A Filter Design Method for Robot Tip Velocity Derivation*

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High performance robot control sometimes requires direct velocity measurement at the end effector as well as at the joint. However, to date, sensing devices are still not readily available for tip velocity measurement on most industrial manipulators and one often has to rely on approximations to derive this velocity. The limitations of these approximations are analyzed in this paper. In order to overcome these limitations and derive the robot tip velocity, end point acceleration measurement is employed and a complementary filtering technique is adopted. The filter design approach is described with some implementation results given.

Key Words : Robot, Sensing, Tip Velocity, Filter, Acceleration Measurement

1. Introduction

Robot end effector velocity, as an external sensor signal, plays an important role in robotic sensing and control. It has been used to obtain relative robot end effector displacements to be input into a Kalman Filter in proximity sensor signal processing⁽¹⁾. It is also useful as a direct measurable variable for control purposes to overcome the undesirable effects of phase lag introduced by a filter which has to be used with a derivative approach to suppress the noise level⁽²⁾.

However, for industrial robots, sensing devices for measuring end effector velocity are still not readily available. Instead, only joint position signals are directly sensed via motor encoders and the joint velocities often have to be calculated based on numerical difference. Theoretically, forward kinematics can be used to formulate the robot end point velocity in Cartesian space^{(3),(4)}. This formulation is based on the assumption of rigid body robot dynamics and perfect

motor-to-link transmissions which do not hold when robot compliances are present. These unmodelled dynamics have non-negligible effects under certain circumstances even for industrial robots like the PUMA with presumably rigid links^{(5),(6)}. This paper shows that using forward kinematics to derive the tip velocity is at most an approximation when a robot moves with low or moderate accelerations. At certain frequencies, the robot tip velocity turned out to be almost anti-phase with the joint velocity. Another problem with the kinematics approach is that joint velocity signals derived from encoder tend to be poor at low velocities. Industrial robots often use an encoders as the only means of joint sensing. The encoder capacity gives rise to quantization problems, restricting the effective range of the velocity signals⁽⁷⁾.

To solve these problems, an end point sensing and complementary filtering technique is adopted. The filter design method is described and experimental results are given regarding the implementation of the complementary filtering approach on an industrial PUMA robot.

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2. Problems with a Pure Kinematics Approach

Given a robot's state in its joint space, the end point velocity in Cartesian space can be obtained through a kinematics approach⁽³⁾ and a closed form solution exists for a PUMA 560 robot⁽⁴⁾. The pure kinematics approach is based on the assumption of rigid arm dynamics and perfect actuation and transmission systems which do not hold under certain circumstances, e.g. when the effect of compliance becomes important^{(8),(9)}.

The following simulation shows this effect. Here the robot is modeled as a mass-spring system in the one-dimensional case as illustrated in Fig. 1.

The robot is divided into two masses m_1 and m_2 by a spring and a damper with values k_2 and b_2 representing the joint compliance effect. F is the input force and v_1 and v_2 represent the joint and end point velocities respectively. The relationship between the joint and tip velocity can be represented in complex frequency domain by

$$\frac{V_2(s)}{V_1(s)} = \frac{b_2 s + k_2}{m_2 s^2 + b_2 s + k_2} \quad (1)$$

The discrepancy between the joint and tip velocity can be seen clearly in the plots of the frequency domain properties of the above transfer function, as shown in Fig. 2.

At low frequencies, the end point velocity appears nearly the same as the joint velocity. However, as frequency increases, the discrepancy becomes more and more obvious with the phase lag reaching almost 180° immediately above resonance, indicating that the end point velocity is out of phase with that derived

Fig. 1 A model of robot arm compliance

Fig. 2 Bode plot of a compliant arm transfer function

from joint sensing. Therefore, using joint sensing and a kinematics approach to derive the end effector velocity is at most an approximate solution when a robot moves with low acceleration. When the acceleration is high, meaning a high rate of change in velocity, the high frequency components in the velocity assume more importance and the effect of compliance is emphasized. The above simulation is based on robot motion in free space. Similar effects have also been observed in a situation when a robot is in contact with an environment⁽¹⁰⁾.

