F_{res}\}, \quad (16)$$

where

$$[A] = \begin{pmatrix} \frac{\partial U_{d1}}{\partial b_1} & \dots & \frac{\partial U_{d1}}{\partial b_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial U_{N_{dep}}}{\partial b_1} & \dots & \frac{\partial U_{N_{dep}}}{\partial b_M} \end{pmatrix} - [C] \begin{pmatrix} \frac{\partial U_{i1}}{\partial b_1} & \dots & \frac{\partial U_{i1}}{\partial b_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial U_{N_{ind}}}{\partial b_1} & \dots & \frac{\partial U_{N_{ind}}}{\partial b_M} \end{pmatrix} \quad (17)$$

$$\{U_d\} = [U_{d1}, U_{d2}, \dots, U_{N_{dep}}]^T \quad (18)$$

$$\{U_i\} = [U_{i1}, U_{i2}, \dots, U_{N_{ind}}]^T \quad (19)$$

$$\{F_{res}\} = [C] \{U_i\} + \{C_0\} - \{U_d\}. \quad (20)$$

Then $\delta \mathbf{b}$, the general solution of Eq. (16), is determined by using Moore-Penrose's generalized inverse matrix.

$$\delta \mathbf{b} = [A]^{-} \{F_{res}\} + ([I_M] - [A]^{-} [A]) \{a\}, \quad (21)$$

where $[A]^{-}$: generalized inverse matrix of $[A]$

$[I_M]$: unit matrix of size, M

$\{a\}$: arbitrary vector.

The necessary and sufficient condition for the existence of a solution in Eq. (16) is represented as

$$([I]_{N_{dep}} - [A][A]^{-}) \{F_{res}\} = \{0\}, \quad (22)$$

where $[I]_{N_{dep}}$ is the unit matrix of $N_{dep} \times N_{dep}$. M should be equal to or greater than N_{dep} since the size of the coefficient matrix in Eq. (17) is $N_{dep} \times M$. Taking only the first term of the right-hand side in Eq. (21) to have the nearest solution,

$$\delta \mathbf{b} = [A]^{-} \{F_{res}\}. \quad (23)$$

If $\delta \mathbf{b}$ approaches zero, it is considered that homology design is realized. Therefore, Eq. (16) is solved successively until the convergence is satisfied. The flow of the suggested algorithms is illustrated in Fig. 2.

3.2 Case II : Specified nodal displacements

A structure may be designed to obtain specified displacements under some designated loading. It is possible to determine the design variables to generate

Fig. 2 Flow of homology design with equality equations

the specified displacements at some nodes if the suggested method is utilized. At this time, the geometrical equality equations are defined as

$$\mathbf{h} = \{U_d\} - \{C_0\}, \quad (24)$$

where

$$\{C_0\} = [C_{01}, C_{02}, \dots, C_{N_{dep}}]^T. \quad (25)$$

Each component of the constant vector $\{C_0\}$ represents a specified displacement at a node. Substituting Eq. (24) into Eq. (14), and transforming into the matrix form,

$$\begin{bmatrix} \frac{\partial U_{d1}}{\partial b_1} & \dots & \frac{\partial U_{d1}}{\partial b_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial U_{N_{dep}}}{\partial b_1} & \dots & \frac{\partial U_{N_{dep}}}{\partial b_M} \end{bmatrix} \delta \mathbf{b} = \begin{Bmatrix} -(U_{d1} - C_{01}) \\ \vdots \\ -(U_{N_{dep}} - C_{N_{dep}}) \end{Bmatrix}. \quad (26)$$

The solution of Eq. (26), $\delta \mathbf{b}$, can be obtained from the method as described in section 3.1. The necessary and sufficient condition for the existence of a solution in Eq. (26) is the same as in case I. Therefore, it is shown that homology design technique can be applied to the design for user-specified responses. However, if the user sets illogical targets, a good design may not be obtained. Equation (26) can be utilized for the evaluation of the given targets.

4. Example Problems

For engineering applications, homology design using equality equations is applied to a seven-bar structure whose geometrical constraint is linear and the back-up structure of a large-scale antenna whose geometrical constraint is quadratic. Also, the results of the suggested method are compared with those of

Table 1 Initial areas and vertical displacements of the seven-bar problem

Element No.	1	2	3	4	5	6	7
Area	100	100	100	100	100	100	100
Node No.	2	3	4				
Vertical displacement	-6.15	-4.76	-2.58				

Fig. 3 Seven-bar structure

the existing method related to the first previous method. The convergence criterion for the examples is taken as

$$\|\delta \mathbf{b}\| \leq 1.0 \times 10^{-7}. \quad (27)$$

The design variables of each example are the areas of all elements in a structure.