3. The Complementary Filtering Scheme

End point sensing is apparently a means for solving the above problems. However, direct velocity measurement is not readily achievable for such a robotic application due to technological limitations. Acceleration measurement, on the other hand, provides an inexpensive and feasible alternative solution to the problem. The accelerometer, however, suffers from its own drawbacks. Its output signal is corrupted by DC offsets and low frequency disturbances⁽¹¹⁾. This has prevented its direct integration in obtaining the velocity⁽¹²⁾.

To solve this problem, a complementary filtering scheme is adopted⁽¹³⁾. This technique involves high-pass filtering of the end point acceleration signals and combining this information with that from joint sensing to reconstruct the tip velocity as shown in Fig. 3.

Using this approach, both the end point and the joint sensing information are taken into account and their limitations can be overcome by appropriately designing the low-pass and high-pass filters. The filtering of the joint velocity is performed on the velocity signal instead of the original encoder reading. This is due to the following facts:

- The position signal from an encoder is normally well behaved and there is not much need to filter it. It is mainly following differentiation of the position signal that noise is introduced.
- Here the joint motor velocity is derived not by

Fig. 3 A complementary filter scheme

software but by hardware via an encoder board.

The filtering of the end point acceleration is mainly for convenience of illustration and design and it can be done after the integration. Indeed, an integrator can be regarded as a kind of filter and its action can be taken into account in the design of the acceleration filter. The overall performance of the complementary filtering is governed by summation of the low-pass and high-pass filtering actions as will be seen in the following section.

4. Filter Design

To realize the complementary filtering proposed above, the low-pass and high-pass filters have to be designed appropriately. The following methods are discussed for such a design.

4.1 Independent design method

The independent filter design method involves separately designing the low-pass and high-pass filters in Fig. 3 using standard filter design techniques. Although each individual filter can be designed as desired, it is difficult to achieve satisfactory overall performance as demonstrated by the example where the results are given in Figs. 4 and 5.

The low-pass and high-pass filter used here are of

Fig. 4 Magnitude responses of standard filters

Fig. 5 Magnitude responses of the overall filter

the elliptic type of the 6th and 5th order respectively. Ripples exit in the overall magnitude response due to the phase shift in the individual filter. The overall filter magnitude is derived by summation of the individual filtering vectors which are shown in the Nyquist plots in Figs. 6 and 7.

4.2 The low-pass based (LPB) design method and its limitations

A low-pass based (LPB) technique was proposed to overcome the above difficulties in an independent design approach⁽¹⁴⁾. The LPB technique designs the low-pass filter part $F_l(s)$ only and the high-pass filter part is formulated by

$$F_h(s) = 1 - F_l(s). \quad (2)$$

Using this design method, suppression of the high frequency disturbance in the joint sensing can be achieved as specified. However, the low frequency disturbance in the acceleration measurement cannot be suppressed as desired. In the independent filter design approach, both the low-pass and the high-pass filter can have similar roll-off slopes in their stop

Fig. 6 Filter vector f_l of low-pass filter

Fig. 7 Filter vector f_h of high-pass filter

bands if the same order and same type filters are used. This feature is lost in the LPB approach. At very low frequencies (near zero frequency), the phase shift of the low-pass filter, designed by the LPB method, tends to be zero. Thus the filter vector of the high-pass filter is

$$\mathbf{f}_h = 1 - \mathbf{f}_l = 1 \angle 0^\circ - 1 \angle -\varepsilon^\circ = \rho \angle 90^\circ, \quad (3)$$

where $\mathbf{f}_h$ and $\mathbf{f}_l$ are the high-pass and low pass filter vectors respectively, ε represents the infinitely small phase shift of the low-pass filter, and ρ is the infinitesimal magnitude of the high pass filter. For the low-pass filter used here, Eq. (3) implies that at very low frequencies, the high-pass filter has a magnitude response with a slope of 20 dB/dec, which is less than that achievable with an independently designed high pass filter of higher than second order.

The low frequency disturbance in the acceleration sensing, having very adverse corrupting effect on the output signal, deserves special attention in the filter design. This leads to the search for an alternative filter design approach.

4.3 The high-pass based (HPB) design method

To overcome the limitations of the LPB design, a high-pass based (HPB) filter design method is put forward. Using this method, the high-pass filter part is designed first using a standard technique. The low-pass filter is then formulated by

$$F_l(s) = 1 - F_h(s). \quad (4)$$

The velocity from joint sensing is express as $v_J = v_E + d_h$ and the measured end point acceleration as $a_E = \dot{v}_E + d_l$, where v_E represents the true value of the end point velocity and d_h and d_l denote the introduced high frequency disturbance existing in the joint sensing and the low frequency disturbance in the acceleration measurement respectively. Referring to Fig. 3, we have

$$\begin{aligned} V_J(s)F_l(s) + A_E(s)F_h(s) &= \frac{1}{s} \\ &= [V_E(s) + D_h(s)]F_l(s) + [sV_E(s) \\ &\quad + D_l(s)]F_h(s)\frac{1}{s}. \end{aligned}$$

Under the assumption of perfect filtering, the above equation becomes

$$V_E(s)F_l(s) + \left[sV_E(s)F_h(s)\frac{1}{s} \right].$$

Replacing $F_l(s)$ with Eq. (4), we then arrive at the exact value of the end point velocity v_E . The HPB approach gives a steeper roll-off for the high-pass filter than its counterpart in the LPB approach. This effect is shown in Fig. 8, where the same second order Butterworth filter was used in designing the first filter, the high pass filter.

This faster roll-off in the high-pass filter is important in suppressing the very low frequency distur-

Fig. 8 Comparison of magnitude responses of a LPB and HPB filter

Fig. 9 Filter vector $\mathbf{f}_h$ of high-pass filter

Fig. 10 Filter vector $\mathbf{f}_l$ of low-pass filter

bances in the acceleration signals. The accelerometer readings are corrupted by low frequency noise, mainly DC offset. Figure 8 indicates a non-trivial improvement in suppressing the DC disturbance by the HPB approach. As the frequency approaches zero, the high-pass filter designed using the HPB approach offers

significantly stronger rejection capability than that designed using the LPB approach.

The filtering vectors in the HPB design are shown in the Nyquist plots in Figs. 9 and 10.

It can be seen that the individual filter vector f_i and f_h may deviate from the ideal case of $1\angle 0^\circ$ in its pass band. But the overall filter vector, governed by $f = f_i + f_h$ remains at the desired value of $1\angle 0^\circ$. The slow roll-off of the resulting low-pass filter does not have a noticeable deteriorating effect on the output of the overall filter. In fact, most electronic components in a robot system are low-pass in nature, and therefore at very high frequencies, the noise is attenuated automatically.

4.4 Filter type selection

In a standard filter design, an elliptic filter offers the steepest roll-off characteristics of all the model-based designs using model functions of the same order. However, this is achieved only at the expense of band ripple⁽¹⁵⁾. This ripple is highly undesirable in HPB complementary filter design as it hampers the suppression of the extremely low frequency disturbance in the acceleration signals which is considered here to be the source of the most harmful noise.

On the other hand, a Butterworth filter, though slower in its roll-off, is attractive feature due to its monotonicity which is beneficial for increasing the filter's low frequency disturbance rejection capacity. Using the HPB approach, enhanced performance of the high-pass filter is achieved without affecting the overall filter characteristics. The overall filter vector is not governed by the filter in the low-pass or high-pass path and remains at its value of $1\angle 0^\circ$ regardless of any deviations in the individual filters' behavior.