4.1 Seven-bar structure

The design objective of the seven-bar structure as shown in Fig. 3 is to determine the design variables of areas in order to make vertical displacements of nodes 2, 3 and 4 have the same value or make them have a specified value. The initial design variables and vertical displacement at each node are given in Table 1. The initial area of each member is 10^{-4} m^2 , and the vertical displacement of nodes 2, 3, and 4 have values of $-6.15 \times 10^{-5} \text{ m}$, $-4.76 \times 10^{-5} \text{ m}$, and $-2.58 \times 10^{-5} \text{ m}$, respectively.

First, homology design is applied in a case where the number of dependent variables is two ($N_{dep}=2$). If it is supposed that there is an identical y coordinate in the structure, the relation between dependent DOFs and independent DOFs is expressed as

$$\begin{Bmatrix} v_3 \\ v_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}, \quad (28)$$

where u is the x -directional displacement, v is the y -directional displacement and each subscript denotes a node number. In Eq. (28), v_3 and v_4 are treated as dependent variables, while the other nodes are treated as independent variables. This means that v_3 and v_4 are determined automatically if v_2 is calculated. The

Table 2 Design variables of the seven-bar problem ($N_{dep}=2$)

Element No.	1	2	3	4	5	6	7
			$(\times 10^{-6} \text{ m}^2)$				
Present study	224.1	138.2	105.2	125.1	181.5	83.8	32.4
Existing study	161.4	116.1	70.0	146.8	91.9	84.0	21.2

Table 3 Vertical displacements of the seven-bar problem ($N_{dep}=2$)

Node No.	2	3	4
	$(\times 10^{-5} \text{ m})$		
Present study	-4.50	-4.50	-4.50
Existing study	-6.24	-6.24	-6.24

Table 4 Design variables of the seven-bar problem (specified displacement)

Element No.	1	2	3	4	5	6	7
			$(\times 10^{-6} \text{ m}^2)$				
Present study	346.6	218.5	146.8	183.4	244.8	81.3	50.2
Existing study	489.6	183.4	115.2	194.5	163.6	59.8	48.0

results at the converged iteration are shown in Tables 2–3. In Table 2, the areas of the present study are different from those of the existing study. This is because the present study utilizes the sensitivities of displacements in the FEM equation. On the other hand, the existing study uses the sensitivities of displacements in the decomposed FEM equation. From Table 3 it is shown that homology design is realized in this structure, since the displacements at the concerned nodes are generated equally.

Second, homology design is applied in a case having the y -directional coordinate specified at some nodes. In this problem, the nodes are nodes 2, 3 and 4, and the specified y -directional displacements are all $-3.0 \times 10^{-5} \text{ m}$. They are determined arbitrarily. The equality equations are represented below.

$$\mathbf{h} = \begin{Bmatrix} v_2 \\ v_3 \\ v_4 \end{Bmatrix} - \begin{Bmatrix} -3.0 \times 10^{-5} \text{ m} \\ -3.0 \times 10^{-5} \text{ m} \\ -3.0 \times 10^{-5} \text{ m} \end{Bmatrix}. \quad (29)$$

The design variables after applying homology design are given in Table 4. The reason the two methods have different solutions is the same as that mentioned in the previous example.

4.2 Back-up structure of an antenna

A reflector is an equipment which gathers or scatters electric waves in a satellite antenna structure. The planar view of the back-up structure⁽⁸⁾ supporting the reflector of a satellite antenna is illustrated in Fig. 4. Nodes 1, 2, 3 and 4 where the reflector is attached are located on the parabola, and $y = 9.0703 \times 10^{-5} x^2$ initially. By characteristics of the antenna, the parabolic shape of the back-up structure must be maintained so that the focal length after deformation

Fig. 4 Back-up structure of antenna

Fig. 5 Configuration of parabola

is adjusted. Therefore, since nodes 1, 2, 3 and 4 in Fig. 4 should remain in a parabolic configuration before and after deformation, the functional relation between the concerned nodes is quadratic. Figure 5 represents two parabolic shapes before and after deformation. In Fig. 4, nodes 1 and 7, where the back-up structure and the hub structure are connected, are fixed as boundary conditions. This is because the stiffness of the hub structure is much larger than that of the back-up structure.

The objective of this problem is to determine the areas so that nodes 1, 2, 3 and 4 make a parabola after deformation. The original equation of the parabola is $y = a_1 x^2$, and the equation of the parabola after deformation is expressed as $y = a_2 x^2 + a_3$, where a_1 is a known value and a_2 and a_3 are constants determined from the design stage. The parabola after deformation goes through a fixed point, node 1 replacing a_3 with $l_y - a_2 l_x^2$, so the constant a_2 is the only unknown value. In this study, the node, k , has to satisfy the equation

$$y_k + v_k - l_y = a_2 [(x_k + u_k)^2 - l_x^2], \quad (30)$$

where x, y are the coordinates, u and v are the x, y directional displacements, l_x and l_y are the coordinates at the fixed point 1, and the subscript k means node k . Also, for an arbitrary node j on the initial parabola, they remain on the parabola passing through node k . That is,

$$y_j + v_j - l_y = a_2[(x_j + u_j)^2 - l_x^2]. \quad (31)$$

Using the condition that the constant a_2 is identical in Eqs. (30)–(31), a_2 is expressed as

$$a_2 = \frac{y_k + v_k - l_y}{[(x_k + u_k)^2 - l_x^2]} = \frac{y_j + v_j - l_y}{[(x_j + u_j)^2 - l_x^2]}. \quad (32)$$

Neglecting second and higher than second order terms in Taylor's expansion, the relation between dependent DOFs and independent DOFs is represented as

$$v_j = \frac{1}{x_k^2 - l_x^2} \left\{ [2x_j(y_k - l_y)2x_k(l_y - y_j)(x_j^2 - l_x^2)] \begin{Bmatrix} u_j \\ u_k \\ v_k \end{Bmatrix} + x_j^2(y_k - l_y) - x_k^2(y_j - l_y) + l_x^2(y_j - y_k) \right\}, \quad (33)$$

where y -directional displacements at node j are treated as dependent DOFs and the other displacements are treated as independent DOFs. The coordinates evaluated from a new parabola equation ($y = a_2x^2 + a_3$), do not correspond to the coordinates from which the original values and displacements are summed. This discrepancy between the coordinates is generated from the fact that second and higher than second order terms are neglected in composing the constraint matrix of Eq. (13). Consequently, an index is required to measure how the homology design is carried out where a geometrical constraint is quadratic such as in the case of the back-up structure. In this study, a homology index, HI , is defined as

$$HI = \text{Max}_{1 \leq j \leq n} |\epsilon_j|, \quad (34)$$

where

$$\epsilon_j = \frac{\text{exact value} - \text{value by homology design}}{\text{exact value}} \times 100, \quad (35)$$

and where n is the number of nodes subjected to homologous deformation. The exact value is the y coordinate from the evaluated equation of a parabola of the above mathematical process. The value by homology design is the y coordinate after real deformation which is developed from the structure designed by the above process. Therefore, it is desirable for this index to approach zero. The design variables in the converged iteration are given in Table 5, and the coordinates of the parabola are given in Table 6. The equation of the parabola using the equality equations is $y = 7.614 \times 10^{-5}x^2 + 16.057$ and HI is 0.288, while the equation of the parabola using the existing method is $y = 7.578 \times 10^{-5}x^2 + 16.448$ and HI is 0.311.

The numerical performances of the two methods

Table 5 Design variables of the back-up structure

Element No.	1	2	3	4	5	6	7	8	9	10
	($\times 10^{-6} \text{m}^2$)									
Initial value	774	564	774	564	774	564	774	774	564	774
Present study	786	449	707	551	981	321	798	982	512	1072
Existing study	794	422	709	547	881	279	805	894	476	946

Table 6 Coordinates of the back-up structure (x, y)

Node No.	1	2	3	4
	($\times 10^{-3} \text{m}, \times 10^{-3} \text{m}$)			
Initial value	(1050,100)	(2338.5,496)	(3373.1,1032)	(4167.1,1575)
Present study	(1050,100)	(2368.5,442)	(3440.6,916)	(4274.1,1407)
Existing study	(1050,100)	(2368.9,440)	(3442.3,913)	(4277.1,1402)

Table 7 Numerical performances of the two methods

Problem type	Design type	Iteration	Norm
7-bar ($N_{dep}=2$)	present study	12	170
	existing study	7	117
7-bar (specified)	present study	13	328
	existing study	39	420
back-up structure	present study	6	505
	existing study	6	413

are presented in Table 7. The two methods use different sensitivities and simultaneous equations. Therefore, comparisons in terms of superiority are not appropriate at this time.

5. Conclusions

The following is concluded from this study.

(1) Homology design using equality equations is developed in an easy way compared to the existing methods. The existing methods have been improved in that in the proposed method simultaneous equations need not be derived for each problem. Also, it is shown that the method developed can be applied to user-specified targets.

(2) The proposed method can be combined with other design methodologies, since the sensitivities of displacements are obtained from the FEM equation.

(3) For the design of a back-up structure in an antenna, a practical solution is evaluated and a homology index is created to measure how it performs.

(4) The existing methods do not have the minimization capability, although a nearest solution from the initial design value can be obtained by using Moore-Penrose's generalized inverse matrix. For future study, it is important to consider the minimization of volume since it is related to the cost of design and production. Then, the sensitivity information of the present study can be utilized in the optimization

process. The homologous deformation can be included as constraints in the optimization process.

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