A method to overcome the slow roll-off of a Butterworth filter without introducing ripple involves using a Chebyshev filter⁽¹⁵⁾. The Chebyshev filter, being monotonic in its stop band and having a steeper roll-off than the Butterworth filter, is considered to be most suited for the HPB design.

Fig. 11 Placement of the accelerometer on the end effector

5. Implementation Results

5.1 Experimental setup

Using joint 1 of a PUMA 560 robot, experiments were conducted to test the HPB complementary filter design method on a real system. The following tests were limited to one-dimensional study only. To measure the linear acceleration of an end point, an accelerometer was attached to the end effector of the robot using wax. The accelerometer and the end effector are shown in Fig. 11.

The accelerometer was a silicon based, piezoresistive, micro-machined type⁽¹¹⁾. This sensor is of light weight (13 grams including the connecting cable) and small in size (0.9×0.9 in) and has a measuring range of ± 10 g (about 98 m/s^2). A built-in amplifier within the sensor head gives an analogue DC voltage output between 0.5 V and 4.5 V which can be directly sent to the A/D converter connected to a transputer network. The precision sensor has an accuracy of 0.2% of its full span. The calibrated resonant frequency is 1260 Hz, with a working frequency range of 0-700 Hz over which the sensitivity is within the rated tolerance. The joint velocity was derived by an encoder board also connected to the transputer network. All the filter algorithms were implemented on the transputer network with Occam as the programming language.

5.2 Implementation results

In the first test, the robot was given a step position trajectory for the joint. The end point velocity and acceleration responses are shown in Figs. 12 and 13, respectively.

In Fig. 12, the dotted line is the calculated tip velocity based on the measured motor velocity and the arm kinematics. The solid line is the tip velocity derived from the complementary filter using the HPB design method. The difference between the filter-derived tip velocity and that derived from the motor

Fig. 12 End point velocities in step position command case

Fig. 13 End point accelerations in step position command case

Fig. 14 End point velocities in high order position command case

Fig. 15 End point acceleration in high order position command case

was clearly observable. As demonstrated in Section 4.3, the filter-derived tip velocity represents the true tip velocity under the assumed conditions. Therefore, the tip velocity cannot be reconstructed correctly if solely based on the motor velocity. In Fig. 13, it can be seen that the high pass filter suppressed the DC disturbance in the measured acceleration. The high-pass filter employed was a second order Chebyshev filter.

Next, a smooth joint trajectory was sent to the robot. There was no discontinuity, whether in the joint position, joint velocity, or joint acceleration command. Figures 14 and 15 show the velocity and acceleration responses respectively.

In this case, the velocity calculated directly from joint sensing and that derived from the complementary filter turned out to be almost identical. This corresponded to the case of low acceleration, with its magnitude limited below $\pm 6 \text{ m/s}^2$ over the whole trajectory. The acceleration spikes at some individual points (shown in Fig. 15), should be attributable to the high frequency noise which was left unattenuated in our filter design but was reduced by the subsequent integrating action. Low acceleration indicates a low

rate of change in velocity and therefore the velocity signal is relatively low-frequency dominant. As discussed in section 2, the kinematically derived tip velocity from joint sensing matches the true tip velocity well in such a case.

6. Conclusion

Theoretical analyses and experimental work have revealed problems in deriving robot end point velocity using a pure kinematics approach. A complementary filtering method that combines both joint sensing and end point measurement is adopted. The HPB filter design is proposed and implemented. The effectiveness of the proposed filtering method was verified by experiments performed on an industrial robot.

The experimental results supported the theoretical analyses. At low accelerations, the motor velocity can acceptably be used for calculating the end point velocity using the kinematics relationship. However, the discrepancy between the motor-derived velocity and the tip velocity arises when the robot acceleration is high in magnitude which is often the case in a robot's transition from free space to contact or vice versa.

For an absolutely rigid robot, the tip velocity derived from the complementary filtering method is equivalent to that derived directly from the motor side sensing, but with improved behavior at low velocity where quantization problems are encountered.

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